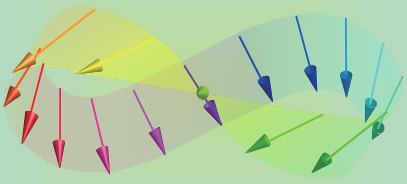
Nanoelectronics 15



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10:00 Monday, 06/March/2023 (P/L 006)

Quick Review over the Last Lecture

	Fermi-Dirac distribution	Bose-Einstein distribution
Function		
Energy dependence		
Quantum particles		
Spins		
Properties	At temperature of 0 K, each energy level is occupied by two Fermi particles with opposite spins. →	At very low temperature, large numbers of Bosons fall into the lowest energy state. →

Contents of Nanoelectonics

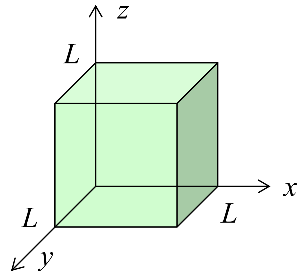
15 Low-Dimensional Nanodevices

- Quantum wells
- Superlattices
- 2-dimensional electron gas
- Quantum nano-wires
- Quantum dots

Schrödinger Equation in a 3D Cube

In a 3D cubic system, Schrödinger equation for an inside particle is written as :

$$\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(x,y,z)+(E-V)\psi(x,y,z)=0$$



where a potential is defined as

$$V=\begin{cases}0 & (\text{inside the box}: 0\le x\le L, 0\le y\le L, 0\le z\le L)\\ +\infty & (\text{outside the box}: x,y,z<0, x,y,z>L)\end{cases}$$

By considering the space symmetry, the wavefunction is

$$\psi(\boldsymbol{r},t)=\psi(x,t)\psi(y,t)\psi(z,t)$$

Inside the box, the Schrödinger equation is rewritten as

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(x,t)\psi(y,t)\psi(z,t)=E\psi(x,t)\psi(y,t)\psi(z,t)$$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\psi(x,t)}\frac{\partial^2}{\partial x^2}\psi(x,t)+\frac{1}{\psi(y,t)}\frac{\partial^2}{\partial y^2}\psi(y,t)+\frac{1}{\psi(z,t)}\frac{\partial^2}{\partial z^2}\psi(z,t)\right)=E_x+E_y+E_z$$

In order to satisfy this equation, 1D equations need to be solved :

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)=E_x\psi(x,t)$$

Schrödinger Equation in a 3D Cube (Cont'd)

To solve this 1D Schrödinger equation, $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)=E_x\psi(x,t)$,

we assume the following wavefunction, $\psi(x,t)=\phi(x)\exp(-i\omega t)$, and substitute into the above equation :

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\phi(x)\exp(-i\omega t)=E_x\phi(x)\exp(-i\omega t)$$

By dividing both sides with $\exp(-i\omega t)$,

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\phi(x)=E_x\phi(x)\quad\rightarrow$$

By substituting into the steady-state Schrödinger equation,

$$k_x=\pm\frac{\sqrt{2mE_x}}{\hbar}$$

For $0\le x\le L$, a general solution can be defined as

$$\phi(x)=A\sin(k_x x)+B\cos(k_x x)$$

At $x=0$, the probability of the particle is 0, resulting $\phi(0)=0$: $B=0$.

Similarly, $\phi(L)=0$: $k_x=\frac{n_x\pi}{L}\quad(n_x=1,2,3,\cdots)$

Schrödinger Equation in a 3D Cube (Cont'd)

By normalising $\phi(x) = A\sin\left(\frac{n_x \pi}{L} x\right)$,

$$\therefore A = \sqrt{\frac{2}{L}}$$

Hence, the wave function on *x* is obtained as

$$\phi(x) = \qquad\qquad \left(k_x = \frac{\sqrt{2mE_x}}{\hbar}\right)$$

By considering the symmetry, a particle in a 3D cubic system can be described with the wavefunction :

$$\psi(x, y, z) = \psi(x, t)\psi(y, t)\psi(z, t) = \left(\quad\right)^{\frac{3}{2}}$$

Here, the energy eigen values are

$$E = \qquad = \frac{\hbar^2}{2m}\left(\qquad\qquad\right) = \frac{\hbar^2}{2m}\left(\quad\right)^2\left(\qquad\qquad\right)$$

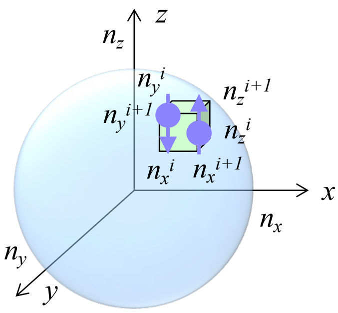
$$(n_x, n_y, n_z = 1, 2, 3, \cdots)$$

Density of States in Low-Dimensions

The available states can be obtained as

$$n_x{}^2 + n_y{}^2 + n_z{}^2 = \frac{2m}{\hbar^2}\left(\frac{L}{\pi}\right)^2 E$$

The available states can be approximated by the volume of the Fermi sphere (*x*, *y*, *z* ≥ 0) :



$$N^2 = \frac{1}{8}\frac{4\pi}{3}\left(\sqrt{\frac{2m}{\hbar^2}\left(\frac{L}{\pi}\right)^2 E}\right)^3 = \frac{L^3}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}$$

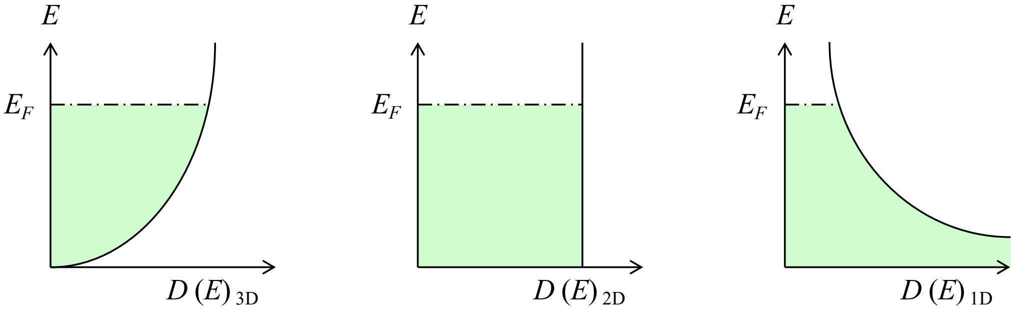
By dividing both sides by L^3 :

$$N(E) = \frac{1}{6\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2} \quad \left[= \qquad\qquad\right]$$

The density of states can be obtained as :

$$D(E) = \frac{dN}{dE} = \frac{2}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}$$

Density of States in Low-Dimensions



$D(E)_{3D} \propto$

$D(E)_{2D} \propto$

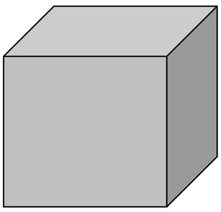
$D(E)_{1D} \propto$

→ Anderson localisation

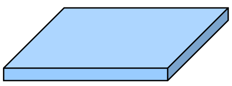
Dimensions of Nanodevices

By reducing the size of devices,

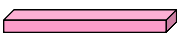
3D :



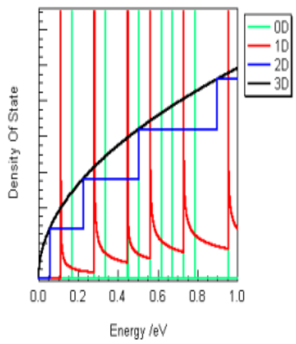
2D :



1D :



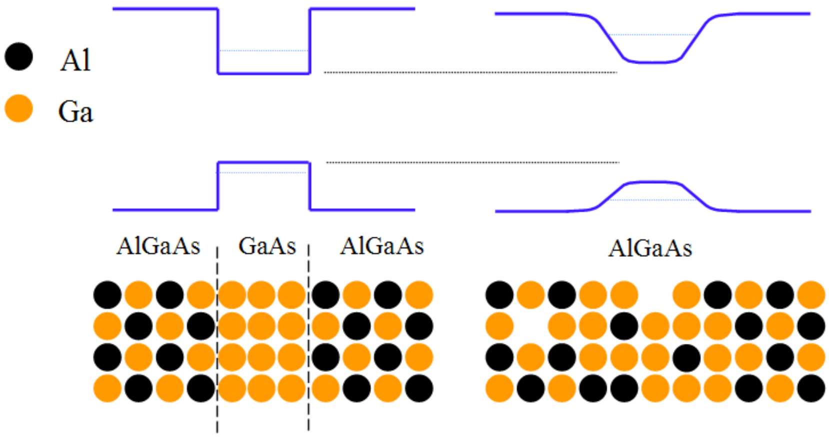
0D :



* http://www.kawasaki.imr.tohoku.ac.jp/intro/themes/nano.html

Quantum Well

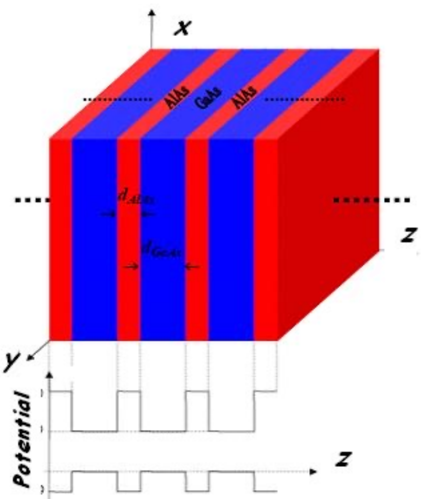
A layer (thickness <) is sandwiched by high potentials :



* http://www.intenseco.com/technology/

Superlattice

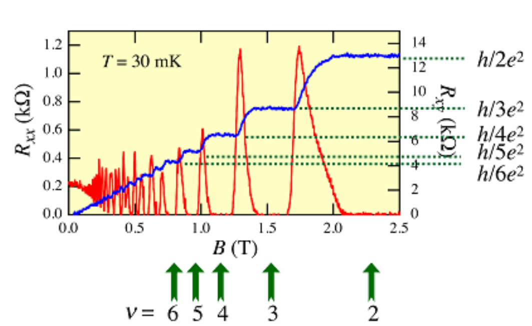
By employing ultrahigh vacuum molecular-beam epitaxy and sputtering growth, periodically alternated layers of several substances can be fabricated :



* http://www.wikipedia.org/

Quantum Hall Effect

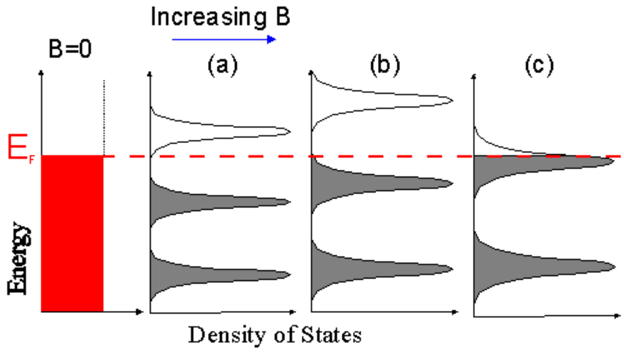
First experimental observation was performed by Klaus von Klitzing in 1980 :



By increasing a magnetic field, the Fermi level in 2DEG changes.

This was theoretically proposed by Tsuneya Ando in 1974 :

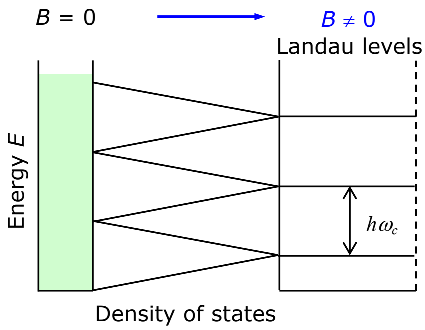
→ Observation of the Landau levels.

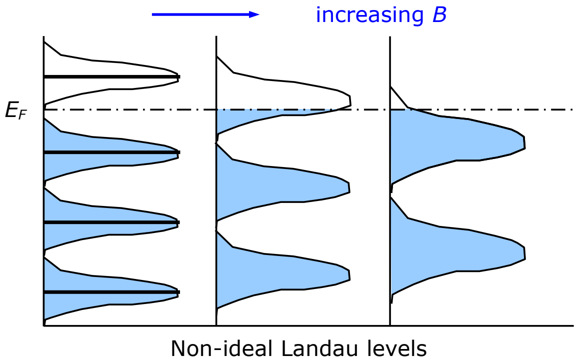


* http://www.wikipedia.org/
** http://qhe.iis.u-tokyo.ac.jp/research7.html
*** http://www.stat.phys.titech.ac.jp/ando/
† http://www.warwick.ac.uk/~phsbm/qhe.htm

Landau Levels

Lev D. Landau proposed quantised levels under a strong magnetic field :

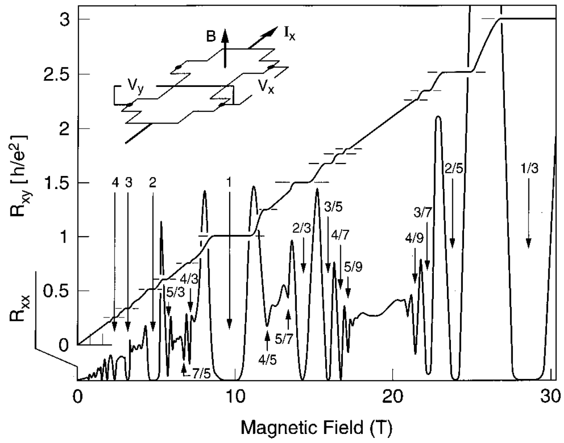


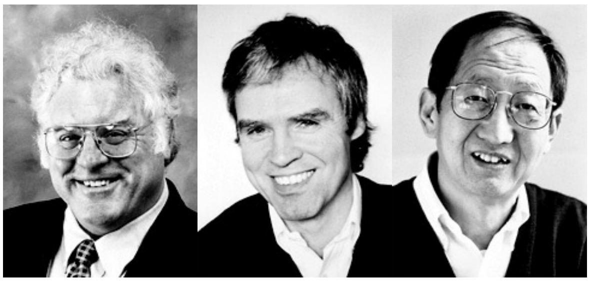


* http://www.wikipedia.org/

Fractional Quantum Hall Effect

In 1982, the fractional quantum Hall effect was discovered by Robert B. Laughlin, Horst L. Strömer and Daniel C. Tsui :





Coulomb interaction between electrons > Quantised potential

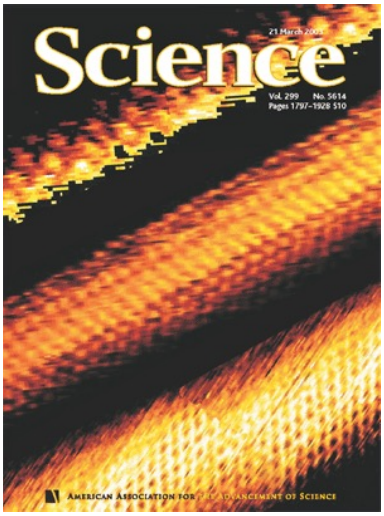
← Improved quality of samples (less impurity)

* http://nobelprize.org/

** H. L. Strömer *et al.*, *Rev. Mod. Phys.* **71**, S298 (1999).

Quantum Nano-Wire

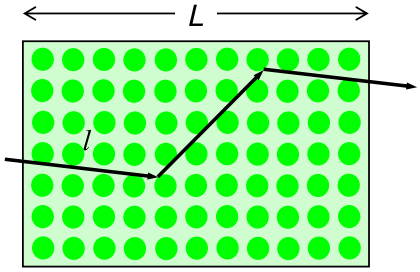
By further reducing another dimension, ballistic transport can be achieved :



* D. D. D. Ma *et al.*, *Science* **299**, 1874 (2003).

Diffusive / Ballistic Transport

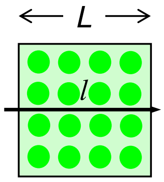
In a macroscopic system, electrons are scattered :



System length (L) > Mean free path (l)

→ transport

When the transport distance is smaller than the mean free path,



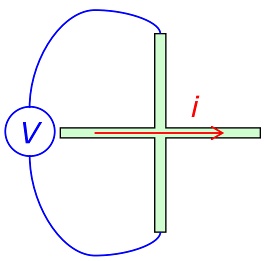
System length (L) ≤ Mean free path (l)

→ transport

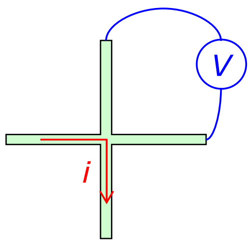
Resistance-Measurement Configurations

In a nano-wire device, 4 major configurations are used to measure resistance :

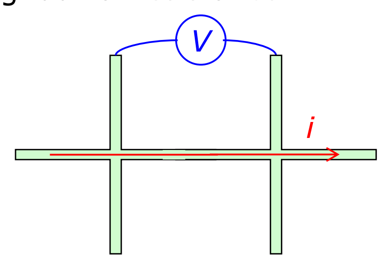
Hall resistance :



Bend resistance :

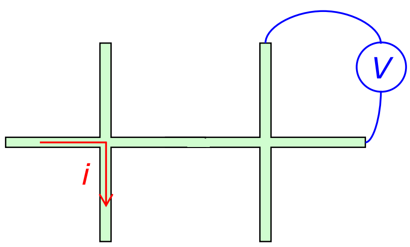


Longitudinal resistance :



method

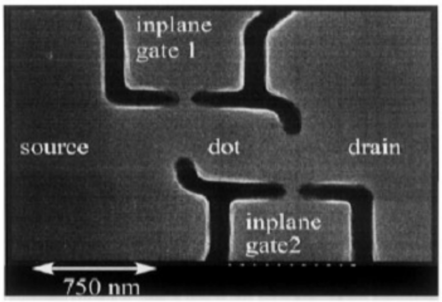
Transmission resistance :



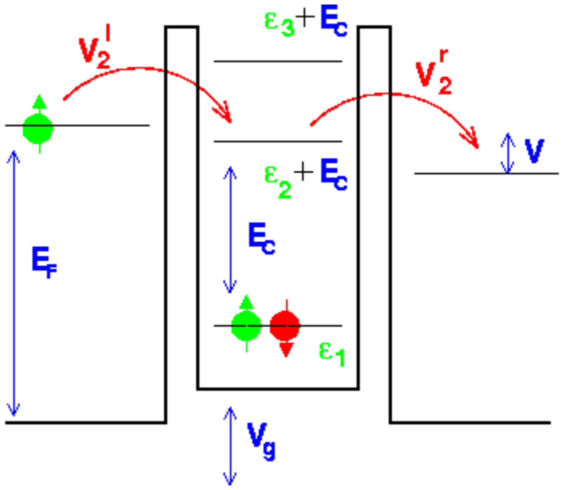
method

Quantum Dot

By further reduction in a dimension, well-controlled electron transport is achieved :



Top view



* http://www.fkf.mpg.de/metzner/research/qdot/qdot.html

Nanofabrication - Photolithography

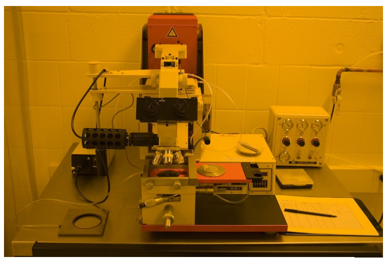
Photolithography :

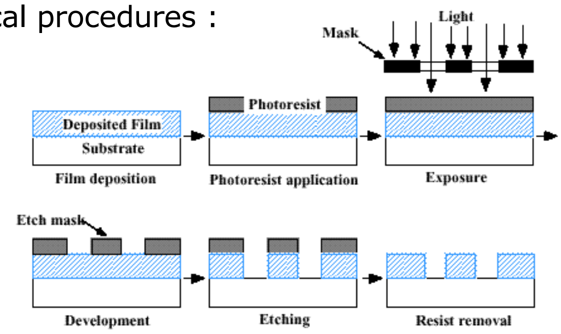
Typical wavelength :

Hg lamp (g-line: 436 nm and i-line: 365 nm),
KrF laser (248 nm) and ArF laser (193 nm)

Resolution : > 50 nm (KrF / ArF laser)

Typical procedures :





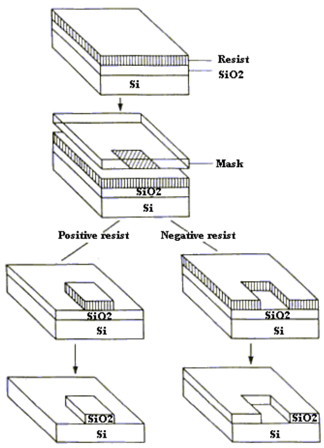


Photo-resists :

negative-type (SAL-601, AZ-PN-100, etc.)
positive-type (PMMA, ZEP-520, MMA, etc.)

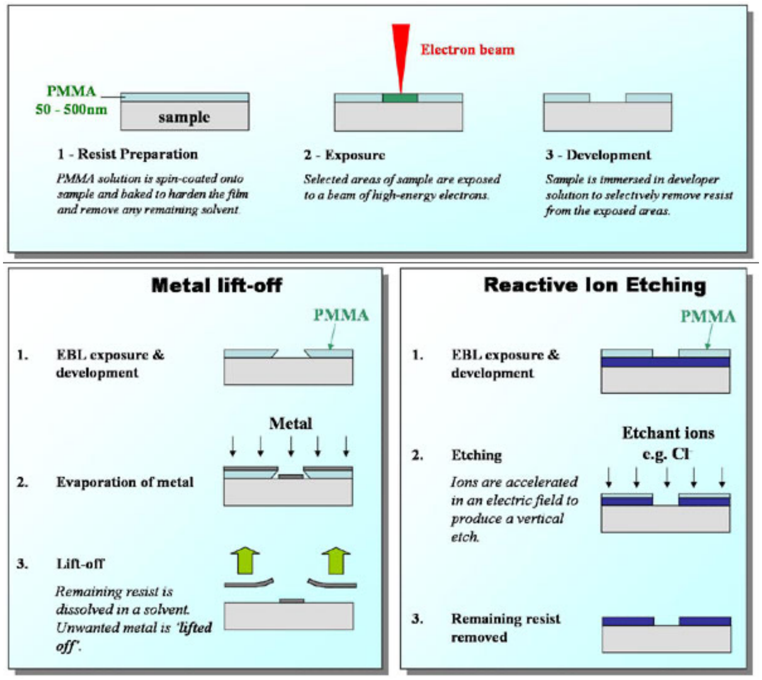
* http://www.wikipedia.org/
** http://www.hitequest.com/Kiss/VLSI.htm
*** http://www.ece.gatech.edu/research/labs/vc/theory/photolith.html

Nanofabrication - Electron-Beam Lithography

Electron-beam lithography :

Resolution : < 10 nm

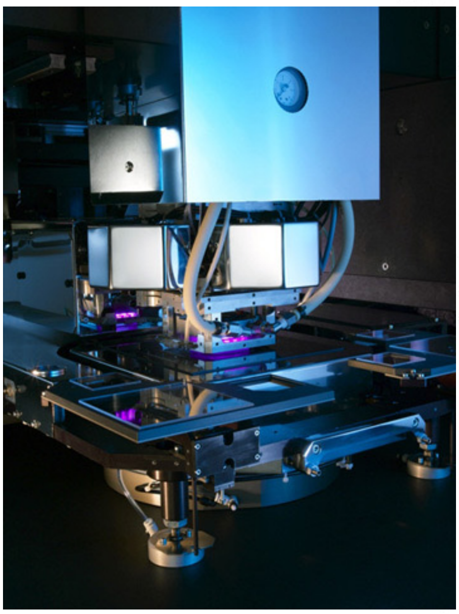
Typical procedures :

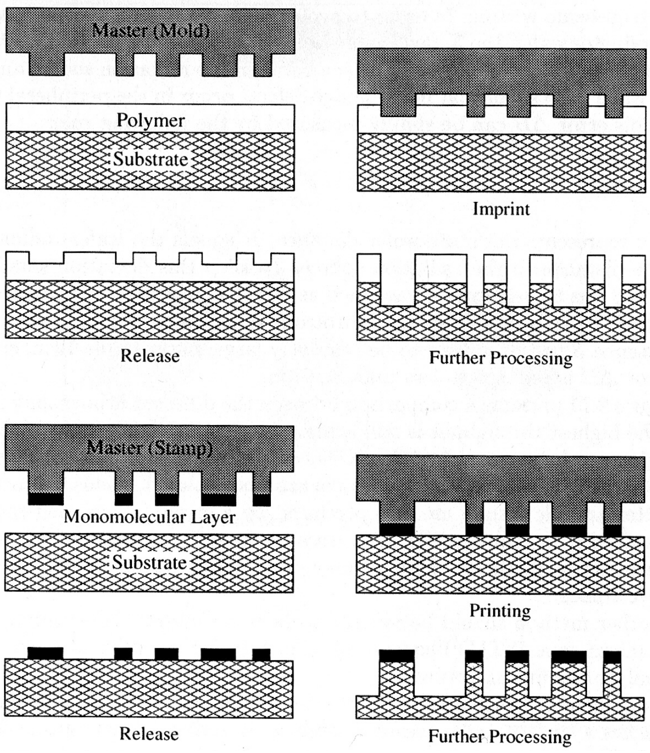


* http://www.shef.ac.uk/eee/research/ebl/principles.html

Nanofabrication

Nano-imprint :



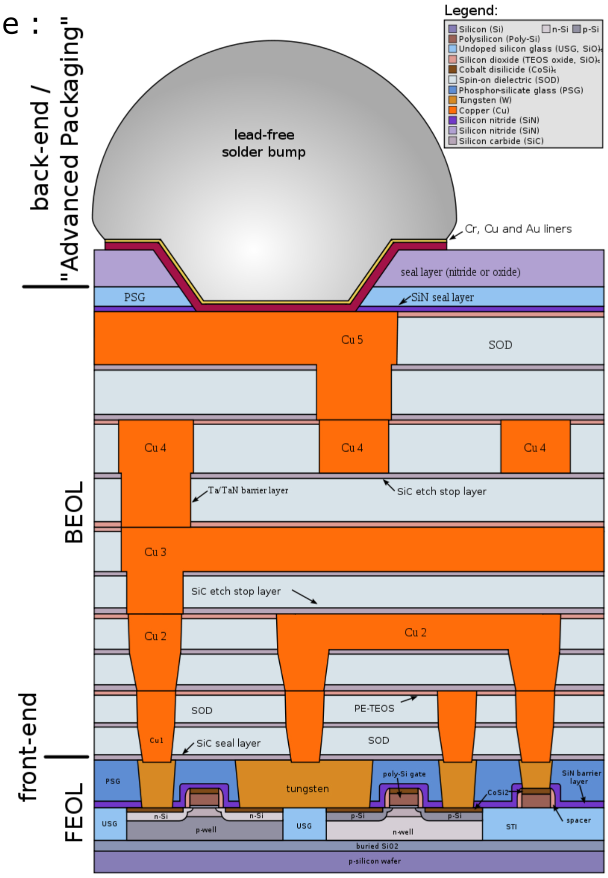


* http://www.leb.eei.uni-erlangen.de/reinraumlabor/ausstattung/strukturierung.php?lang=en

** K. Goser, P. Glosekotter and J. Diestuhl, *Nanoelectronics and Nanosystems* (Springer, Berlin, 2004).

Current Process Technology

Front / back end of line :



* http://www.wikipedia.org/

Next-Generation Nanofabrication

Development of nanofabrication techniques :

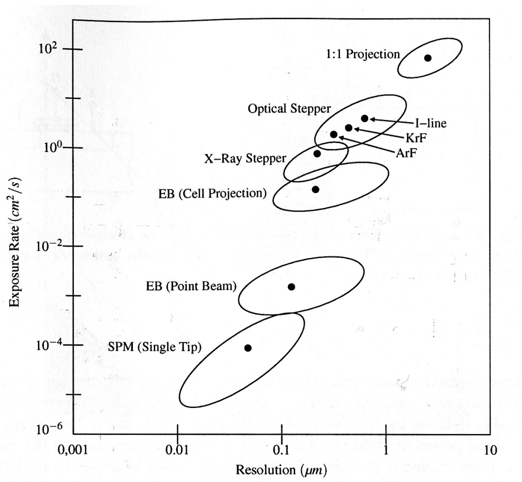
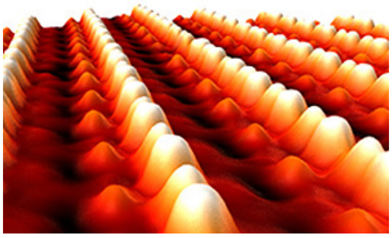


Fig. 2.11. Comparison of different lithography techniques from the production point of view

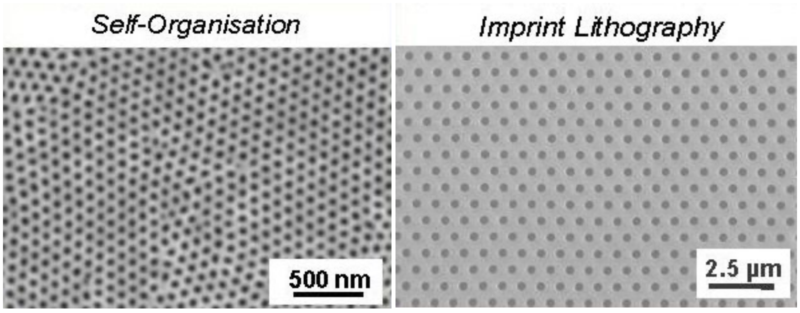
* K. Goser, P. Glosekotter and J. Diestuhl, *Nanoelectronics and Nanosystems* (Springer, Berlin, 2004).

Self-organisation (self-assembly) without any control :

Pt nano-wires



Polymer



* http://www.omicron.de/
** http://www.nanomikado.de/projects.html