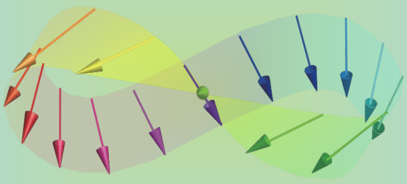
Nanoelectronics
02



Atsufumi Hirohata

Department of Electronic Engineering

THE UNIVERSITY *of York*

12:00 Thursday, 19/January/2023 (ENV 005)

Quick Review over the Last Lecture

Nano-scale miniaturisation :

- ✓ reduction of (   )
- ✓ reduction of (   )
- → (   ) operation
- ✗ nano-fabrication ;
  - (   ) processes
  - (   ) cost
  - (   ) distributions in device properties
- ✗ (   ) current
- ✗ (   ) heating
- ✗ electron (   )

Electron transport :

- (   ) transport
  - → (   )
- (   ) transport
  - → (   )

Contents of Nanoelectronics

Lecture notes and files can be found at
http://www-users.york.ac.uk/~ah566/

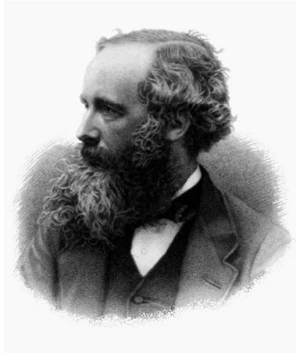
02 Maxwell Equations

- Electromagnetic field
- Origins of an electromagnetic field
- Boundary conditions of an electromagnetic field

Maxwell Equations

Maxwell equations are proposed in 1864 :

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \mathrm{div}\,\boldsymbol{D} = \rho \\ \mathrm{div}\,\boldsymbol{B} = 0 \end{cases} \qquad \begin{cases} \nabla \times \boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} \\ \nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{D} = \rho \\ \nabla \cdot \boldsymbol{B} = 0 \end{cases}$$



$\boldsymbol{E}$: electric field, $\boldsymbol{B}$: magnetic flux density,
$\boldsymbol{H}$: magnetic field, $\boldsymbol{D}$: electric flux density,
$\boldsymbol{J}$: current density and ρ : charge density

Supplemental equations for materials :

$$\begin{cases} \boldsymbol{D} = \varepsilon \boldsymbol{E} & \rightarrow \\ \boldsymbol{B} = \mu \boldsymbol{H} & \rightarrow \\ \boldsymbol{J} = \sigma \boldsymbol{E} & \rightarrow \end{cases}$$

ε : permittivity, μ : magnetic permeability,
and σ : conductivity

* http://www.wikipedia.org/

Maxwell Equations - Origins of an electromagnetic field

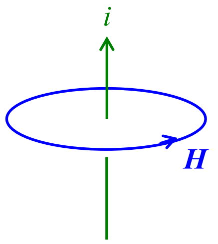
Maxwell equations :

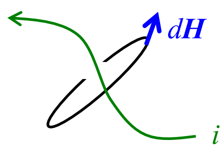
$$\begin{cases} \boxed{\mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t}} \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \boxed{\mathrm{div}\,\boldsymbol{D} = \rho} \\ \mathrm{div}\,\boldsymbol{B} = 0 \end{cases} \qquad \begin{cases} \boldsymbol{D} = \varepsilon \boldsymbol{E} \\ \boldsymbol{B} = \mu \boldsymbol{H} \\ \boldsymbol{J} = \sigma \boldsymbol{E} \end{cases}$$

For a time-independent case,

$\mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J}$

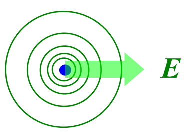
$\rightarrow$ law $\rightarrow$ law





law :

An electrical charge induces an electric field.

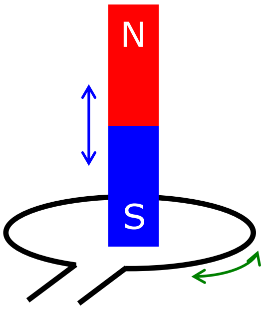


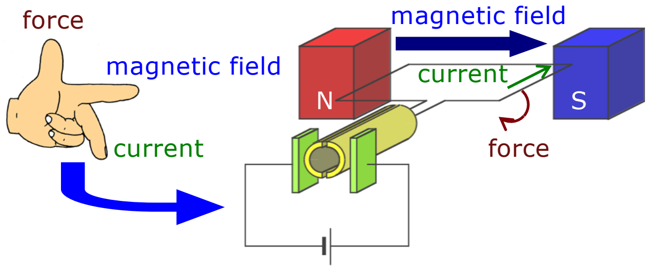
Maxwell Equations - Boundary conditions of an electromagnetic field

Maxwell equations :

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} \\ \boxed{\mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}} \\ \mathrm{div}\,\boldsymbol{D} = \rho \\ \boxed{\mathrm{div}\,\boldsymbol{B} = 0} \end{cases} \qquad \begin{cases} \boldsymbol{D} = \varepsilon \boldsymbol{E} \\ \boldsymbol{B} = \mu \boldsymbol{H} \\ \boldsymbol{J} = \sigma \boldsymbol{E} \end{cases}$$

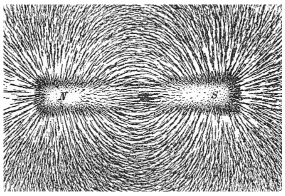
law of :





law for :

Conservation of magnetic flux



* http://www.wikipedia.org/

Maxwell Equations in Free Space

Maxwell equations :

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \mathrm{div}\,\boldsymbol{D} = \rho \\ \mathrm{div}\,\boldsymbol{B} = 0 \end{cases} \qquad \begin{cases} \boldsymbol{D} = \varepsilon \boldsymbol{E} \\ \boldsymbol{B} = \mu \boldsymbol{H} \\ \boldsymbol{J} = \sigma \boldsymbol{E} \end{cases}$$

In free space (no electron charge, and ε, μ and σ : constant),

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \sigma \boldsymbol{E} + \varepsilon \dfrac{\partial \boldsymbol{E}}{\partial t} \\ \mathrm{rot}\,\boldsymbol{E} = -\mu \dfrac{\partial \boldsymbol{H}}{\partial t} \\ \mathrm{div}\,\boldsymbol{E} = \rho / \varepsilon \\ \mathrm{div}\,\boldsymbol{H} = 0 \end{cases}$$

By differentiating the first equation with t and substituting the second equation,

$$\frac{\partial}{\partial t}\left(\mathrm{rot}\,\boldsymbol{H}\right) = \frac{\partial}{\partial t}\left(\sigma \boldsymbol{E}\right) + \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial \boldsymbol{E}}{\partial t}\right)$$

Maxwell Equations in Free Space (Cont'd)

Here, the left term can be rewritten as

$$-\operatorname{rot}\operatorname{rot}\boldsymbol{E} = -\operatorname{grad}\operatorname{div}\boldsymbol{E} + \nabla^2\boldsymbol{E} = \nabla^2\boldsymbol{E}$$

Similarly,

$$\nabla^2\boldsymbol{H} = \sigma\mu\frac{\partial\boldsymbol{H}}{\partial t} + \varepsilon\mu\frac{\partial^2\boldsymbol{H}}{\partial t^2}$$

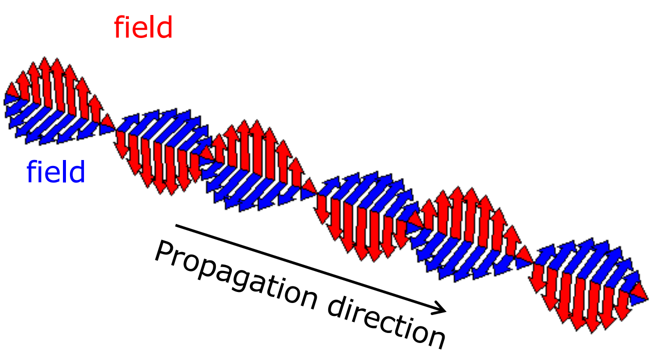
For an ideal insulating matrix,

$$\begin{cases}\nabla^2\boldsymbol{E} = \varepsilon\mu\dfrac{\partial^2\boldsymbol{E}}{\partial t^2}\\ \nabla^2\boldsymbol{H} = \varepsilon\mu\dfrac{\partial^2\boldsymbol{H}}{\partial t^2}\end{cases}$$

$\rightarrow$ Electromagnetic wave

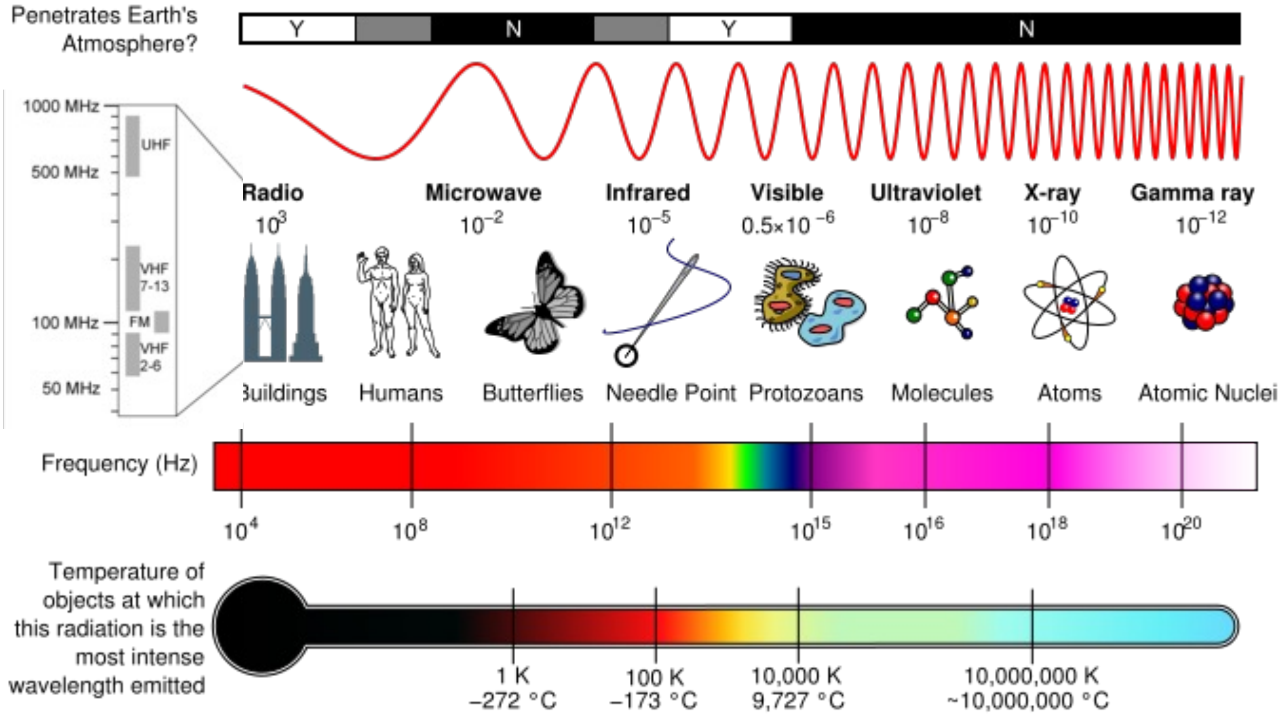
propagation speed :

in a vacuum,



* http://www.molphys.leidenuniv.nl/monos/smo/index.html

Electromagnetic Wave



* http://www.wikipedia.org/

Essence of the Maxwell Equations

Maxwell equations unified electronics and magnetism :

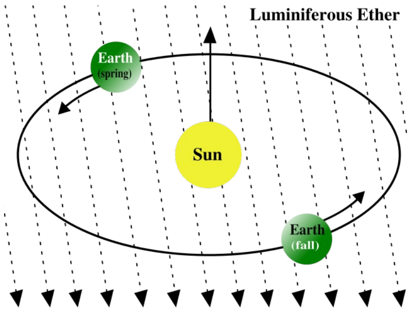
Electronics		Magnetism
Electron charge	Source	Magnetic dipole moment
	Force (Coulomb's law)	
	Field	
	Potential	
	Flux (Gauss' law)	

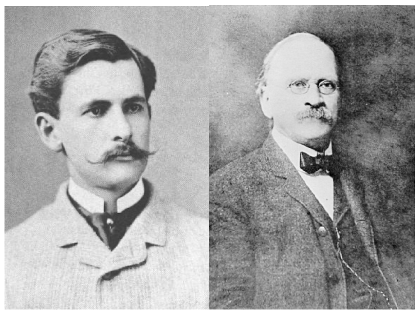
→ Further unification with the other forces

→ Einstein's theory of relativity

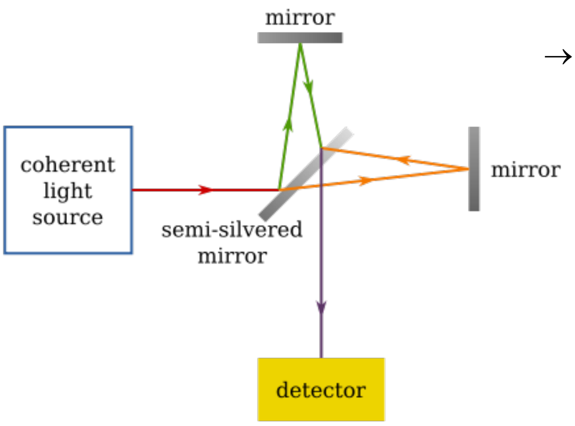
Michelson-Moley Experiment

In 1881, Albert A. Michelson and Edward W. Morley precisely designed experiment to prove the presence of Ether :





Ether was believed exist as a matrix to transfer an electromagnetic wave.



→ No interference between
parallel / perpendicular to Ether flow

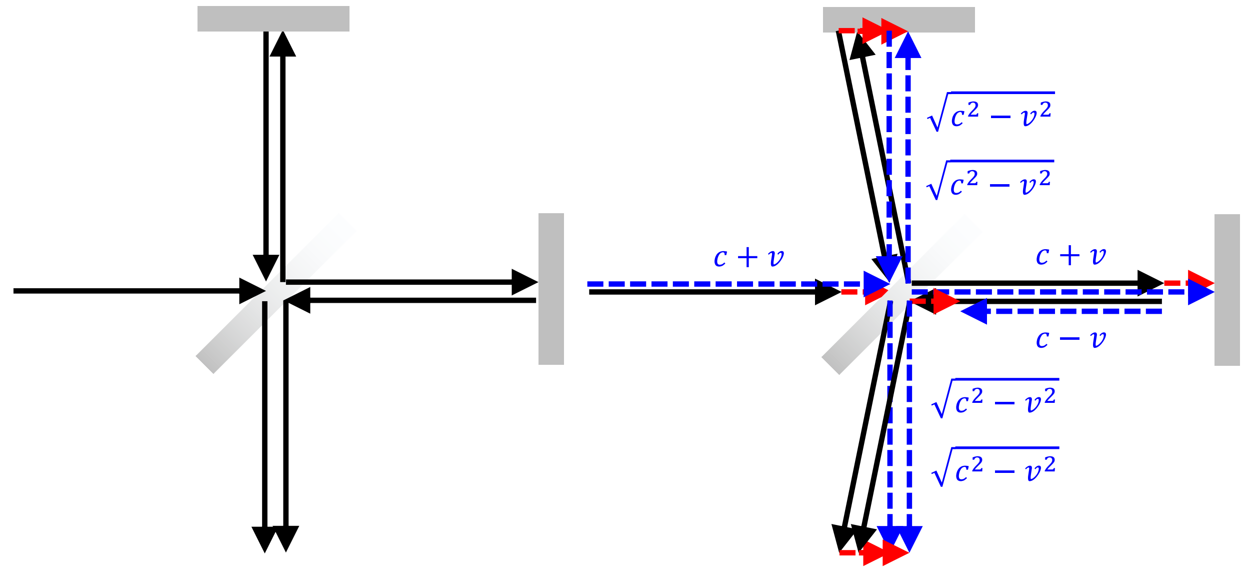
→

→ !

* http://www.wikipedia.org/

Michelson-Moley Experiment

When the system is still with the Ether flow at the speed of *v* :



→ The horizontal path is shrunk by √

→

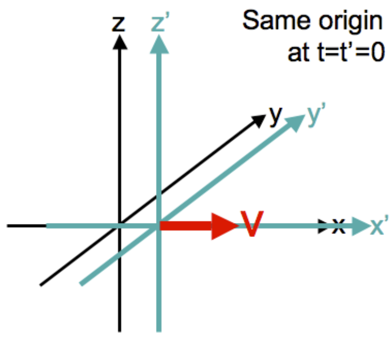
Einstein's Theory of Relativity

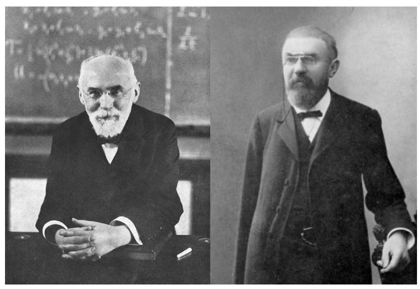
In 1905, Albert Einstein proposed the theory of special relativity :

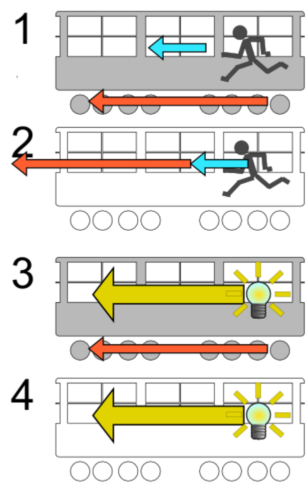
Lorentz invariance for Maxwell's equations (1900)

Poincaré proved the Lorentz invariance for dynamics.

→ Lorentz invariance in any inertial coordinates







Speed of light (electromagnetic wave) is constant.

* http://www.wikipedia.org/

Unified Theory beyond the Maxwell Equations