Applied Biostatistics

Proportions, risk ratios and odds ratios

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Than uniciciice	Risk	difference
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Cough dur	-	-	r at ni before	-	ge 14 and
Cough at		chitis	before	2	Total
age 14	Yes		No	2	
Yes	26	9.5%	44	4.2%	70
No	247	90.5%	1002	95.8%	1249
Total	273	100.0%	1046	100.0%	1319
Want an estimate of the size of the bronchitis effect.					

Difference between proportions:

0.095 - 0.042 = 0.0539.5% - 4.2% = 5.3 percentage points.

or

Standard error for difference = 0.019, 95% CI:

0.053 - 1.96 × 0.019 to 0.053 + 1.96 × 0.019 = 0.016 to 0.090.

Cough du	ring the day bronchiti		-	ge 14 and
Cough at age 14		s before	age 5	Total
Yes	26 9.5	5% 44	4.2%	70
No	247 90.5	5% 1002	95.8%	1249
Total	273 100.0	0% 1046	100.0%	1319
Difference bet	nate of the size tween proporti - 0.042 = 0.05	ons: 3	onchitis e points.	effect.

Difference is absolute risk difference.

Risk ratio

Cough dur:	ing the day or a	at night at a	ge 14 and
	bronchitis be	fore age 5	
Cough at	Bronchitis be	fore age 5	Total
age 14	Yes	No	
Yes	26 9.5%	44 4.2%	70
No	247 90.5%	1002 95.8%	1249
Total	273 100.0%	1046 100.0%	1319

Want an estimate of the size of the bronchitis effect.

Proportion who cough is called the $\ensuremath{\textit{risk}}$ of cough for that population.

Difference is absolute risk difference.

Risk ratio = 0.095/0.042 = 2.26.

Also called relative risk, RR.

Risk ratio

Cough during the day or at night at age 14 and				
	bronchitis	before age 5		
Cough at	Bronchitis	before age 5	Total	
age 14	Yes	No		
Yes	26 9.5%	44 4.2%	70	
No	247 90.5%	1002 95.8%	1249	
Total	273 100.0%	1046 100.0%	1319	

Risk ratio = 0.095/0.042 = 2.26.

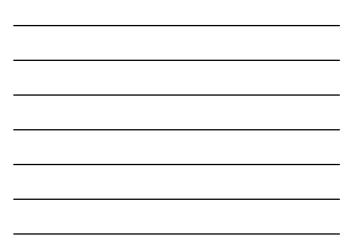
Because risk ratio is a ratio, it has a very awkward distribution.

If we take the log of the risk ratio, we have something which is found by adding and subtracting log frequencies.

The distribution becomes approximately Normal.

Provided frequencies are not small, simple standard error.

Risk ratio					
Cough dur	ing the day or at n bronchitis befor	night at age 14 and e age 5			
Cough at age 14		e age 5 Total No			
Yes	26 9.5% 44	4.2% 70			
No	247 90.5% 1002	2 95.8% 1249			
Total	273 100.0% 1046	5 100.0% 1319			
Risk ratio = 0.095/0.042 = 2.26.					
$\log_{e}(RR) = 0.81$	7.				
SE for $\log_{e}(RR)$	= 0.238.				
95% CI for log _e (RR) = 0.817 - 1.96× 0.238 to 0.817 + 1.96 × 0.238 = 0.351 to 3.607.					
95% CI for RR =	exp(0.351) to exp(1	.283) = 1.42 to 3.61.			



Risk ratio

Cough during the day or at night at age 14 and bronchitis before age 5				
Cough at	Bronchitis	before age 5	Total	
age 14	Yes	No		
Yes	26 9.5%	44 4.2%	70	
No	247 90.5%	1002 95.8%	1249	
Total	273 100.0%	1046 100.0%	1319	

log(RR) = 0.817, 95% CI = 0.351 to 1.283.

Risk ratio = 2.26, 95% CI = 1.42 to 3.61.

RR is not in the middle of its confidence interval.

The interval is symmetrical on the log scale, not the natural scale.

OddsCough 26 9.5%TotalBronchitis26 9.5%247 90.5%273 100%Risk of cough = 26/273 = 0.095Odds of cough = 26/247 = 0.105Risk = number experiencing event divided by number who could.Odds = number experiencing event divided by number who did not experience event.

Odds

Cough No cough Total Bronchitis 26 9.5% 247 90.5% 273 100%

Risk of cough = 26/273 = 0.095

Odds of cough = 26/247 = 0.105

Risk: for every child, 0.095 children cough, for every 100 children, 9.5 children cough.

Odds: for every child who does not cough, 0.105 children cough, for every 100 children who do not cough, 10.5 children

cough.

Odds ratio

Cough during the day or at night at age 14 and bronchitis before age 5				
Cough at	Bronchitis	before age 5	Total	
age 14	Yes	No		
Yes	26 9.5%	44 4.2%	70	
No	247 90.5%	1002 95.8%	1249	
Total	273 100.0%	1046 100.0%	1319	

Odds of cough given bronchitis: 26/247 = 0.105.

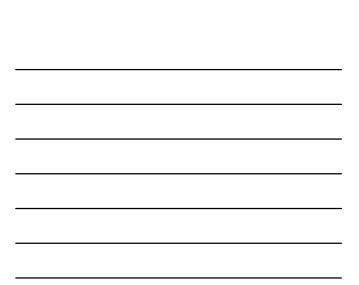
Odds of cough given no bronchitis: 44/1002 = 0.044.

Odds ratio = (26/247)/(44/1002) = 0.105/0.044 = 2.397.

For every child who does not cough, 2.397 times as many will cough with a history bronchitis as will cough with no history of bronchitis.

Odds ratio					
Cough dur		r at night at before age 5	age 14 and		
Cough at age 14	Bronchitis b Yes	pefore age 5 No	Total		
Yes	26 9.5%	44 4.2%	70		
No	247 90.5%	1002 95.8%	1249		
Total	273 100.0%	1046 100.0%	1319		
Odds ratio, OR = (26/247)/(44/1002) = 2.397.					
Like RR, OR ha odds ratio.	s an awkward c	distribution. We	e use the log		
The distribution	becomes appro	oximately Norm	al.		
Provided freque	Provided frequencies are not small, simple standard error.				

Odds ratio Cough during the day or at night at age 14 and bronchitis before age 5 Cough at Bronchitis before age 5 Total age 14 Yes No 26 9.5% 247 90.5% 273 100.0% 44 4.2% 1002 95.8% 1046 100.0% Yes 70 No 1249 Total 1319 Odds ratio, OR = (26/247)/(44/1002) = 2.397. $\log_{e}(OR) = 0.874$ $SE \log_{e}(OR) = 0.257$ 95% CI for log_e(OR) = 0.874 - 1.96 × 0.257 to 0.874 + 1.96 × 0.257 = 0.370 to 1.379. 95% CI for OR = exp(0.370) to exp(1.379) = 1.45 to 3.97.



Odds ratio

Cough during the day or at night at age 14 and bronchitis before age 5				
Cough at	Bronchitis	before age 5	Total	
age 14	Yes	No		
Yes	26 9.5%	44 4.2%	70	
No	247 90.5%	1002 95.8%	1249	
Total	273 100.0%	1046 100.0%	1319	

 $\log_{\rm e}({\rm OR})$ = 0.874, 95% CI = 0.370 to 1.379.

OR = 2.397, 95% CI = 1.45 to 3.97.

OR is not in the middle of its confidence interval.

The interval is symmetrical on the log scale, not the natural scale.

Odds ratio			
Cough dur	ing the day or bronchitis	r at night at before age 5	age 14 and
Cough at age 14		efore age 5 No	Total
Yes	26 9.5%	44 4.2%	70
No	247 90.5%	1002 95.8%	1249
Total	273 100.0%	1046 100.0%	1319
Odds ratio for co	ough = (26/247)	/(44/1002) = 2.	.397.
Doesn't matter w	which way round	d we do it.	
Odds ratio for br	onchitis =(26/4	4)/(247/1002)	= 2.397.
Both OR = (26×	1002)/(44×247)	= 2.397.	
Ratio of cross p	roducts.		
Not true for relat	ive risk.		

Odds ratio					
Cough du		or at night at before age 5	age 14 and		
Cough at	Bronchitis	before age 5	Total		
age 14	Yes	No			
No	247 90.5%	1002 95.8%	1249		
Yes	26 9.5%	44 4.2%	70		
Total	273 100.0%	1046 100.0%	1319		
Switching the rows or columns inverts the odds ratio.					
Odds ratio for no cough given a history of bronchitis: OR = (247/26)/(1002/44) = 0.417 = 1/2.397.					
There are only	two possible or	dds ratios.			

On the log scale, equal and opposite.

 $\log_{e}(2.397) = 0.874, \, \log_{e}(0.417) = -0.874.$