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Letter

On the application, estimation and interpretation of coherence and pooled coherence

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Coherence and related functions are powerful tools in the study of neurophysiological data. Amjad et al. (1997) introduced two measures which allow an arbitrary number of independent coherence functions to be evaluated for statistically significant differences and combined into a single population measure. This latter measure they termed 'pooled coherence'. Baker (2000) presents an example of the application of pooled coherence to a population of simulated data which does not describe accurately the correlation structure within the data.

The estimation of pooled coherence, as defined by Eq. (2.11) in Amjad et al. (1997) and Eq. (1) in Baker (2000), is equivalent to joining the separate records into a single long record and estimating the coherence for this single pooled record using a periodogram approach (e.g. Halliday et al., 1995). Thus, once the context of an analysis has been determined, the terms coherence and pooled coherence are interchangeable. Asymptotic confidence limits allow coherence estimates to be assessed for the presence of significant correlation at each Fourier frequency. In the case of periodogram estimates formed from disjoint sections this confidence limit can be estimated from a simple expression which depends on the number of sections averaged (Amjad et al., 1997). This approach is based on the extensively developed theory for the properties of Fourier transforms of stationary stochastic data (e.g. Brillinger, 1981). Why should a coherence estimate and confidence limit which are appropriate for single records fail to describe the correlation structure in a single record formed from 100 separate sections (Baker 2000)?

Consider a plot of the two simulated signals constructed in Baker (2000), each consisting of 100 sections, with the amplitude of each section scaled by a normally distributed factor of 10 ± 8 (mean ± 2 SD). The fine detail will be lost, but the two traces will exhibit step changes in their envelope at transition points between sections. In statistical terms the data exhibits a nonstationarity, in fact nonstationarity is built into the construction of such data. Second order stationarity (invariance of second order moments) is one of the assumptions underlying the application of coherence analysis (e.g. Brillinger, 1981). As described below, it is the violation of the assumption of stationarity which underlies the result reported in Baker (2000).

The analogy between spectrum analysis and analysis of variance has long been recognized (e.g. Tukey, 1961). The spectrum of a signal can be interpreted as providing a measure of the fraction of the total variance (power) at a Fourier frequency. Scaling the amplitude of different sections of data by a constant is equivalent to scaling the variance of each signal resulting in an increase or decrease in the relative power at each frequency. Following this line of reasoning we can see that a pooled coherence estimate (calculated from pooled cross and auto spectra) will be dominated by the sections with the largest variance, and the sampling fluctuations will be determined mostly by the subset of sections with the largest variance. A confidence limit

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constructed on the basis of the total number of sections is not appropriate for such a pooled coherence estimate, and will give misleading results.

In many situations it is not possible to control the magnitude of the signals being pooled, resulting in non constant standard deviation (SD) between different sections to be pooled. In a pooled coherence study involving tremor recordings from 13 subjects with RMS values of 1–37 cm/s^2, Halliday et al. (1999) propose normalising the RMS value or SD between records prior to estimating pooled parameters, Baker (2000) reiterates this suggestion. In situations where there is considerable variation in the SD between records, this corrective action is advisable prior to a pooled analysis.

Coherence is defined as the magnitude squared of the coherency. If we denote the coherency estimate between two processes a and b as $\hat{R}_{ab}(\lambda)$, the variance of the Fisher transform, Tan h^{-1}, of this can be estimated as

$$\mathrm{var}\{\mathrm{Tan}\ h^{-1}\ |\hat{R}_{ab}(\lambda)|\} \approx \frac{1}{2L} \quad (1)$$

where L is the number of periodograms averaged to estimate the coherency function (Brillinger, 1981). It is this result which underpins the difference of coherence (Rosenberg et al., 1989) and extended difference of coherence test (Amjad et al., 1997), allowing independent coherence estimates to be compared in a rigorous statistical manner. The evaluation of the extended difference of coherence requires an estimate of the common mean of the Fisher transformed coherency estimates for individual records, $\bar{z}$, see Eq. (2.8) in Amjad et al. (1997). This quantity can be transformed back to the domain of coherence: (Tan h $(\bar{z}))^2$, and used as an alternative estimate of the pooled coherence. Since this estimate is based on pooling coherency rather than pooling auto and cross spectra, its use may be preferable in situations where it is not possible to correct for non constant SD of individual records (e.g. spike train data). The advantage of using the method based on pooling spectra is that it provides estimates of pooled spectra, from which a pooled time domain measure can be constructed (Amjad et al., 1997).

Baker (2000) proposes a similar approach to the problem of comparing and combining coherence estimates. This involves combining Fisher transformed coherency estimates from separate experiments (Kilner et al., 1999). It is not clear that the method set out in Kilner et al. (1999) has the statistical rigour of the extended difference of coherence and pooled coherence approach. The variance of Eq. (2) in Kilner et al. (1999) is approximately $(1/2L)^2$ and not 1 as stated, and the method violates the assumption of independent pairs of processes (see Amjad et al., 1997), without inclusion of a covariance term. The extended difference of coherence test and pooled coherence estimate set out in Amjad et al. (1997) are analogous to standard techniques for comparison of correlation coefficients for random variables derived from large samples (e.g. Rao, 1973, Ch 6). Notwithstanding the cautionary note raised in Baker (2000), they can be expected to exhibit a similar level of robustness when applied appropriately to neurophysiological data.

References

Amjad AM, Halliday DM, Rosenberg JR, Conway BA. An extended difference of coherence test for comparing and combining several independent coherence estimates — theory and application to the study of motor units and physiological tremor. J Neurosci Meth 1997;73:69–79.

Baker SN. Pooled coherence, can overestimate the significance of coupling in the presence of inter-experiment variability. J Neurosci Meth 2000;96:171–2.

Brillinger DR. Time Series — Data Analysis and Theory. San Francisco: Holden Day, 1981.

Halliday DM, Rosenberg JR, Amjad AM, Breeze P, Conway BA, Farmer SF. A framework for the analysis of mixed time series/point process data — theory and application to the study of physiological tremor, single motor unit discharges and electromyograms. Prog Biophys Mol Biol 1995;64:237–78.

Halliday DM, Conway BA, Farmer SF, Rosenberg JR. Load independent contributions from motor unit synchronization to human physiological tremor. J Neurophysiol 1999;82:664–75.

Kilner JM, Baker SN, Salenius S, Jousmäki V, Hari R, Lemon RN. Task-dependent modulation of 15–30 Hz coherence between rectified EMGs from human hand and forearm muscles. J Physiol 1999;516:559–70.

Rao CR. Linear Statistical Inference and its Applications. New York: Wiley, 1973.

Rosenberg JR, Amjad AM, Breeze P, Brillinger DR, Halliday DM. The Fourier approach to the identification of functional coupling between neuronal spike trains. Prog Biophys Mol Biol 1989;53:1–31.

Tukey JW. Discussion, emphasizing the connection between analysis of variance and spectrum analysis. Technometrics 1961;3:191–219.