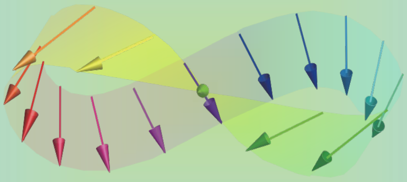
Nanoelectronics

03



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09:00 Thursday, 26/January/2023 (P/T 005A)

Quick Review over the Last Lecture

Maxwell equations :

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} & \rightarrow \text{Time-independent case : (} \qquad\qquad \text{) law} \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} & \rightarrow \text{(} \qquad\qquad \text{) law of induction} \\ \mathrm{div}\,\boldsymbol{D} = \rho & \rightarrow \text{(} \qquad\qquad \text{) law} \\ \mathrm{div}\,\boldsymbol{B} = 0 & \rightarrow \text{(} \qquad\qquad \text{) law for magnetism} \end{cases}$$

( )

( )

$$\begin{cases} \boldsymbol{D} = \varepsilon \boldsymbol{E} \\ \boldsymbol{B} = \mu \boldsymbol{H} \\ \boldsymbol{J} = \sigma \boldsymbol{E} \end{cases}$$

( )

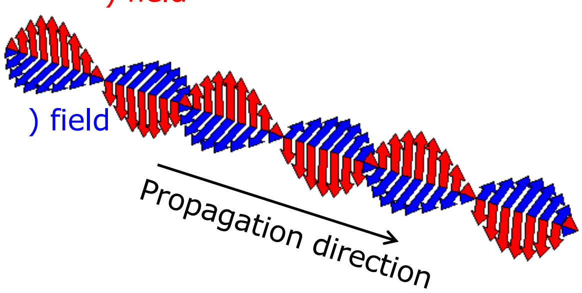
Electromagnetic wave :

propagation speed :

in a vacuum, $v =$

( ) field

( ) field



Contents of Nanoelectronics

03 Scalar and Vector Potentials

- Scalar potential ϕ
- Vector potential A
- Lorentz transformation

Maxwell Equations

Maxwell equations :

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \mathrm{div}\,\boldsymbol{D} = \rho \\ \mathrm{div}\,\boldsymbol{B} = 0 \end{cases} \qquad \begin{cases} \nabla \times \boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} \\ \nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{D} = \rho \\ \nabla \cdot \boldsymbol{B} = 0 \end{cases}$$

$\boldsymbol{E}$: electric field, $\boldsymbol{B}$: magnetic flux density,
$\boldsymbol{H}$: magnetic field, $\boldsymbol{D}$: electric flux density,
$\boldsymbol{J}$: current density and ρ : chatge density

Supplemental equations for materials :

$$\begin{cases} \boldsymbol{D} = \varepsilon \boldsymbol{E} & \rightarrow \text{Definition of an electric flux density} \\ \boldsymbol{B} = \mu \boldsymbol{H} & \rightarrow \text{Definition of an magnetic flux density} \\ \boldsymbol{J} = \sigma \boldsymbol{E} & \rightarrow \text{Ohm's law} \end{cases}$$

Electromagnetic Potentials

Scalar potential ϕ and vector potential $\boldsymbol{A}$ are defined as

$$\begin{cases} \boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A} \\ \boldsymbol{E} = -\dfrac{\partial \boldsymbol{A}}{\partial t} - \mathrm{grad}\,\phi \end{cases}$$

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} & (1) \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} & (2) \\ \mathrm{div}\,\boldsymbol{D} = \rho & (3) \\ \mathrm{div}\,\boldsymbol{B} = 0 & (4) \end{cases}$$

By substituting these equations into Eq. (2),

$$\mathrm{rot}\,\boldsymbol{E} = \mathrm{rot}\left(-\frac{\partial \boldsymbol{A}}{\partial t} - \mathrm{grad}\,\phi\right)$$

Here,

$$\mathrm{rot}\left(\mathrm{grad}\,\phi\right)_x = \frac{\partial}{\partial y}\left(\mathrm{grad}\,\phi\right)_z - \frac{\partial}{\partial z}\left(\mathrm{grad}\,\phi\right)_y = \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial y}\right) = 0$$

$\rightarrow$ Similarly, y- and z-components become 0.

Also,

$$\mathrm{rot}\left(-\frac{\partial \boldsymbol{A}}{\partial t}\right) =$$

$\rightarrow$ Satisfies Eq. (2).

Electromagnetic Potentials (Cont'd)

$$\begin{cases} \boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A} \\ \boldsymbol{E} = -\dfrac{\partial \boldsymbol{A}}{\partial t} - \mathrm{grad}\,\phi \end{cases}$$

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} & (1) \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} & (2) \\ \mathrm{div}\,\boldsymbol{D} = \rho & (3) \\ \mathrm{div}\,\boldsymbol{B} = 0 & (4) \end{cases}$$

By substituting these equations into Eq. (4),

$$\mathrm{div}(\mathrm{rot}\,\boldsymbol{A}) =$$

$$=$$

$\rightarrow$ Satisfies Eq. (4).

Electromagnetic Potentials (Cont'd)

By assuming x as a differentiable function, we define

$$\begin{cases} \boldsymbol{A}' = \boldsymbol{A} + \mathrm{grad}\,x \\ \phi' = \phi - \dfrac{\partial x}{\partial t} \end{cases}$$

$\rightarrow$ Gauge transformation

Here, $\boldsymbol{A}'$ and ϕ' provide $\boldsymbol{E}$ and $\boldsymbol{B}$ as the same as $\boldsymbol{A}$ and ϕ.

Therefore, electromagnetic potentials $\boldsymbol{A}$ and ϕ contains uncertainty of x.

In particular, when $\boldsymbol{A}$ and ϕ satisfies the following condition :

$\rightarrow$ Laurenz gauge

Under this condition, Eqs. (1) and (3) are expressed as

$$\begin{cases} \left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{A} = \mu \boldsymbol{J} \\ \left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = \dfrac{1}{\varepsilon}\rho \end{cases}$$

$$\begin{cases} \mathrm{rot}\,\boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t} & (1) \\ \mathrm{rot}\,\boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} & (2) \\ \mathrm{div}\,\boldsymbol{D} = \rho & (3) \\ \mathrm{div}\,\boldsymbol{B} = 0 & (4) \end{cases}$$

$\rightarrow$ Equation solving at home

Einstein's Theory of Relativity

In 1905, Albert Einstein proposed the theory of special relativity :

Speed of light (electromagnetic wave) is constant.

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Lorentz factor :

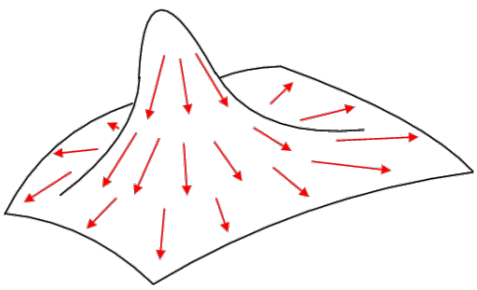
Lorentz transformation $\rightarrow$ Galilean transformation when $\frac{v}{c} \rightarrow 0$.

$$\begin{cases} x' = \\ t' = \end{cases} \qquad \begin{cases} x' = \\ t' = \end{cases}$$

Scalar Potential ϕ

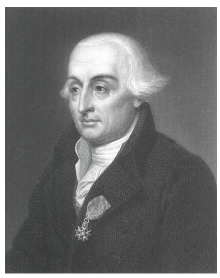
Scholar potential holds the following relationship with a force :

$$\boldsymbol{F} = -\,\mathrm{grad}\,\phi$$



The concept was first introduced by Joseph-Louis Lagrange in 1773,

and named as scalar potential by George Green in 1828.

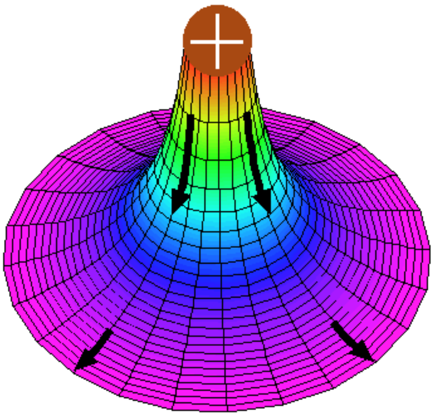


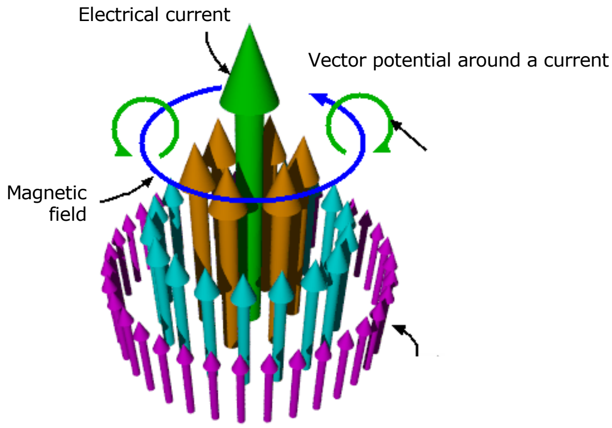
* http://www12.plala.or.jp/ksp/vectoranalysis/ScalarPotential/

** http://www.wikipedia.org/

Scalar Potential ϕ and Vector Potential A

Scalar potential holds the following relationship with a force :





Electric field $\boldsymbol{E}$
(induced along the direction to decrease the voltage potential)

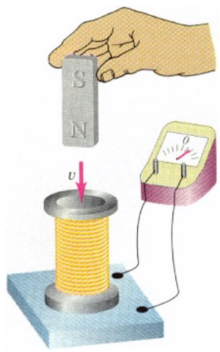
Magnetic field $\boldsymbol{H}$
(induced along the axis of the rotational flux of the vector potential, $\mathrm{rot}\,\boldsymbol{A}$)

* http://www.phys.u-ryukyu.ac.jp/~maeno/cgi-bin/pukiwiki/index.php

Vector Potential A

Faraday found electromagnetic induction in 1831 :

Faraday considered that an "electronic state" of the coil
can be modified by moving a magnet.
→ induces current flow.



In 1856, Maxwell proposed a theory
with using a vector potential instead of "electronic state."

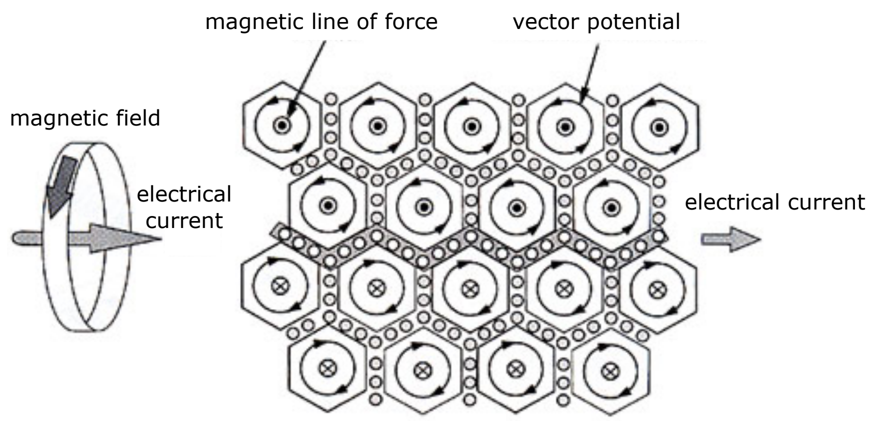
$$\begin{cases} \boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A} \\ \boldsymbol{E} = -\dfrac{\partial \boldsymbol{A}}{\partial t} \end{cases}$$

→ Rotational spatial distribution of $\boldsymbol{A}$ generates a magnetic flux $\boldsymbol{B}$.

→ Time evolution of $\boldsymbol{A}$ generates an electric field $\boldsymbol{E}$.

* http://www.physics.uiowa.edu/~umallik/adventure/nov_06-04.html

Maxwell's Vector Potential



→ By the rotation of the vector potentials in the opposite directions,
rollers between the vector potentials move towards one direction.

→ Ampère's law

After the observation of a electromagnetic wave,

E and ***B*** : physical quantities

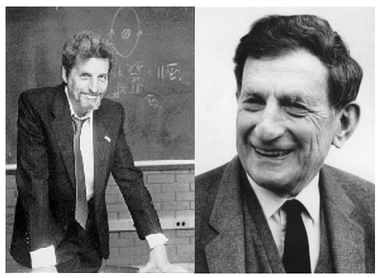
A : mathematical variable

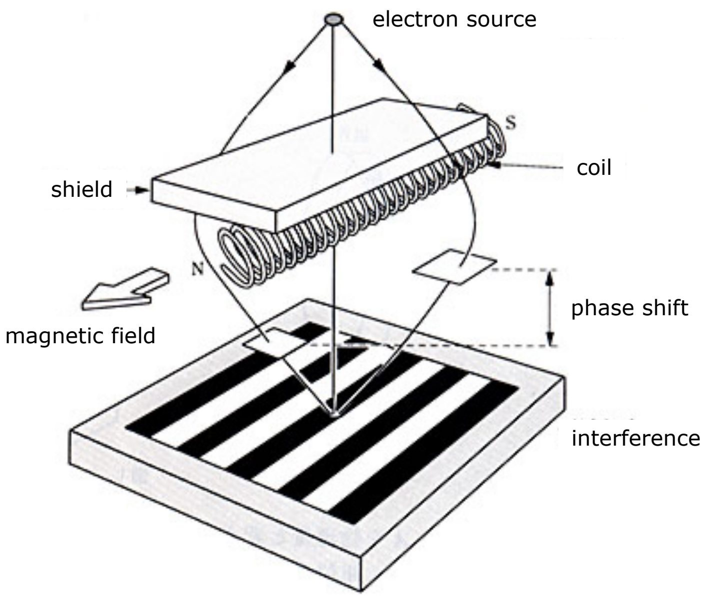
* http://www.ieice.org/jpn/books/kaishikiji/200012/20001201-2.html

Aharonov-Bohm Effect

Yakir Aharonov and David Bohm theoretically predicted in 1959 :

Electron can modify its phase
without any electrical / magnetic fields.



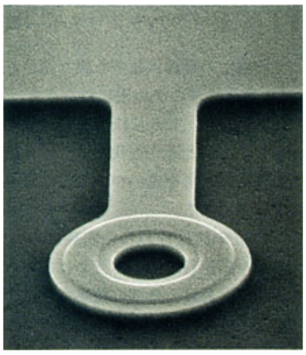


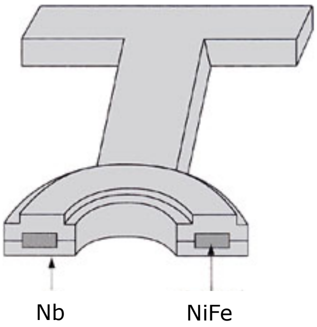
* http://www.wikipedia.org/; http://www.physics.sc.edu/~quantum/People/Yakir_Aharonov/yakir_aharonov.html

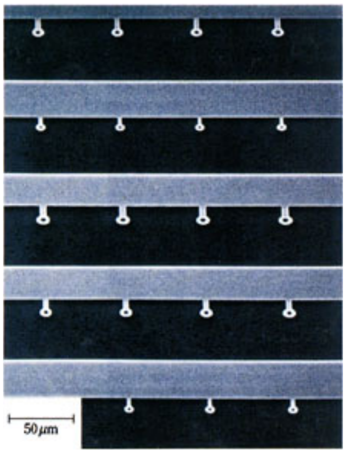
** http://www.ieice.org/jpn/books/kaishikiji/200012/20001201-3.html

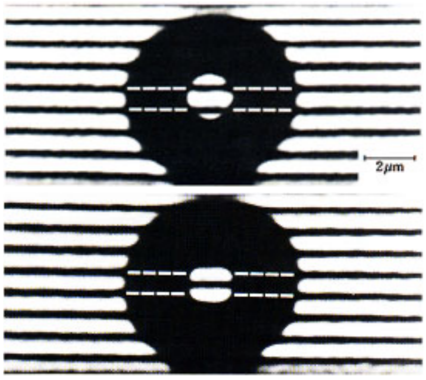
Observation of the Vector Potential

In 1982, Akira Tonomura proved the Aharonov-Bohm effect :









No phase shift

Phase shift =

* http://www.nanonet.go.jp/japanese/mailmag/2003/009a.html
** http://www.ieice. org/jpn/books/kaishikiji/200012/20001201-4.html