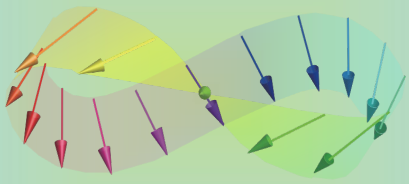
Nanoelectronics
05



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09:00 Thursday, 02/February/2023 (P/T 005A)

Quick Review over the Last Lecture

Light quantum :

E = (   )

(   ) scattering

Electron (   )

→ (   ) wave

λ =

Contents of Nanoelectonics

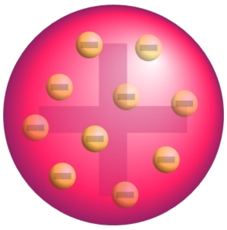
05 History of Quantum Mechanics 2

- Rutherford's model
- Bohr's model
- Balmer series
- Uncertainty principle

Early Models of an Atom

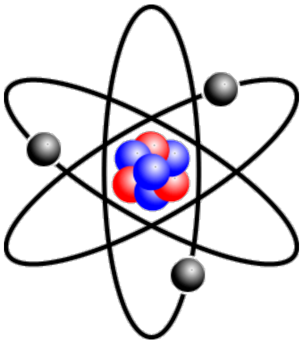
In 1904, J. J. Thomson proposed a   model :

Negatively charged "plums" (electrons) are surrounded by positively charged "pudding."



In 1904, Hantaro Nagaoka proposed a   model :

Negatively charged electrons rotate around positively-charged core.



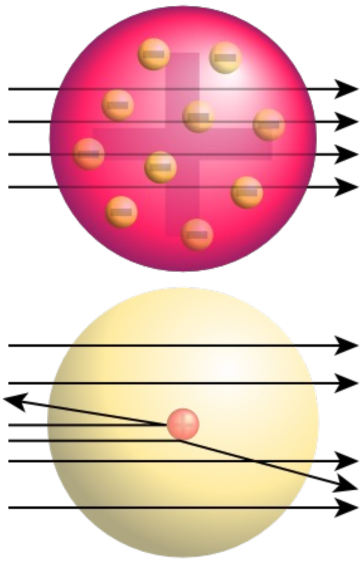
* http://www.wikipedia.org/
** http://www.nararika.com/butsuri/kagakushi/genshi/genshiron.htm

Rutherford's Model

In 1909, Ernest Rutherford carried out a Au foil experiment :

α-ray was introduced onto a very thin Au foil.

→   was observed.



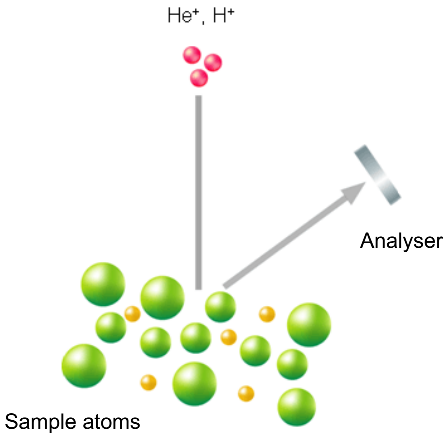
cannot be explained by the plum pudding model, and the Saturn model was adopted.

The size of the core is estimated to be
                    m.

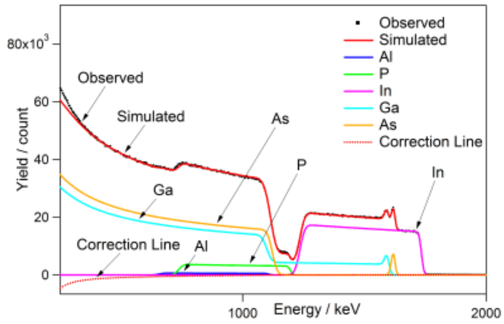
* http://www.wikipedia.org/

Rutherford Back Scattering

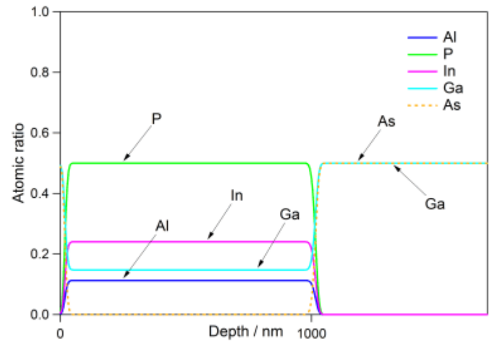
Powerful tool for materials analysis :



Example : InAlGaP (1000nm) / GaAs-sub.



RBS Spectrum



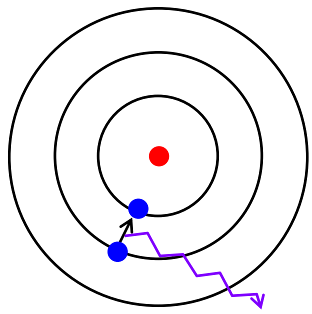
Depth Profile

* http://www.toray-research.co.jp/kinougenri/hyoumen/hyo_006.html

Limitation of Rutherford's Model

In classical electromagnetism,

An electron rotating around the core loses its energy
by irradiating electromagnetic wave,
and falls into the positively-charged core.



In 1913, Niels H. D. Bohr proposed a quantum rule :

An electron can permanently rotate around the core
when occupying an orbital with

m_e : electron mass, v : electron speed, r : orbital radius,
n : quantum number and h : Planck constant

$\rightarrow$ stable state

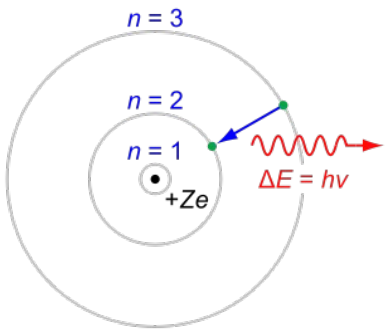
$\rightarrow$ $(n = 1, 2, 3, \ ...)$

* http://www.wikipedia.org/

Bohr's Model

Allowed transitions between the energy levels :

From the energy level of E_n to that of $E_{n'}$,
a photon is absorbed when $E_{n'} - E_n > 0$.
a photon is released when $E_{n'} - E_{n'} < 0$.



Meaning of the quantum rule :

De Broglie wave length is defined as

By substituting this relationship into the quantum rule,

$$m_e vr = p_e r = n \frac{h}{2\pi}$$

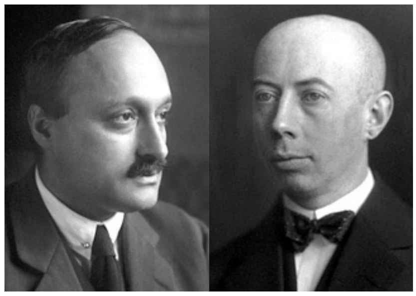
→ Electron as a standing wave in an orbital.

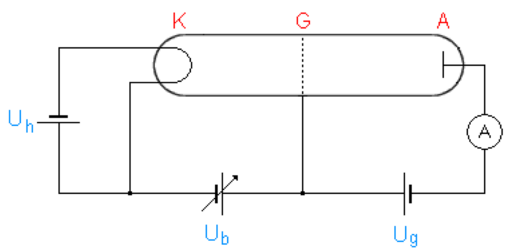
* http://www.wikipedia.org/

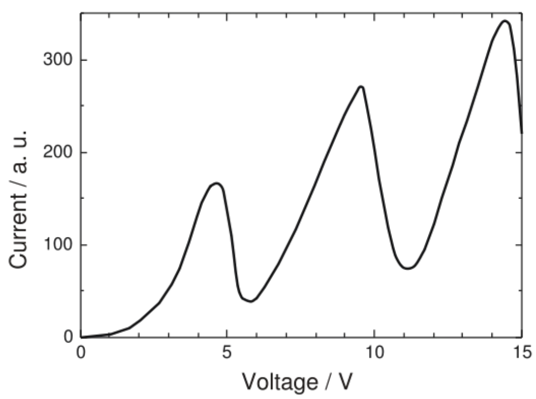
Proof of Bohr's Model

In 1914, James Franck and Gustav L. Hertz proved discrete energy levels :

An acceleration voltage is tuned to allow
diluted gas to absorb the energy.







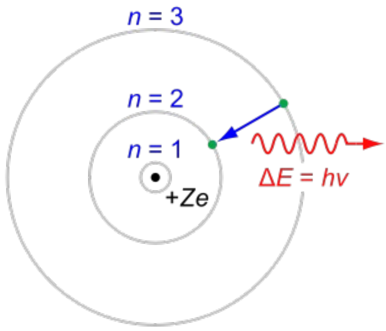
→ A proof of the

* http://nobelprize.org/
** http://www.wikipedia.org/

Balmer Series in a Hydrogen Atom

In 1885, Johann Jakob Balmer proposed an empirical formula :

Balmer series observation :



Balmer formula :

$$\lambda = B\left(\frac{m^2}{m^2 - n^2}\right) = B\left(\frac{m^2}{m^2 - 2^2}\right)$$

λ : wavelength, B : constant (364.56 nm), $n = 2$ and m : an integer ($m > n$)

→ A proof of the discrete

* http://www.wikipedia.org/

Rydberg Formula

In 1888, Johannes R. Rydberg generalised the Balmer formula :

Rydberg formula for Hydrogen :

$$\frac{1}{\lambda} = \frac{4}{B}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) = R_{\mathrm{H}}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \quad (m < n,\ n = 3,4,5,\ldots)$$

R_{H} : Rydberg constant (10973731.57 m^{-1})

Rydberg formula for atoms :

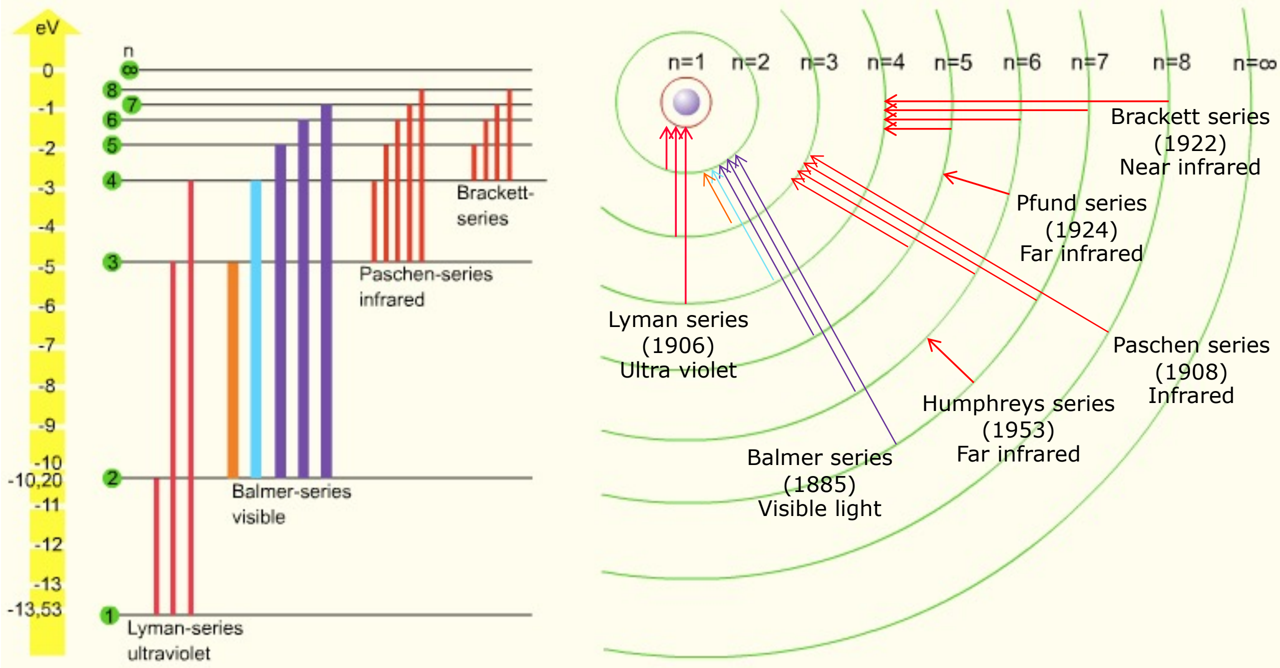
$$\frac{1}{\lambda_{\mathrm{vac}}} = R_{\mathrm{H}} Z^2\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \quad (m < n,\ n = 3,4,5,\ldots)$$

λ_{vac} : wavelength of the light emitted in a vacuum,
Z : atomic number, m and n : integers

* http://www.wikipedia.org/

Balmer Series in a Hydrogen Atom

Other series were also found :

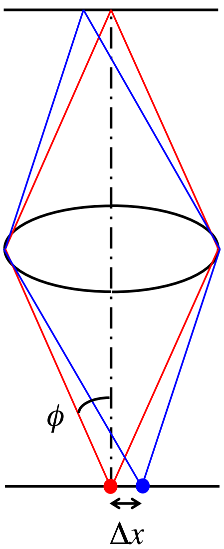


* http://www.bigs.de/en/shop/htm/termsch01.html

Heisenberg's Thought Experiment

In 1927, Werner Karl Heisenberg proposed the uncertainty principle :

Resolution of a microscope is defined as $\Delta x = \dfrac{\lambda}{2\sin\phi}$



In order to minimise Δx,

Larger ϕ

Small λ

Here, $p = \dfrac{h}{\lambda}$

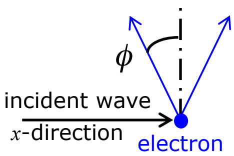
Small $\lambda \rightarrow$ large p (damage to samples)

Also, larger $\phi \rightarrow$ difficult to identify paths

In addition, incident wave can be reflected within ϕ

Momentum along x is within

$$\pm \frac{h}{\lambda}\sin\phi$$



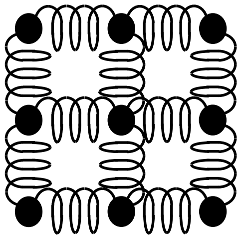
$\Delta x \Delta p =$

* http://www.wikipedia.org/

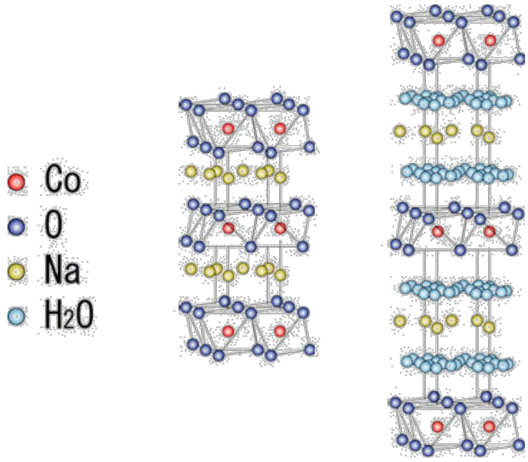
Observation of the Uncertainty Principle

Zero-point motion :

He does not freeze near 0 K under atmospheric pressure.



Spin fluctuation in an itinerant magnet :



* http://www.e-one.uec.ac.jp/~kuroki/cobalt.html

Precise Quantum Physical Definition

Relationship between an observation error in position ε and disturbance in momentum :

$$\Delta x \Delta p = \varepsilon(x)\eta(p) \geq \frac{\hbar}{2}$$

Heisenberg's uncertainty principle using operators :

$$\varepsilon(A)\eta(B) \geq \frac{1}{2}\left|\langle [A,B] \rangle\right|$$

Here, communication relation :

$$[A,B] = AB - BA$$

If *A* and *B* are commutative operators, the right-hand side is 0. This leads the error and disturbance to be 0.

However, *A* and *B* are not commutative operators, the right-hand side is not 0. This leads the error and disturbance to have a trade-off.

Heisenberg's uncertainty principle using standard deviations :

$$\sigma(A)\sigma(B) \geq \frac{1}{2}\left|\langle [A,B] \rangle\right| \quad \rightarrow$$

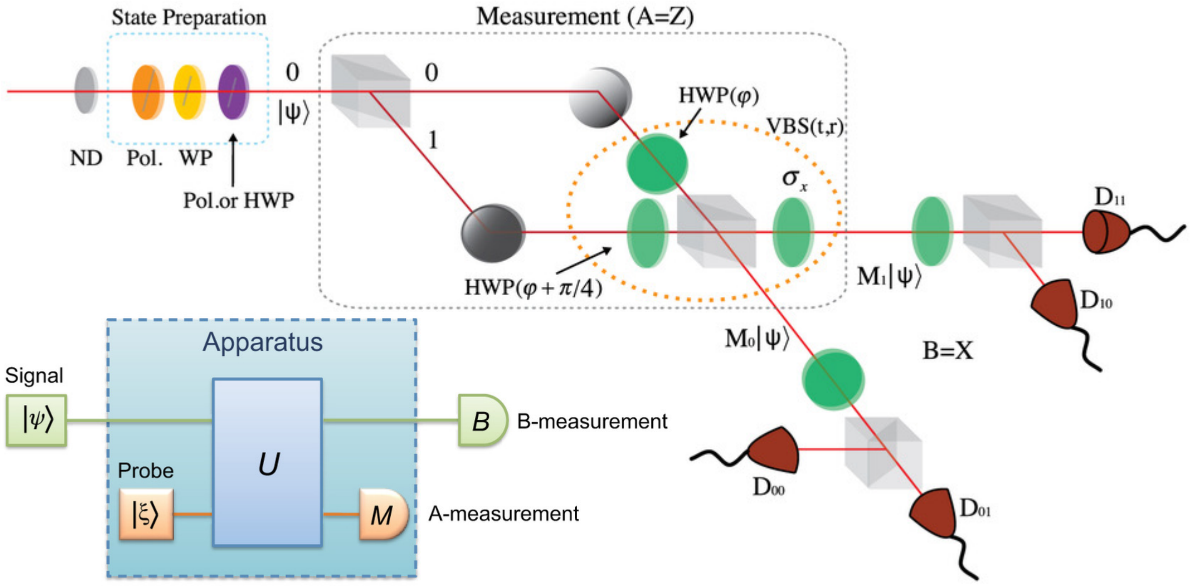
* https://www.nikkei-science.com/?p=16686

Corrections to Heisenberg's Uncertainty Principle

Ozawa's relationship : *

$$\varepsilon(A)\eta(B)+\varepsilon(A)\sigma(B)+\sigma(A)\eta(B)\geq\frac{1}{2}\left|\langle[A,B]\rangle\right|$$

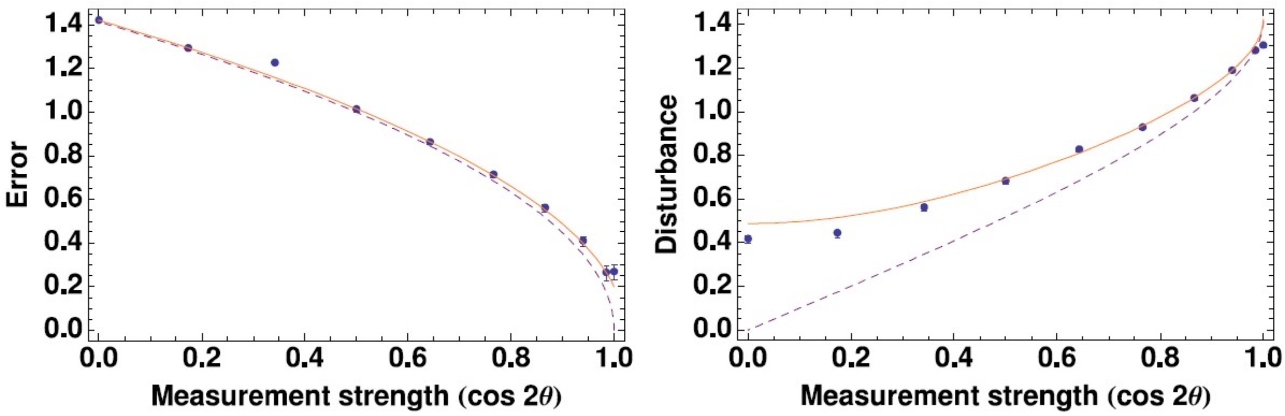
Optical proof : **



* M. Ozawa, *Phys. Rev. A* **67**, 042105 (2003);
** S.-Y. Baek *et al.*, *Sci. Rep.* **3**, 2221 (2013).

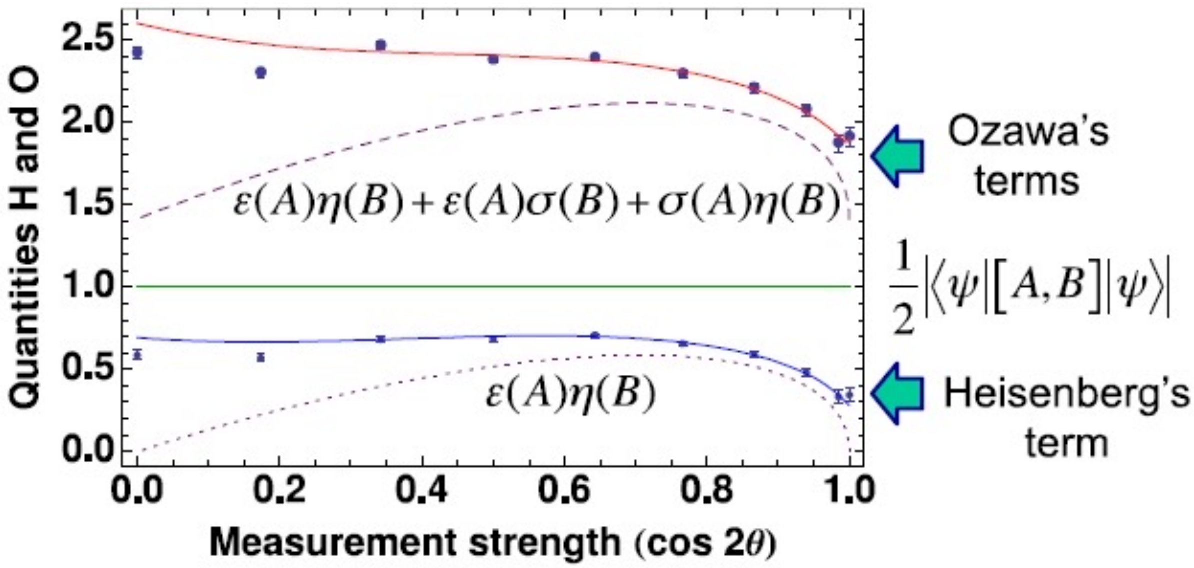
Experimental Data

Error and disturbance measured :



** S.-Y. Baek *et al.*, *Sci. Rep.* **3**, 2221 (2013).

Ozawa's relationship



** S.-Y. Baek *et al.*, *Sci. Rep.* **3**, 2221 (2013).