Nanoelectronics

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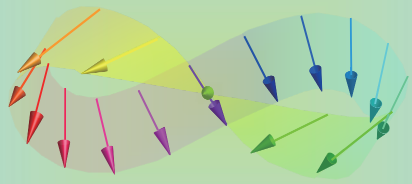

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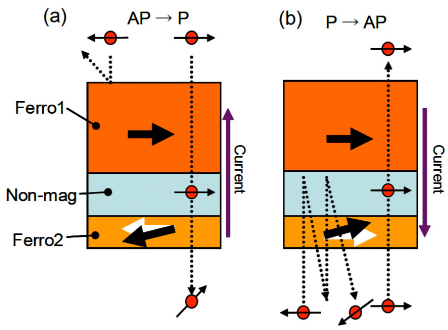
09:00 Thursday, 02/March/2023 (P/T 005A)

Quick Review over the Last Lecture



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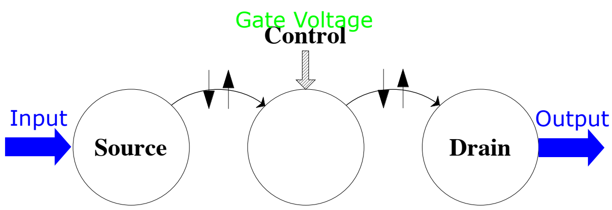


	FM / SC hybrid Structures	Magnetic tunnel junctions (MTJ)	All metal and spin valve structures
Interface			
Spin carriers			
Device applications	FM / 2DEG Schottky diodes Spin FET Spin LED Spin RTD	MOS junctions Coulomb blockade structures SP-STM Supercond. point contacts Spin RTD	Johnson transistors Spin valve transistors

* *After* M. Johnson, *IEEE Spectrum* **37**, 33 (2000).

Contents of Nanoelectonics

13 Quantum Statistics 1

- Classical distribution
- Fermi-Dirac distribution
- Bose-Einstein distribution

Classical Distribution

Maxwell-Boltzmann distibution :

In the classical theory, the same types of particles can be distinguished.

→ Every particle independently fills each quantum state.

In order to distribute N particles to the states $1, 2,$ ... with numbers of $N_1, N_2,$ ...

$$\frac{N!}{N_1!N_2!N_3!\cdots}$$

To distribute N_i particles to the states G_i is G_i^{Ni}, and hence the whole states are

$$G_1^{N_1}G_2^{N_2}G_3^{N_3}\cdots$$

Microscopic states in the whole system W are

$$W = \frac{N!}{N_1!N_2!N_3!\cdots}\cdot G_1^{N_1}G_2^{N_2}G_3^{N_3}\cdots$$

By taking logarithm for both sides and assuming N_i and G_i are large, we apply the Stirling formula.

$$\ln W = N\ln N - \sum_i N_i \ln\frac{N_i}{G_i} = \left(\sum_i N_i\right)\ln\left(\sum_i N_i\right) - \sum_i N_i \ln\frac{N_i}{G_i} \quad \begin{cases}\ln G_i \cong G_i \ln G_i - G_i \\ \ln N_i! \cong N_i \ln N_i - N_i\end{cases}$$

Classical Distribution (Cont'd)

In the equilibrium state,

$$\frac{\partial}{\partial N_i}\left(\ln W - \alpha\sum_i N_i - \beta\sum_i E_i N_i\right) = 0$$

Again, by applying the Stirling formula, $\begin{cases}\ln G_i \cong G_i \ln G_i - G_i \\ \ln N_i! \cong N_i \ln N_i - N_i\end{cases}$

$$\ln\frac{N_i}{G_i} - \ln N + \alpha + \beta E_i = 0$$

Here, with using

$$\ln W = \sum_i\left[\pm G_i \ln G_i \mp (G_i \mp N_i)\ln(G_i \mp N_i) - N_i \ln N_i\right]$$

$$\therefore \frac{N_i}{G_i} = N\exp(-\alpha - \beta E_i)$$

By using $N = \sum_i N_i$ and $\dfrac{N_i}{G_i} = \dfrac{1}{\exp(\alpha + \beta E_i) \pm 1}$

$$N = N\exp(-\alpha)\sum_i G_i \exp(-\beta E_i)$$

Now, the partition function is

$$Z = \sum_i G_i \exp(-\beta E_i)$$

Classical Distribution (Cont'd)

Therefore,

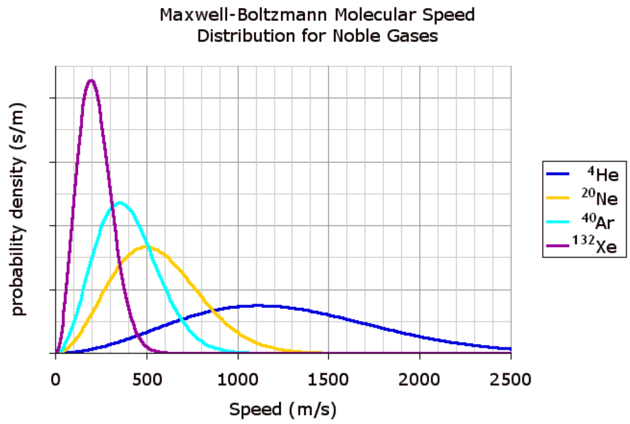
$$\exp(-\alpha) = \frac{1}{Z}$$

By substituting this relationship into $\frac{N_i}{G_i} = N \exp(-\alpha - \beta E_i)$

$$\frac{N_i}{N} = \frac{G_i \exp(-\beta E_i)}{Z}$$

Here, $\beta = 1/k_B T$

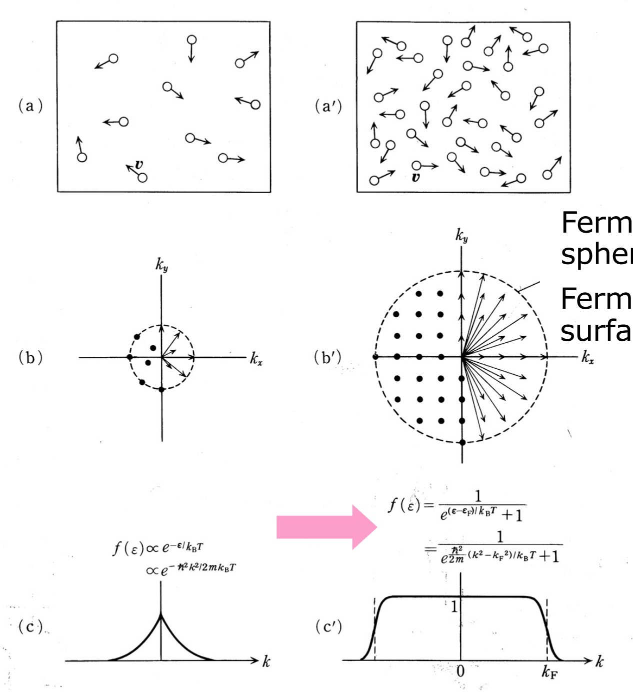
G_i : degree of degeneracy of eigen enrgy E_i



* http://www.wikipedia.org/

Fermi-Dirac / Maxwell-Boltzmann Distribution

Electron number density :



Fermi sphere :
sphere with the radius k_F

Fermi surface :
surface of the Fermi sphere

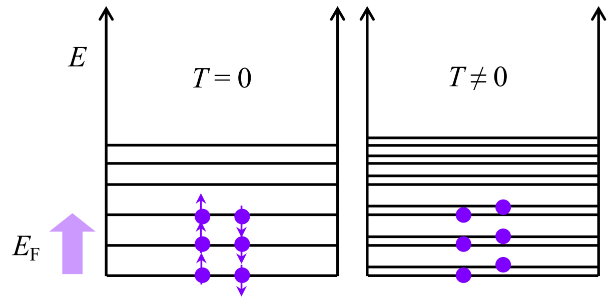
distribution (electron number density)

distribution (electron number density)

* M. Sakata, *Solid State Physics* (Baifukan, Tokyo, 1989).

Fermi Energy

Fermi-Dirac distribution :



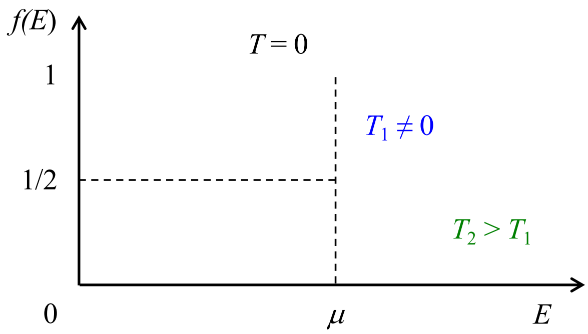
Pauli exclusion principle

At temperature T, probability that one energy state E is occupied by an electron :

$f(E) =$

μ : chemical potential
(= Fermi energy E_F at $T = 0$)

k_B : Boltzmann constant



Fermi velocity and Mean Free Path

Fermi wave number k_F represents E_F :

Fermi velocity : $v_F = \sqrt{\dfrac{2E_F}{m}}$

$$k_F = \frac{m}{\hbar} v_F = \frac{\sqrt{2mE_F}}{\hbar}$$

Under an electrical field :

Electrons, which can travel, has an energy of ~ E_F with velocity of v_F

For collision time τ, average length of electron path without collision is

$\ell = v_F \tau$

Density of states :

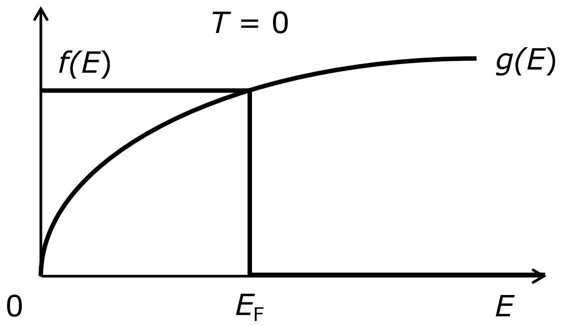
Number of quantum states at a certain energy in a unit volume

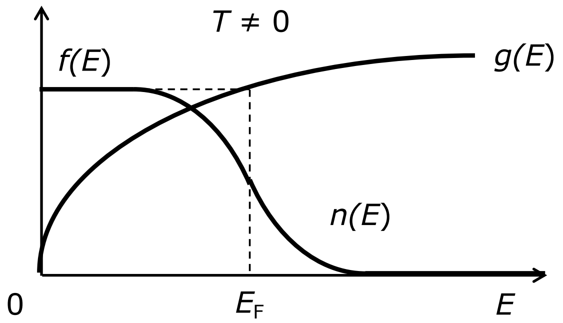
$$g(E) = 2 \frac{1}{(2\pi)^3} \frac{4\pi}{2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \sqrt{E} dE$$

Density of States (DOS) and Fermi Distribution

Carrier number density n is defined as :

$$n = \int f(E)g(E)dE$$





Fermi-Dirac Distribution

In the Fermi-Dirac distribution, 1 quantum state can be filled by 1 particle.

→ Fermi particle (spin 1/2)

For the entire wavefunction :

$$\psi(\cdots, x_i, \cdots, x_j, \cdots) = -\psi(\cdots, x_j, \cdots, x_i, \cdots)$$

→ Particle exchange induces a sign change of the wavefunction.

For a 2-Fermi-particle system, due to the exchange characteristics, the wavefunction is expressed as

$$\psi(x_1, x_2) = \phi(x_1)\chi(x_2) - \phi(x_1)\chi(x_2)$$

If the 2 particles satisfy the same wavefunction,

$$\psi(x_1, x_2) = \phi(x_1)\chi(x_2) - \phi(x_1)\chi(x_2) = 0$$

→ principle

Bose-Einstein Distribution

In the Bose-Einstein distribution, 1 quantum state can be filled by many particles.

→ Bose particle (spin 1,2,...)

For the entire wavefunction :

$$\psi(\cdots,x_i,\cdots,x_j,\cdots)=\psi(\cdots,x_j,\cdots,x_i,\cdots)$$

→ Particle exchange does not induce a sign change of the wavefunction.

For a 2-Bose-particle system, due to the exchange characteristics,
the wavefunction is expressed as

$$\psi(x_1,x_2)=\phi(x_1)\chi(x_2)+\phi(x_1)\chi(x_2)$$

If the 2 particles satisfy the same wavefunction,

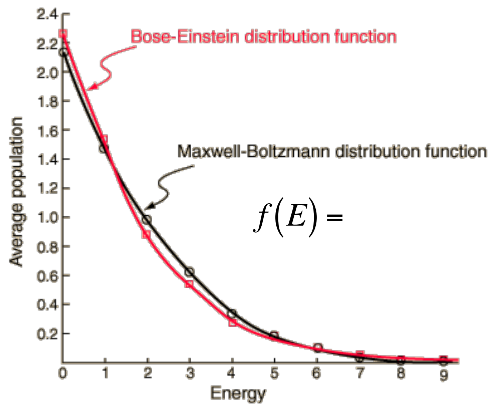
$$\psi(x_1,x_2)=\phi(x_1)\chi(x_2)$$

$$f(E)=$$

Bose-Einstein Distribution (Cont'd)

Difference between Bose-Einstein and Maxwell-Boltzmann distributions :

$$f(E)=$$



* http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/disbex.html

In quantum mechanics, the same types of particles cannot be distinguished :

By exchanging coordinate systems, Hamiltonian (operator) does not change :

$HP = PH$

P : coordinate exchange operator

$\rightarrow$ Diagonalization between H and P

By assuming the eigenvalue for P to be λ,

$P\psi = \lambda\psi$

Since twice exchange of the coordinate systems recover the original state,

$P^2 = 1$

$\therefore P^2\psi = \psi = \lambda^2\psi$

$\therefore \lambda^2 = 1 \qquad \therefore \lambda = \pm 1$

Therefore,

$\psi = \psi$ (symmetric exchange) $\rightarrow$ Bose-Einstein statistics

$\rightarrow$ spins

$\psi = -\psi$ (asymmetric exchange) $\rightarrow$ Fermi-Dirac statistics

$\rightarrow$ spins