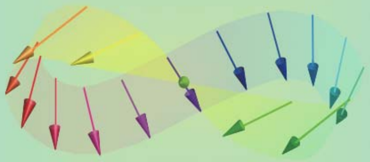
Introductory Nanotechnology
~ Basic Condensed Matter Physics ~



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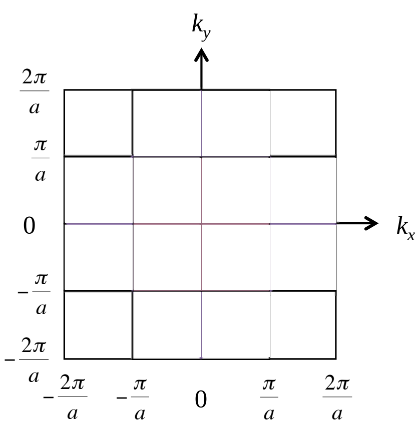
Quick Review over the Last Lecture

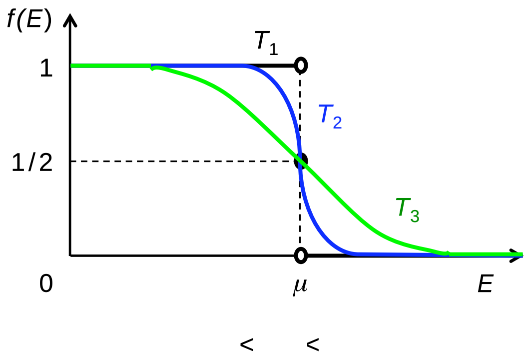
Wave / particle duality of an electron :

	Particle nature	Wave nature
Kinetic energy		
Momentum		

Brillouin zone (1st & 2nd) :



Fermi-Dirac distribution (*T*-dependence) :



Contents of Introductory Nanotechnology

First half of the course :

Basic condensed matter physics

1. Why *solids* are *solid* ?
2. What is the *most common atom* on the earth ?
3. How does an electron travel in a material ?
4. How does lattices vibrate thermally ?
5. What is a *semi*-conductor ?
6. How does an electron tunnel through a barrier ?
7. Why does a magnet attract / retract ?
8. What happens at interfaces ?

Second half of the course :

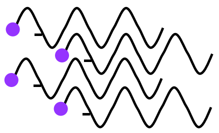
Introduction to nanotechnology (nano-fabrication / application)

How Does an Electron Travel in a Material ?

- Group / phase velocity
- Effective mass
- Hall effect
- Harmonic oscillator
- Longitudinal / transverse waves
- Acoustic / optical modes
- Photon / phonon

How Fast a Free Electron Can Travel ?

Electron *wave* under a uniform ***E*** :



Wave packet

Group velocity :

$$\boldsymbol{v}_{\mathrm{g}} = \frac{d\omega}{d\boldsymbol{k}}$$

Here, energy of an electron wave is

$$E = h\nu = \hbar\omega$$

Accordingly,

$$\boldsymbol{v}_{\mathrm{g}} = \frac{1}{\hbar}\frac{dE}{d\boldsymbol{k}}$$

Therefore, electron wave velocity depends on gradient of energy curve $E(\boldsymbol{k})$.

Phase-travel speed in an electron wave :

Phase velocity :

$$\boldsymbol{v}_{\mathrm{p}} = \frac{\omega}{\boldsymbol{k}}$$

Equation of Motion for an Electron with *k*

For an electron *wave* travelling along ***E*** :

$$\frac{d\boldsymbol{v}_{\mathrm{g}}}{dt} = \frac{1}{\hbar}\frac{d}{dt}\left(\frac{dE}{d\boldsymbol{k}}\right) = \frac{1}{\hbar}\frac{d}{d\boldsymbol{k}}\left(\frac{dE}{d\boldsymbol{k}}\right)\frac{d\boldsymbol{k}}{dt} = \frac{1}{\hbar}\frac{d^2E}{d\boldsymbol{k}^2}\frac{d\boldsymbol{k}}{dt}$$

Under ***E***, an electron is accelerated by a force of $-q\boldsymbol{E}$.

In Δt, an electron travels $\boldsymbol{v}_{\mathrm{g}}\Delta t$, and hence ***E*** applies work of $(-q\boldsymbol{E})(\boldsymbol{v}_{\mathrm{g}}\Delta t)$.

Therefore, energy increase ΔE is written by

$$\Delta E = -q\boldsymbol{E}\boldsymbol{v}_{\mathrm{g}}\Delta t = -q\boldsymbol{E}\frac{1}{\hbar}\frac{dE}{d\boldsymbol{k}}\Delta t$$

At the same time $\Delta\boldsymbol{E}$ is defined to be

$$\Delta E = \frac{dE}{d\boldsymbol{k}}\Delta\boldsymbol{k}$$

From these equations,

$$\Delta\boldsymbol{k} = -\frac{1}{\hbar}q\boldsymbol{E}\Delta t \qquad \therefore \frac{d\boldsymbol{k}}{dt} = -\frac{1}{\hbar}q\boldsymbol{E} \qquad \therefore \hbar\frac{d\boldsymbol{k}}{dt} = -q\boldsymbol{E}$$

→ Equation of motion for an electron with ***k***

Effective Mass

By substituting $\hbar \frac{d\boldsymbol{k}}{dt} = -q\boldsymbol{E}$ into $\frac{d\boldsymbol{v}_g}{dt} = \frac{1}{\hbar} \frac{d^2 E}{d\boldsymbol{k}^2} \frac{d\boldsymbol{k}}{dt}$

$$\frac{d\boldsymbol{v}_g}{dt} = -\frac{1}{\hbar^2} \frac{d^2 E}{d\boldsymbol{k}^2} q\boldsymbol{E}$$

By comparing with acceleration for a free electron :

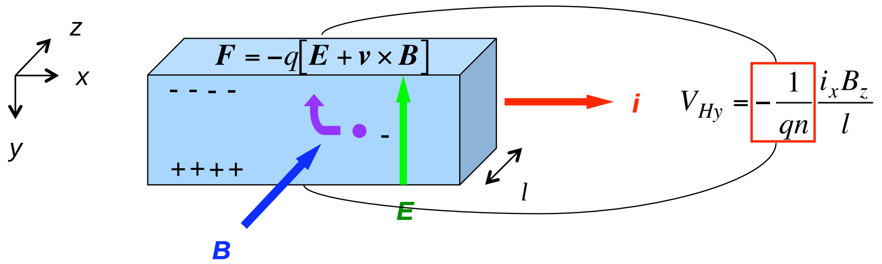
$$\frac{d\boldsymbol{v}}{dt} = -\frac{1}{m} q\boldsymbol{E}$$

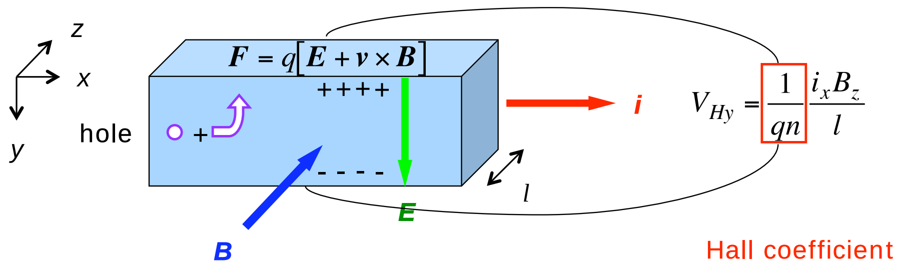
$$m^* = \hbar^2 \Big/ \left(\frac{d^2 E}{d\boldsymbol{k}^2} \right)$$

$\rightarrow$ Effective mass

Hall Effect

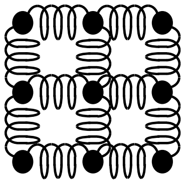
Under an applications of both a electrical current ***i*** an magnetic field ***B*** :

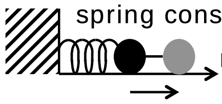




Harmonic Oscillator

Lattice vibration in a crystal :





Hooke's law :

$$M\frac{d^2u}{dt^2} = -kx$$

Here, we define

$$\omega = \sqrt{\frac{k}{M}} \qquad \therefore \frac{d^2u}{dt^2} = -\omega^2 u$$

$$\therefore u(t) = A\sin(\omega t + \alpha)$$

→ 1D harmonic oscillation

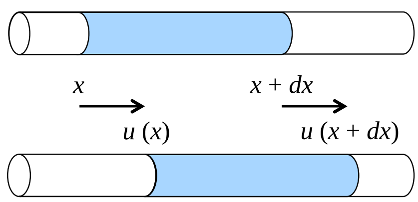
Strain

Displacement per unit length :

$$\delta = \frac{(\partial u/\partial x)dx}{dx} = \frac{\partial u}{\partial x}$$

Young's law (stress = Young's modulus × strain) :

$$\frac{\boldsymbol{F}}{S} = \boldsymbol{E}_{\mathrm{Y}}\frac{\partial u}{\partial x} \qquad S \text{ : area}$$



Here,

$$+\boldsymbol{F}(x+dx) = \boldsymbol{F}(x) + (\partial \boldsymbol{F}/\partial x)dx + \cdots$$

For density of ρ,

$$\rho S dx \frac{\partial^2 u}{\partial t^2} = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}}dx$$

$$\therefore \frac{\partial^2 u}{\partial t^2} = \frac{\boldsymbol{E}_{\mathrm{Y}}}{\rho}\frac{\partial^2 u}{\partial x^2} \equiv v_\ell \frac{\partial^2 u}{\partial x^2}$$

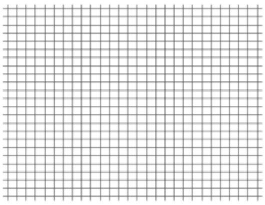
→ Wave eqution in an elastomer

Therefore, velocity of a strain wave (acoustic velocity) :

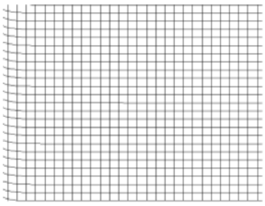
$$v_\ell \equiv \sqrt{\frac{\boldsymbol{E}_{\mathrm{Y}}}{\rho}}$$

Longitudinal / Transverse Waves

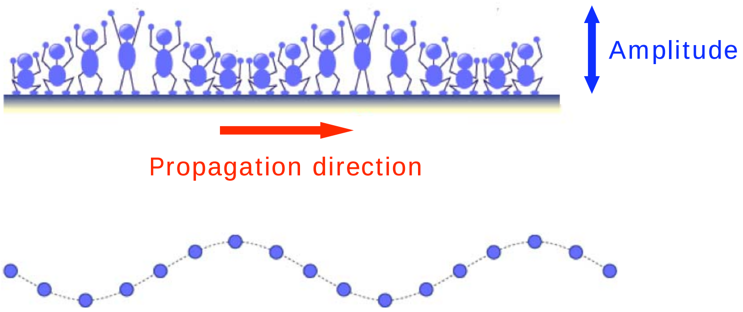
Longitudinal wave : vibrations along or parallel to their direction of travel



Transverse wave : vibrations perpendicular to their direction of travel

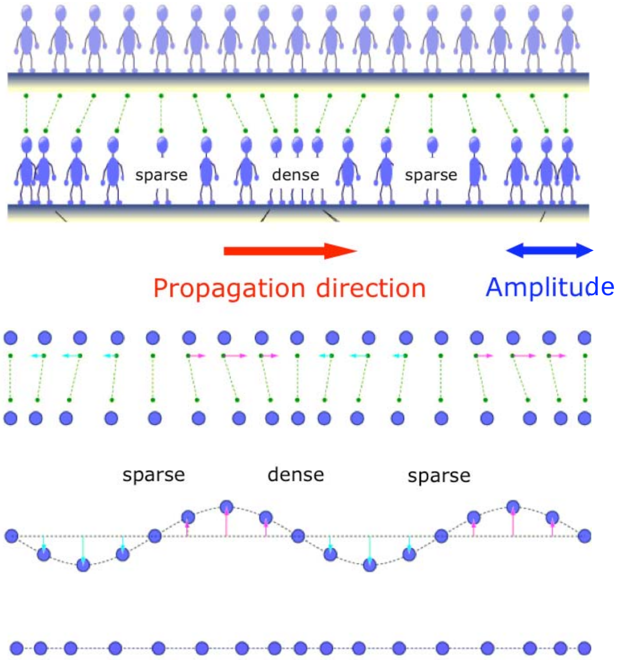


Transverse Wave



* http://www12.plala.or.jp/ksp/wave/waves/

Longitudinal Wave

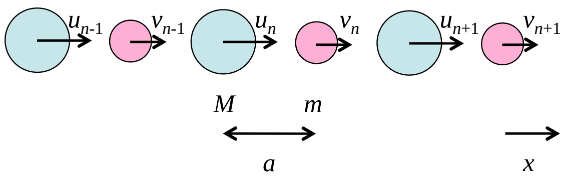


* http://www12.plala.or.jp/ksp/wave/waves/

Acoustic / Optical Modes

For a crystal consisting of 2 elements (k : spring constant between atoms) :

$$\begin{cases} M\dfrac{d^2u_n}{dt^2} = k\left\{\left(v_n - u_n\right) + \left(v_{n-1} - u_n\right)\right\} \\ m\dfrac{d^2v_n}{dt^2} = k\left\{\left(u_{n+1} - v_n\right) + \left(u_n - v_n\right)\right\} \end{cases}$$



By assuming,

$$\begin{cases} u_n(na,t) = A\exp\left\{i(\omega t - qna)\right\} \\ v_n(na,t) = B\exp\left\{i(\omega t - qna)\right\} \end{cases}$$

$$\begin{cases} -M\omega^2 A = kB\left\{1 + \exp(iqa)\right\} - 2kA \\ -m\omega^2 B = kA\left\{\exp(-iqa) + 1\right\} - 2kB \end{cases}$$

$$\therefore \begin{cases} \left(2k - M\omega^2\right)A - k\left\{1 + \exp(iqa)\right\}B = 0 \\ -k\left\{\exp(-iqa) + 1\right\}A + \left(2k - m\omega^2\right)B = 0 \end{cases}$$

$$\therefore \begin{vmatrix} 2k - M\omega^2 & -k\left\{1 + \exp(iqa)\right\} \\ -k\left\{\exp(-iqa) + 1\right\} & 2k - m\omega^2 \end{vmatrix} = 0$$

Acoustic / Optical Modes - Cont'd

Therefore, $\omega^2 = k\left(\frac{1}{M}+\frac{1}{m}\right) \pm k\sqrt{\left(\frac{1}{M}+\frac{1}{m}\right)^2 - \frac{4}{Mm}\sin^2\frac{qa}{2}}$

For $qa = 0$,

$$\begin{cases} \omega_+ = \sqrt{2k\left(\frac{1}{M}+\frac{1}{m}\right)} \\ \omega_- = 0 \end{cases}$$

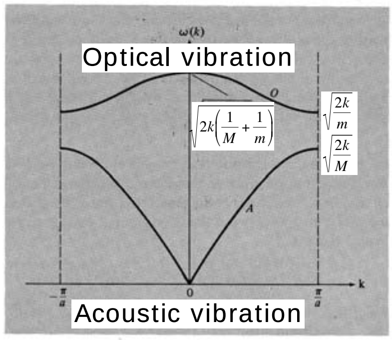
For $qa \sim 0$,

$$\begin{cases} \omega_+ \approx \sqrt{2k\left(\frac{1}{M}+\frac{1}{m}\right)} \\ \omega_- \approx \sqrt{\frac{k/2}{M+m}}\,qa \end{cases}$$

For $qa = \pi$,

$$\begin{cases} \omega_+ = \sqrt{\frac{2k}{m}} \\ \omega_- = \sqrt{\frac{2k}{M}} \end{cases}$$

Figure 22.10
Dispersion relation for the diatomic linear chain. The lower branch is the acoustic branch and has the same structure as the single branch present in the monatomic case (Figure 22.8). In addition, there is now an optical branch (upper branch.)



* N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Thomson Learning, London, 1976).

Why Acoustic / Optical Modes ?

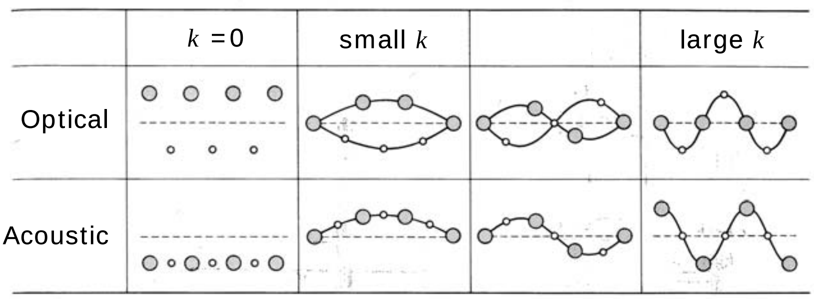
Oscillation amplitude ratio between M and m (A / B):

Optical mode : $\frac{m}{M}$

Neighbouring atoms changes their position in opposite directions,
of which amplitude is larger for m and smaller for M,
however, the cetre of gravity stays in the same position.

Acoustic mode : 1

All the atoms move in parallel.



* M. Sakata, *Solid State Physics* (Baifukan, Tokyo, 1989).

Photon / Phonon

Quantum hypothesis by M. Planck (black-body radiation) :

$$E = \frac{1}{2}h\nu + nh\nu \qquad (n = 0,1,2,\ldots)$$

Here, $h\nu$: energy quantum (photon)

mass : 0, spin : 1

Similarly, for an elastic wave, quasi-particle (phonon) has been introduced by P. J. W. Debye.

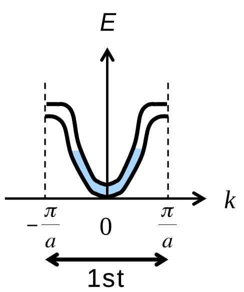
$$E = \frac{1}{2}\hbar\omega + n\hbar\omega \qquad (n = 0,1,2,\ldots)$$

Oscillation amplitude : larger → number of phonons : larger

What is a conductor ?

Number of electron states (including spins) in the 1st Brillouin zone :

$$\int_{-\pi/a}^{\pi/a} 2\frac{L}{2\pi}dk = \frac{2L}{a} = 2N \qquad \left(N = \frac{L}{a}\right)$$



Here, N : Number of atoms for a monovalent metal

As there are N electrons, they fill half of the states.

By applying an electrical field ***E***, the occupied states become asymmetric.

- ***E*** increases asymmetry.
- Elastic scattering with phonon / non-elastic scattering decreases asymmetry.

→ Stable asymmetry

→ Constant current flow

→ Conductor :

Only bottom of the band is filled by electrons with unoccupied upper band.