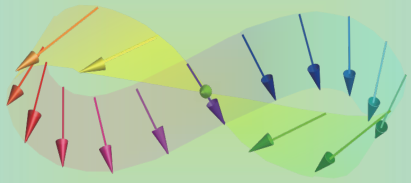
Nanoelectronics
04



Atsufumi Hirohata

Department of Electronic Engineering

THE UNIVERSITY *of York*

12:00 Thursday, 26/January/2023 (P/T 005A)

Quick Review over the Last Lecture

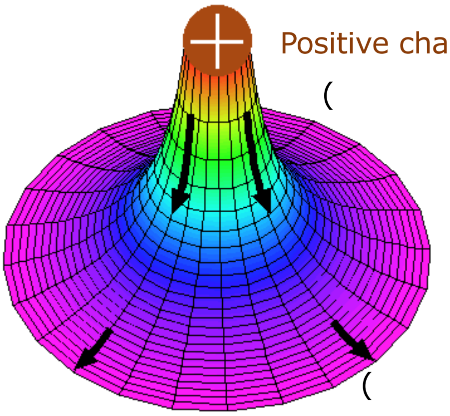
Maxwell equations with ( ) potential ( ) and ( ) potential ( ) :

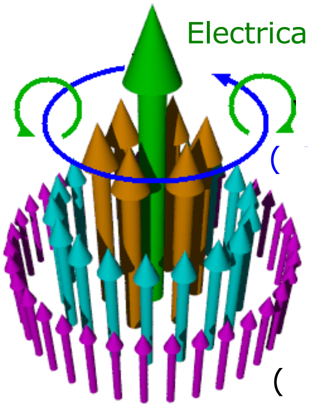
$$\begin{cases} \boldsymbol{B} = \mathrm{rot}\,\boldsymbol{A} \\ \boldsymbol{E} = -\dfrac{\partial \boldsymbol{A}}{\partial t} - \mathrm{grad}\,\phi \end{cases}$$

$$\begin{cases} \left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \nabla^2\right)\boldsymbol{A} = \mu \boldsymbol{J} \\ \left(\dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = \dfrac{1}{\varepsilon}\rho \end{cases}$$

( ) gauge :

$$\mathrm{div}\,\boldsymbol{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$





Contents of Nanoelectonics

I. Introduction to Nanoelectronics (01)
  - 01 Micro- or nano-electronics ?

II. Electromagnetism (02 & 03)
  - 02 Maxwell equations
  - 03 Scalar and vector potentials

III. Basics of quantum mechanics (04 ~ 06)
  - 04 History of quantum mechanics 1
  - 05 History of quantum mechanics 2
  - 06 Schrödinger equation

IV. Applications of quantum mechanics (07, 10, 11, 13 & 14)

V. Nanodevices (08, 09, 12, 15 ~ 18)

---

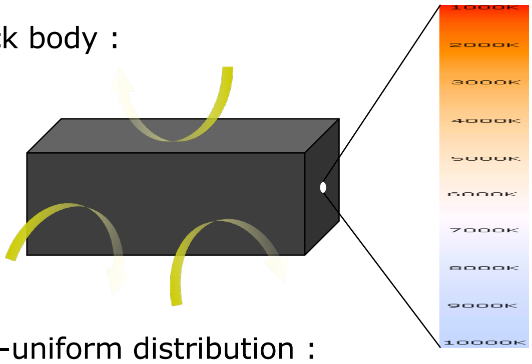
04 History of Quantum Mechanics 1

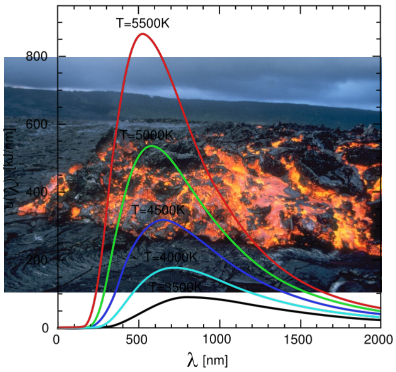
- Black body radiation
- Light quantum
- Photoelectric effect
- Compton scattering
- De Broglie wave

Black Body Radiation

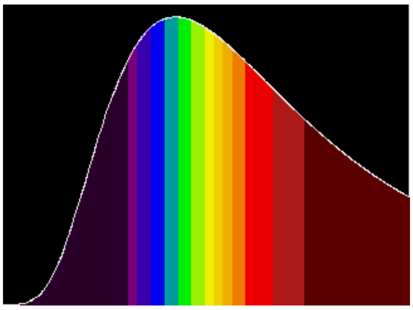
In 1900, Max Planck reported energy unit of $h\nu$ between light and a material :

Black body :





Non-uniform distribution :



cannot be explained by statistical mechanics.

in an equilibrium state,

each degree of freedom holds $E = k_B T / 2$

→ distribution

If the energy unit is $h\nu$,

for $k_B T < h\nu$,

for $k_B T > h\nu$,

→ distribution

* http://www.wikipedia.org/

Einstein's Light Quantum Hypothesis

In 1905, Albert Einstein explained that each photon has the energy of $h\nu$:

→

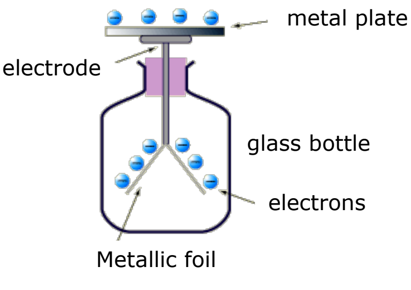
* http://www.wikipedia.org/

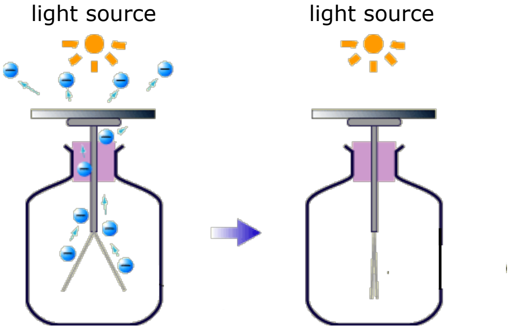
Photoelectric Effect - Experiments

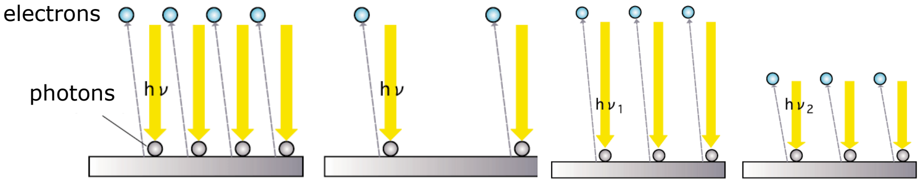
In 1887, Heinrich R. Hertz found ultraviolet light encourages discharge from a negative metallic electrode :

In 1888, W. L. F. Hallwacks observed electron radiation by light.

In 1902, Philipp E. A. von Lenard found radiated electron energy is independent of light intensity but dependent upon ν.







* http://www.wikipedia.org/

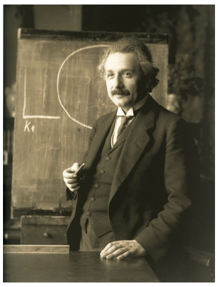
** http://www12.plala.or.jp/ksp/quantum/photoelectric1/

*** http://homepage2.nifty.com/einstein/contents/relativity/contents/relativity3005.html

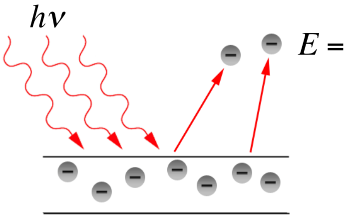
---

Photoelectric Effect - Theory

In 1905, Albert Einstein proposed theories of light with using a light quantum :



Light consists of a light quantum (photon).



W : work function for an electron to be released

Light holds *both wave and particle nature*.

Momentum of a photon is predicted to be .

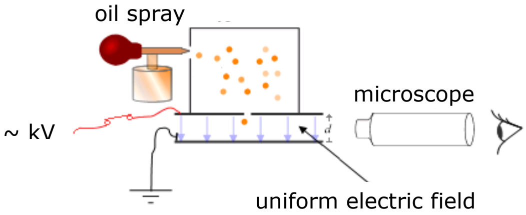
* http://www.wikipedia.org/

Milikan's Oil-Drop Experiment

In 1916, Robert A. Millikan and Harvey Fletcher measured the Planck constant :

In 1912, an electron charge measured to be :

1.592×10^{-19} C ($1.60217653 \times 10^{-19}$ C)



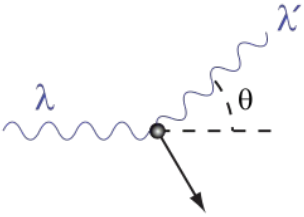
A critical voltage, at which no electron motion (no photocurrent) is realised :

$E =$

* http://www.wikipedia.org/

Compton Scattering

In 1923, Arthur H. Compton measured the momentum of a photon :



$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

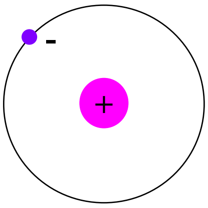
λ : wavelength of a photon before scattering,
λ' : wavelength of a photon after scattering,
m_e : electron mass, θ : scattered photon angle,
h : Planck constant and c : speed of light

* http://www.wikipedia.org/

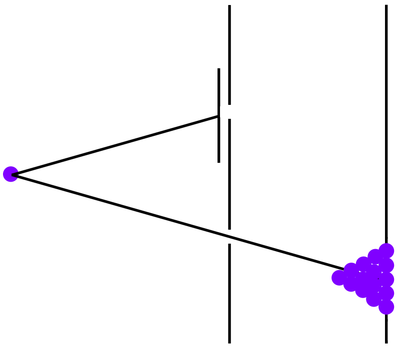
Electron Interference

Davisson-Germer experiment in 1927 :

Electrons are introduced to a screen through two slits.



Electron as a particle
should not interfere.
≠ Photon (light) as a wave



→ observed !

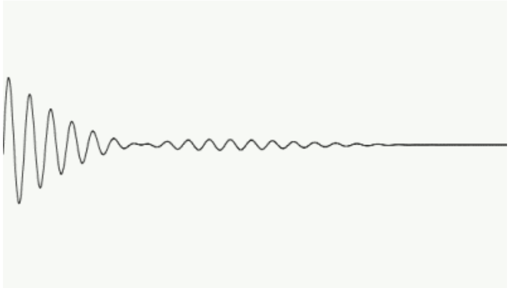
→

* http://www.wikipedia.org/

De Broglie Wave

Wave packet :

contains number of waves, of which amplitude describes probability of the presence of a particle.



where λ : wave length, h : Planck constant
and m_0 : mass of the particle.

→ de Broglie hypothesis
(1924 PhD thesis → 1929 Nobel prize)

According to the mass-energy equivalence :

$E =$

where p : momentum and ν : frequency.
By using $E = h\nu$,

$\lambda =$

* http://www.wikipedia.org/

Schrödinger Equation

In order to express the de Broglie wave, Schrödinger equation is introduced in 1926 :

$$\frac{\hbar^2}{2m}\nabla^2\psi + (E - V)\psi = 0 \qquad \left(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

E : energy eigenvalue and ψ : wave function

Wave function represents probability of the presence of a particle $|\psi|^2 = \psi^*\psi$

ψ^* : complex conjugate (*e.g.*, $z = x + iy$ and $z^* = x - iy$)

Propagation of the probability (flow of wave packet) :

$$\boldsymbol{j} = \frac{\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right)$$

Operation = observation :

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2}\boxed{\psi} = (\boxed{E} - V)\psi \qquad\qquad |\psi|^2 = 1 :$$

Scrödinger's Cat

Thought experiment proposed by Erwin R. J. A. Schrödinger in 1935

The observer cannot know

- if a radioactive atom has decayed.
- if the vial has been broken and the hydrocyanic acid has been released.
- if the cat is killed.

→ The cat is both dead and alive according to quantum law :

superposition of states

The superposition is lost :

- only when the observer opens the box and learn the condition of the cat.
- then, the cat becomes dead or alive.

→ quantum

* http://www.wikipedia.org/

Comparison between Classical and Quantum Mechanics

In order to express the de Broglie wave, Schrödinger equation is introduced in 1926 :

Classical mechanics		Quantum mechanics
	Coordinate	
	Momentum	
	Energy	
	Variables	
	Equation	
	Amplitide / wavefunction	
	Energy / probability	