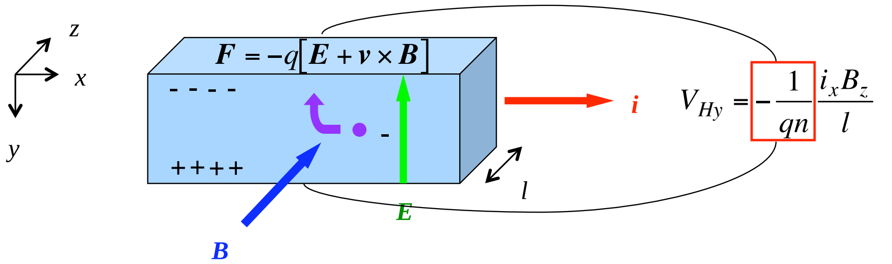
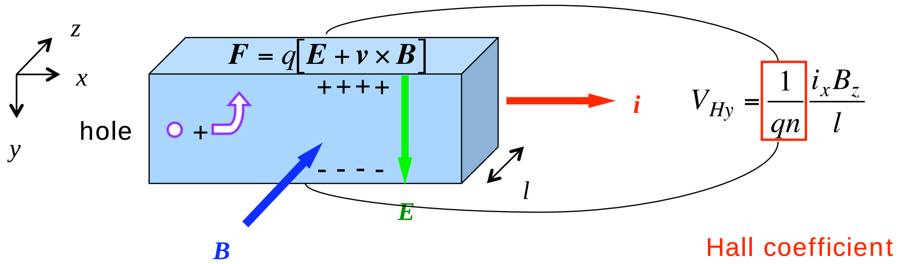
Question - Hall Effect

Under an applications of both a electrical current $\boldsymbol{i}$ an magnetic field $\boldsymbol{B}$, derive the Hall voltage for both electrons and holes from the Lorentz force $\boldsymbol{F}$.

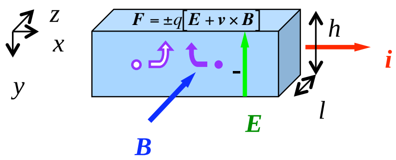




Answer - Hall Effect

Lorentz force for both holes and electrons :

$$\boldsymbol{F} = \pm q[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}] \quad (+ : \text{holes and} - : \text{electrons})$$



y-component is obtained to be

$$F_y = \pm q\left\{E_y + [\boldsymbol{v} \times \boldsymbol{B}]_y\right\} = \pm q\left\{E_y + \left(v_z \underline{B_x} - v_x B_z\right)\right\}$$

$= 0$

For a equilibrium state, $F_y = 0$

$$\pm q\left(E_y - v_x B_z\right) = 0 \qquad \therefore E_y = v_x B_z$$

Current density can be independently defined for both holes and electrons,

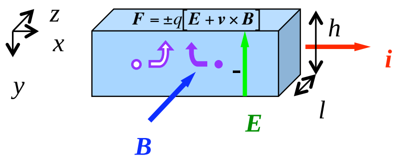
$$\begin{cases} J_x = +qpv_x & (\text{holes; } v_x > 0,\ p : \text{hole number density}) \\ J_x = -qnv_x & (\text{electrons; } v_x < 0,\ n : \text{electron number density}) \end{cases}$$

$$\therefore \begin{cases} E_y = +\dfrac{J_x}{qp} B_z & (\text{holes}) \\ E_y = -\dfrac{J_x}{qn} B_z & (\text{electrons}) \end{cases}$$

Answer - Hall Effect (Cont'd)

Here, $V_y = hE_y$ and $i_x = hlJ_x$,

$$\begin{cases} V_y = +\frac{h}{qp}B_z \cdot \frac{i_x}{hl} = \boxed{+\frac{1}{qp}}\frac{i_x B_z}{l} & \text{(holes)} \\ V_y = -\frac{h}{qn}B_z \cdot \frac{i_x}{hl} = \boxed{-\frac{1}{qn}}\frac{i_x B_z}{l} & \text{(electrons)} \end{cases}$$



Hall coefficient

Hall coefficient R_H ,

$$E_y = R_H J_x B_z \qquad \therefore R_H = \frac{E_y}{J_x B_z}$$

Therefore ,

$$V_y = R_H \frac{i_x B_z}{l} \qquad \therefore R_H = \frac{V_y l}{i_x B_z}$$

← Experimentally measured