Applying the Right Statistics: Analyses of Measurement Studies

J Martin Bland and Douglas G Altman

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Abstract

The study of measurement error, observer variation and agreement between different methods of measurement are frequent topics in the imaging literature. We describe the problems of some applications of correlation and regression methods to these studies, using recent examples from this literature. We use a simulated example to show how these problems and misinterpretations arise. We describe the 95% limits of agreement approach and a similar, appropriate, regression technique. We discuss the difference versus mean plot, and the pitfalls of plotting difference against one variable only. We stress that these are questions of estimation, not significance tests, and show how confidence intervals can be found for these estimates.

Introduction

Many research papers in imaging concern measurement. This is a topic which in the past has been much neglected in the medical research methods literature. In this paper we discuss the estimation of the agreement between two methods of measurement, and the estimation of the agreement between two measurements by the same method, also called repeatability. In both cases we shall be concerned with the question of interpreting the individual clinical measurement. For agreement between two different methods of measurement, we shall be asking whether we can use measurements by these two method interchangeably, i.e. can we ignore the method by which the measurement was made. For two measurements by the same method, we shall be asking how variable measurements on a patient can be if the true value of the quantity does not change and what this measurement tells us about the patient's true or average value. In some studies repeated observations are made by the same observer or many different observers and are treated as repeated observations of the same thing. In others a small number of observers, often two, are used and systematic differences between them are explored. In this case the analysis is like that for comparing two different methods of measurement.

We shall avoid all mathematics, except for one formula near the end. Instead we shall show what happens when we apply some simple statistical methods to a set of randomly generated data, and then show how this informs the interpretation of these methods when they are used to tackle measurement problems. We shall illustrate these methods by examples drawn from the imaging literature. For some of these examples, rather than bother the original authors for their data, we have digitised them approximately from the published graphs, and our figures differ slightly but not in any important way from those originally published.

We shall start with a typical example of a measurement study. Borg *et al.* [1] compared single X-ray absorptiometry (SXA) with single photon absorptiometry (SPA). They produced a scatter plot for arm bone mineral density similar to Figure 1. This looks like good agreement, with a tight cloud of points and a high value for the correlation coefficient, r=0.98. The points cluster quite closely around the line drawn through them, the regression line. But should this make us think we could use bone mineral densities measured by SXA and SPA interchangeably? In Figure 2 we have added the line of equality, the line points would lie on if the two measurements were the same. Nearly all the points lie to the left of the line of equality. There is a clear bias: the SXA measurements tend to exceed the SPA measurements by 0.02 g/cm^2. In this paper we shall explain why the correlation coefficient does not reflect this bias and go on to explore the interpretation of the regression line. To do this we shall show what happens when we apply these methods to artificially generated data, where we know what the interpretation should be. We shall then describe a simple alternative approach, limits of agreement, which avoids these problems.

The interpretation of correlation coefficients

To illustrate the interpretation of correlation, we shall start with some artificial, randomly generated data. This is not because we do not have any real data, but because with randomly generated data we know the answers to our questions. We generated 100 observations from a Normal distribution to represent the true value of the quantity being measured. Now, we want to compare two different measurements of this true quantity. We generated two measurements, X and Y, by adding to the true value some measurement error, from a Normal distribution and independent of the true value.

This gave us artificial data which represent two observations on each of a group of subjects. These observations might be measurements by two different methods, by the same method but different observers, or by the same method and the same observer. We know that they are closely related and that there is no consistent bias or tendency for X to be greater or less than Y. Figure 3 shows the artificial data, with the line of equality.

A natural approach to data like those of Figure 3 is to calculate a correlation coefficient. For these data the correlation between X and Y is r=0.86, P<0.0001. There are two problems with this analysis. First, correlation depends on the range and distribution of the variables and hence on the way the sample of subjects was chosen.

Second, correlation ignores any systematic bias between the two variables. To show that correlation depends on the range of the variables, we consider only subjects in a restricted range. For subjects whose true measurement would be 10 or less we get *r*=0.60 and for subjects whose true measurement would be greater than 10 we get *r*=0.62 (Figure 4). Both of these are less than *r*=0.86 for all the data. If we take several pairs of measurements on the same subject, the correlation can be zero but we should not conclude from this that the two methods do not agree. It is what we should expect, because there is no variation at all in the true value. (Altman and Bland [2] discuss this and give an example.) So the correlation coefficient depends on the group of subjects selected. It should be used only if we have a representative sample of the patient population we wish to study. In measurement studies, we often have samples chosen to contain far more subjects with extreme values than would a representative sample. This is often a desirable feature in such studies, as we want to explore the differences between methods over the full range of measurements, but it affects the interpretation of some statistics, including correlation coefficients. Figure 5 shows an example comparing final fetal weight estimated by three-dimensional ultrasound with actual birthweight.[3] The sample contains far more small birthweights than a representative sample of births would do. The actual birthweight has mean 2930g and standard deviation 1050g, compared to mean 3300g and standard deviation 560g observed in an unselected UK sample. [4] It is thus much more variable than a representative sample and would produce a higher correlation coefficient.

The second problem with correlation is that it looks at the degree of association, not agreement. If we have a third measurement Z, obtained by adding 2.0 to X, this will consistently overestimate the true value by 2 units. However, the correlation of Y with Z is the same as its correlation with X, 0.86 (Figure 6). The correlation between Y and Z is the same as between Y and X, but the agreement is not. Thus high correlation does not imply close agreement, it is blind to the possibility of bias. The clear bias in Figure 2 is a good example. (In a real example we would not know which method is biased compared to the true value, of course.)

Bias can be very large indeed. For example, Bakker *et al.* [5] investigated the agreement between renal volume measurements by ultrasound and magnetic resonance imaging. Their data for 40 kidneys are shown in Figure 7, in the form which they used. There is a clear and significant mean difference of about 25% between the two methods. A correlation coefficient would completely miss this difference and thus be highly misleading. We think that the consistent difference is even clearer if the data are presented as a scatter diagram (Figure 8).

If we are dealing with intra-observer variation using the same method of measurement, where the repeated observations are made by the same observer on the same subject, there should not be any consistent bias. We can then use correlation, again provided the sample is representative. But if we are comparing two different methods of measurement, there may well be a consistent bias, and the correlation coefficient could be quite misleading. Correlation will tell us something about whether the two methods are measuring the same underlying quantity, i.e. about the validity of the two methods, but not about their agreement and whether they can be used interchangeably.

Correlation is thus inappropriate for the study of agreement between different methods of measurement. Despite this, people do it.

Regression lines

Some applications of regression are also inappropriate. It is often thought that, as the data should cluster around the line of equality for good agreement, the regression line should be similar to the line of equality. This is not so. For the randomly generated data, which do lie about the line of equality, the regression line of Y on X is shown in Figure 9. The equation is Y = 1.49 + 0.84 X. The regression line does not coincide with the line of equality, which has equation Y = 0.0 + 1.0 X. It does not go through the origin and its slope is less than one. The 95% confidence interval (CI) for the slope is 0.74 to 0.94. The slope is therefore significantly different from 1.0. Similarly, the intercept has 95% CI 0.51 to 2.47 and is significantly different from 0.0. The cause of the discrepancy is that regression attempts to predict the observed Y from the observed X, not the true Y from the true X. Measurement errors in X reduce the slope of the line and so raise the lower end of the line and lower the upper end, so that the intercept is increased above zero,. Figure 9 also shows the line for the regression with X as dependent variable and Y as independent, X = 1.12 +0.89 Y. Again the slope is significantly less than one (95% CI 0.78 to 0.99) and the intercept significantly greater than zero (95% CI 0.09 to 2.15). On the scatter diagram the line is plotted with Y as dependent variable, and so its slope is shown as 1/0.89 and appears greater than 1.0, but for a method comparison a regression analysis is expected to give a slope less than 1.0.

So when we regress measurements by one method on measurements by another, we expect that the slope will be less than one and intercept greater than 0, whichever way round we do the regression. Slope less than 1.0 and intercept greater than 0.0 thus do not tell us anything. This is not always understood by researchers. For example, Bankier *et al.* [6] compared subjective visual grading versus objective quantification with macroscopic morphometry and thin-section CT densitometry in pulmonary emphysema (Table 1). These measurements were not on the same scale, but all scales had a common point at zero. Bankier *et al.* interpret this table thus: ‘All but one of the CIs did not contain zero, which is suggestive of systematic overestimation of emphysema when compared with objective measurements.’ [6] We disagree. This is what we would expect to see if there were no such bias.

Others have tested the null hypothesis that the slope is equal to 1.0. For example, Tothill *et al* [7] studied absorptiometers used for measuring total body bone and soft tissue. Table 2 shows some of their results looking at the relationship between the known density in a phantom (artificial model) with the measured density. The tests of significance are irrelevant and unnecessary, because the null hypotheses of zero intercept and unit slope are not expected to be true. Tothill *et al* [7] also cite Prior *et al* [8] who compared body composition measurements by a Hologic QDR1000W with a four-compartment hydrodensiometry model. The regression equation of per cent fat for the four compartment model (y) on percent fat measured by the Hologic 1000 (x) was $y = 3.30 + 0.85x$. This was interpreted as 'the [Hologic] 1000 underestimated fat in the leanest women and overestimated it in the fattest'. [7] As we have seen, we expect the intercept to exceed zero and the slope to be less than one when there is no relationship between the error and the magnitude, as for our artificial data. This interpretation is therefore incorrect. This is a well-known instance of regression towards the mean. [9 10] We shall consider an appropriate application of regression later.

The 95% limits of agreement

When we wrote our first paper on comparing methods of measurement, [2] we were mainly concerned with exposing these potentially misleading approaches. However, we thought that we should suggest a valid method of analysis too. Our proposal was the 95% limits of agreement method. We start with the differences between measurements by the two methods, which we thought we should summarize. We calculated the mean and standard deviation of these differences. Then we calculated the mean difference plus and minus 2 (or, more precisely, 1.96) standard deviations. We then expect 95% of differences between measurements by the two methods to lie between these limits.

For the artificial X Y data of Figure 3, the differences Y−X have mean −0.06 and standard deviation 1.46. Hence the 95% limits are $-0.06 - 1.96\times1.46 = -2.92$ and $-0.06 + 1.96\times1.46 = 2.80$. Hence we can say that for 95% of individuals a measurement by Method Y would be between 2.92 units less than a measurement by Method X and 2.80 greater. We thought that this approach was so obvious and so clearly answering the question as to need no justification. We therefore did not go into detail. In a later paper [11] we elaborated the idea and gave a worked example.

For a recent practical example, Cicero *et al.* [12] compared cervical length at 22-24 weeks of gestation measured by transvaginal and transperineal-translabial ultrasonography. Their data are shown in Figure 10. The limits of agreement were quoted as −5.8 mm to 6.1 mm. [12] Interestingly, these authors also quoted r=0.934, P<0.0001, and Figure 10 shows the regression line, not the line of equality. [12] Old habits die hard.

How small the limits of agreement should be for us to conclude that the methods agree sufficiently is a clinical, not statistical decision. This decision should be made in advance of the analysis.

The 95% limits of agreement depend on some assumptions about the data, that the mean and standard deviation of the differences are constant throughout the range of measurement, and that these differences are from an approximately Normal distribution. To check these assumptions we proposed two plots: a scatter diagram of the difference against the average of the two measurements and a histogram of the differences. [2] For the X-Y data, these are shown in Figure 11. The mean and standard deviation appear uniform through the range of measurement and the differences appear to follow a Normal distribution, as they were artificially generated to do.

We also suggested adding the 95% limits of agreement and the mean difference to the scatter plot, as shown in Figure 12. About 95% of points should lie within the limits. In Figure 12, 93% are within the 95% limits and 7% outside. Cicero *et al.* [12] showed such a plot (Figure 13). In this graph there are many overlapping points, and there are in fact 15/231 = 93.5% of the points within the 95% limits.

To our chagrin, the histogram does not seem to have been adopted with the same enthusiasm, but the scatter plot alone is a reasonable check. Also to our chagrin, many researchers seem to think that the plot is the analysis. It is not, of course, but only a check on the assumptions of the limits of agreement.

The assumptions are not always met and checking is essential. In their study of fetal lung volume measurement using three-dimensional·ultrasound, Bahmaie *et al.* [13] produced a difference against mean plot for measurements by two different observers (Figure 14). This shows a divergence as the magnitude increases, making the limits of agreement suspect. They would be too wide for small measurements and too narrow for large ones. Often the differences increase in size proportionally to the size of the measurement. We can resolve this difficulty by analysing the logarithm of the measurement rather than the measurement itself. This leads to limits of agreement in the form of proportions of the measurement rather than in the original units. [11] Another, similar solution is to find the 95% limits for the difference as a percentage of the average of the two methods. [14]

The reason for plotting the difference against the average, rather than either of the measurements singly, is that when there is no real relationship between difference and magnitiude, Y−X and X will be negatively correlated. Subjects for whom the X measurement is larger than their true value will tend to have negative Y−X differences, while subjects for whom the X measurement is smaller than their true value will tend to positive Y−X differences. The subjects with the largest X measurements are likely to include those whose X measurement is above the true value, subjects with small X are likely to include those with X below the true value. Hence Y−X will go down as X goes up.

Similarly, Y−X and Y will be positively correlated. However, when X and Y have the same standard deviation, as they should if they are measurements of the same thing, Y−X and Y+X should not be correlated at all in the absence of a true relationship. [15] For the X−Y data, where is no relationship between difference and magnitude, these correlations are:

	Correlation with difference Y−X	
Average of X & Y	−0.06	P=0.6
X	−0.31	P=0.002
Y	+0.21,	P=0.04

In this example, Y−X and X are negatively correlated and Y−X and Y are positively correlated, and these correlations are both statistically significant. By contrast, Y−X and the average of Y and X have a very small correlation, which is not significant. We can see this in the plots of difference against X and against Y (Figure 15).

Plots of difference against one measurement can be seriously misleading. Figure 5 shows final fetal weight estimated by three-dimensional ultrasound against actual birthweight. [3] There is no evidence of any tendency for the points to divert from the line of equality in the scatter diagram. The authors also plotted the percentage error (100×(ultrasound-actual)/actual, essentially the difference) against the actual birthweight (Figure 16). The downward trend in the graph is readily apparent. If there were really a relationship such that ultrasound over-estimated for low actual weights and underestimated for high ones, as Figure 16 suggests, this would be apparent in Figure 5 also. It is not. Consider the following from a comparison of MR with conventional arthrography:

> 'For all parameters, the difference between the measurements of the two modalities depended on the magnitude of the measurements. By using arthrography as the standard, a slope test indicated overestimation with MR imaging at small measurements and an underestimation at large measurements (all P<0.001).' [16]

If the difference MR minus arthrography were regressed on arthrography, we would expect them to be negatively related even in the absence of a true relationship between difference and magnitude. [15]

The 95% limits of agreement method has been widely cited and quite widely used, [17] though many citers do not appear to have read the paper. For example, in the MR vs arthrography study cited above, the authors state in the methods section:

> 'For each parameter, agreement between MR imaging and arthrography was investigated using the method of Bland and Altman [1986]. Arthrography was considered to be the standard and differences between methods were calculated and plotted. A slope test was used to assess whether these differences varied systematically over the range of measurements.'[16]

The results section of the paper contains no limits of agreement, but rather correlation and rank correlation coefficients with P values! As for plotting difference against a standard measurement, we actually wrote:

> 'It would be a mistake to plot the difference against either value separately because the difference will be related to each, a well-known statistical artefact.' [11]

Appropriate use of regression

We mentioned earlier that there is an appropriate use of regression in the evaluation of agreement. This is particularly useful when the two methods of measurement have different units, as in the study of subjective visual grading versus objective quantification with thin-section CT densitometry [6] described above. In this case we could not simply replace a measurement by one method with a measurement by the other, as they are not measuring the same quantity. However, we could predict what the measurement by the old method would be given the new method. If this method agrees well with the old method measurement, then the two methods give similar information and we could replace the new by the old. We start by regressing the measurement by the old method on the measurement by the new. We can use this regression equation to estimate a predicted old method measurement for any observed value by the new method. Of course, this will gives the mean old method value for subjects with this new method value; it does not take the variation between subjects into account. We take this in account by calculating a range of possible values for the old method value on this subject, called a 95% prediction interval. This gives us something akin to the limits of agreement. The problem is that it is not constant, being smallest near the middle of the range and wider as we get further towards the extremes. This effect is quite marked for small samples, but not for large. For the simulated X Y data, regarding X as the old or standard method and Y as the new, we get Figure 17. Here the spreading out is very small and hard to see. The average width of this prediction interval is 5.7, 2.85 on either side of the prediction. This is very similar to the width of the 95% limits of agreement, −2.92 to 2.80. If the 95% prediction interval has the width which we would find acceptable in the 95% limits of agreement, we could switch to the new method of measurement.

Measurements made using the same method

Repeated measurements may be made by a single observer using the same method to investigate the measurement error, intra-observer variation, or by different observers to investigate the variation between them, inter-observer variation. Inter-observer variation is a complex subject and we shall not go into details here. We may have two observers and analyse them as if they were two different methods of measurement, with the potential for a consistent bias between them, as in Figure 14. We may have many observers and estimate the variance between them.

Repeated measurements by the same method can be analysed in a manner analogous to the limits of agreement. The main difference is that the average difference should be zero, which simplifies things. If there is a consistent difference, between the first and second measurement for example, these are not true replicates of the same measurement and we cannot use them to investigate measurement error. Because there is no consistent bias, correlation can be used in the analysis of such data, provided there is a population from which the sample can be regarded as a representative sample. This is often not the case in the study of clinical measurements, where samples are often chosen to include more subjects with extremely high or low values than would a representative sample, but it is often the case in the study of measurements derived from questionnaire scales as found in psychology. In this case, the correlation coefficient is a measure of the information content of the measurement. [18] However, even when it is appropriate the correlation coefficient does not help us to interpret a clinical measurement on a given patient. To do this we need to consider the variability between repeated measurements on the same subject. If we calculate the standard deviation of the differences between pairs of repeated measurements, we can calculate 1.96 times this standard deviation. This gives the repeatability coefficient, which is the difference which will be exceeded by only 5% of pairs of measurements on the same subject. [19] It is thus directly comparable to the 95% limits of agreement. Thus we can use this to compare the agreement which a new method of measurement would have with a standard method, with the agreement which the new method would have with itself.

In our 1986 paper we advocated a design where each method would be used twice on each subject, so that limits of agreement between the two methods and coefficients of repeatability for each method separately could be compared. We regret that this has not been more widely adopted by researchers.

Confidence intervals for the 95% limits of agreement

Another feature which we stressed in the 1986 paper was that agreement is a question of estimation, not hypothesis testing. Estimates are usually made with some sampling error, and limits of agreement are no exception. We showed how to estimate confidence intervals for the limits of agreement. Another regret is that these confidence intervals are seldom quoted. For the data of Cicero *et al.*, [12] the mean difference was 0.2 mm with standard deviation 3.0 mm, giving 95% limits of agreements −5.8 mm to +6.1 mm. There were 231 cases. The standard error of the limits is approximately $\sqrt{3s^2/n}$. This give $\sqrt{3\times 3.0^2/231}=0.34$. The 95% confidence interval for the limits of agreement is given by ±1.96 standard errors = 0.67, so for the lower limit the confidence interval is −6.5 to −5.1 and for the upper limit the 95% confidence interval is +5.4 to +6.8. Not so hard, really!

Many studies are done using many fewer subjects than that of Cicero *et al.* [12] and confidence intervals would be much wider than these.

Conclusions

The limits of agreement approach is fundamentally very simple and direct. Provided its assumptions of uniform mean and standard deviation are met, it can be carried out by anyone with basic statistical knowledge. It provides statistics which are easy to interpret in a meaningful way. It can be extended to many more complex situations, [20] when distributions are not Normal, when difference is related to magnitude, when there are repeated measurements on the same subject, either paired or not, when there are varying numbers of observations on subjects, and there is also a non-parametric version.

Acknowledgements

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Figure 1. Arm bone mineral density measured by single X-ray absorptiometry (SXA) and single photon absorptiometry (SPA) [1]

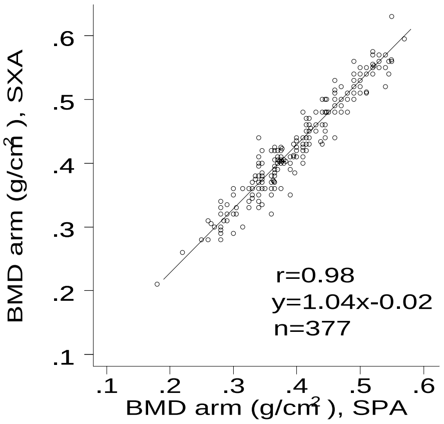

Figure 2. Data of Figure 1 with the line of equality added.

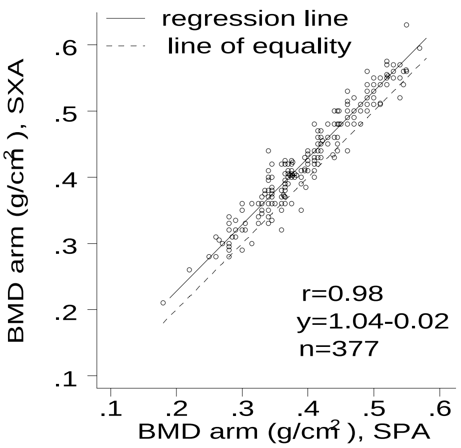

Figure 3. Artificial measurements X and Y with line of equality

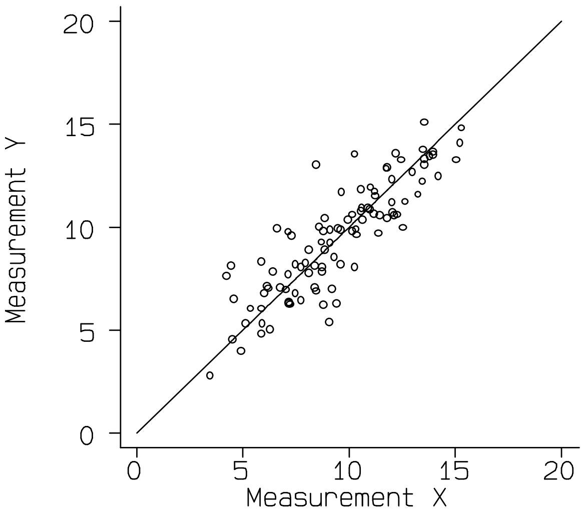

Figure 4. Artificial measurements X and Y, split by the true value above or below 10.

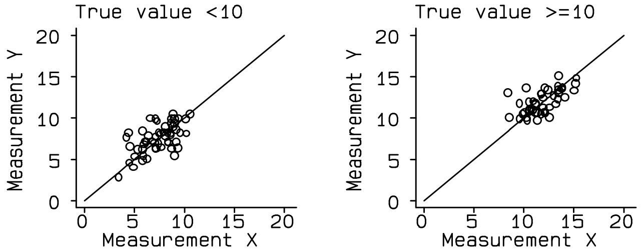

Figure 5. Final fetal weight estimated by three-dimensional ultrasound plotted against actual birth weight.[3]

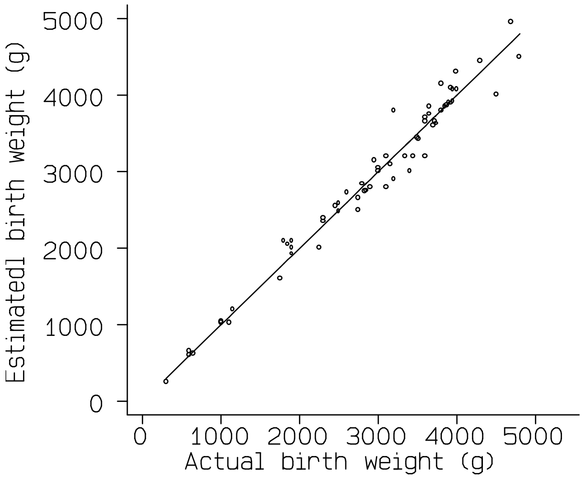

Figure 6. Artificial measurements Y and Z with the line of equality.

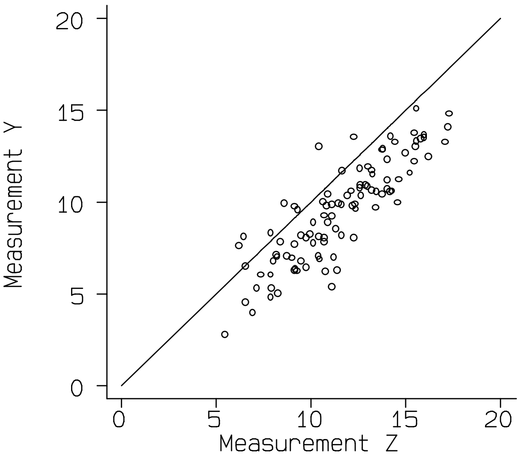

Figure 7. Renal volume measurements by ultrasound and magnetic resonance imaging [5]

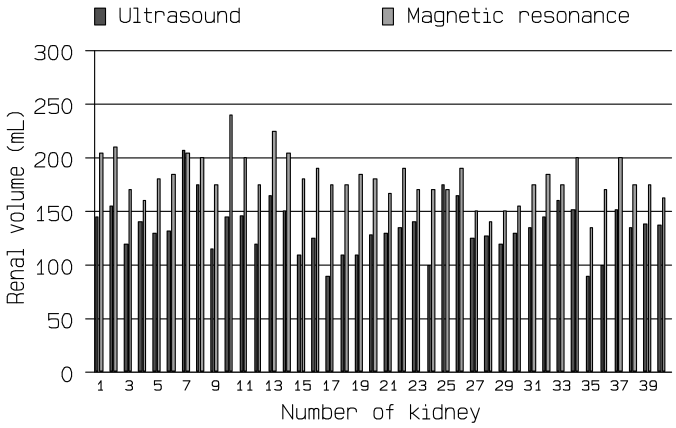

Figure 8. Renal volume measurements by ultrasound and magnetic resonance imaging [5] presented as a scatter diagram with the line of equality

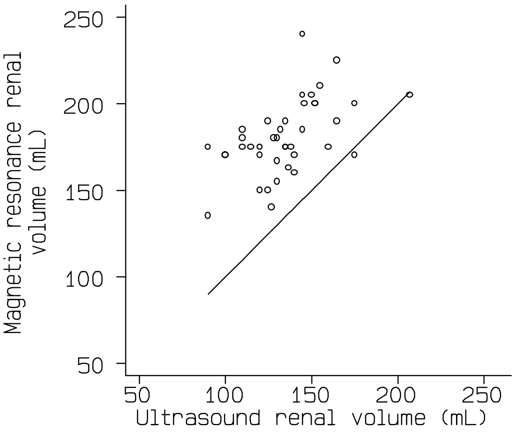

Figure 9. Artificial measurements X and Y with line of equality and both regression lines.

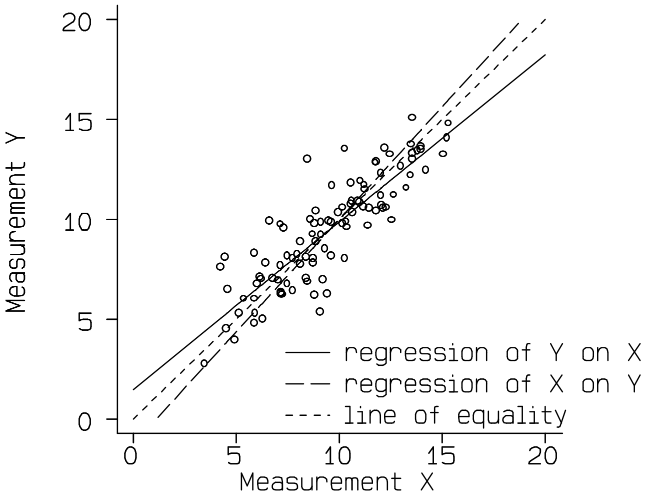

Figure 10. Cervical length at 22-24 weeks of gestation measured by transvaginal and transperineal-translabial ultrasonography [12]

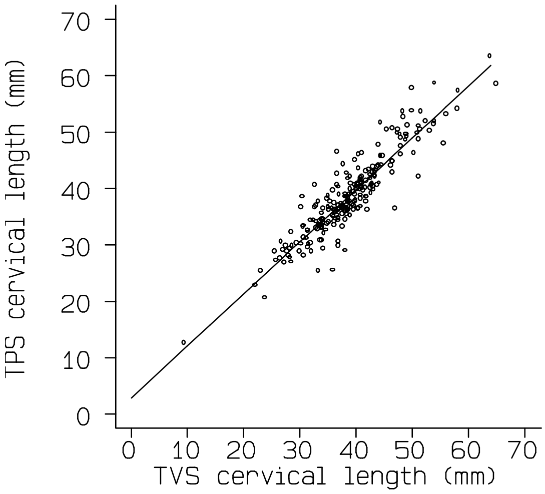

Figure 11. Plots of difference against mean and histogram of differences for the artificial data X and Y.

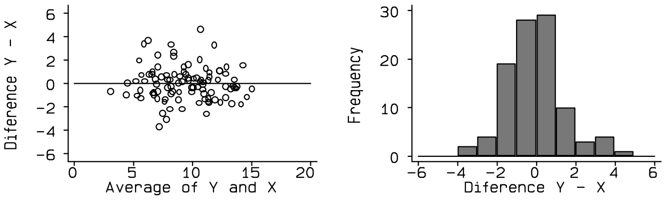

Figure 12. Plot of difference against mean for the artificial data X and Y, with mean difference and 95% limits of agreement.

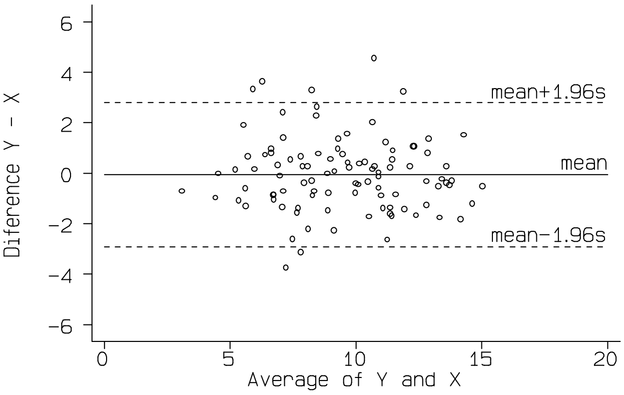

Figure 13. Difference against mean plot for cervical length measured by transvaginal and transperineal-translabial ultrasonography [12]

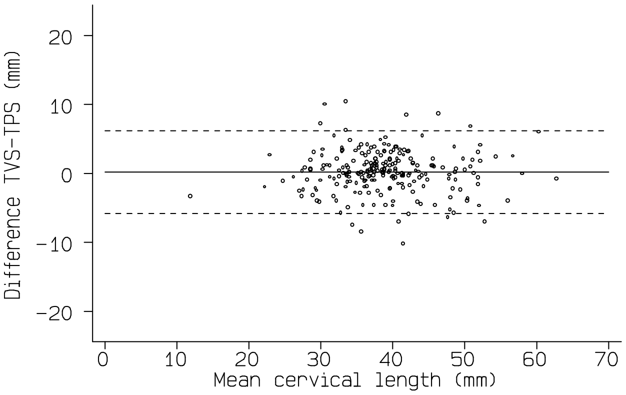

Figure 14. Difference against mean plot for measurements of fetal lung volume using three-dimensional·ultrasound by two different observers [13]

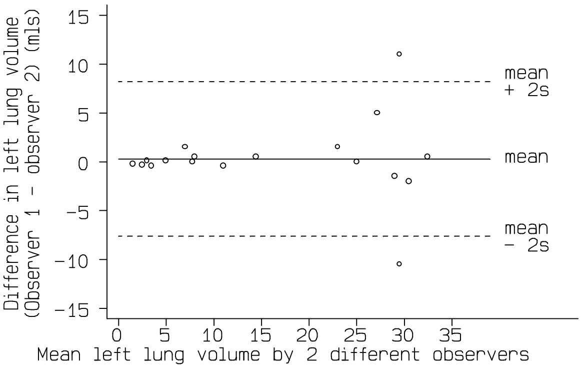

Figure 15. Plots of difference against each measurement for the artificial data X and Y, with mean difference and 95% limits of agreement.

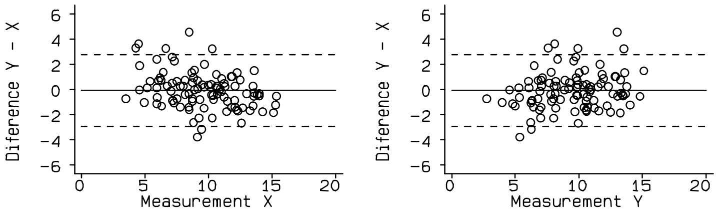

Figure 16. Fetal weight measured by three-dimensional ultrasound and actual birth weight: percentage error (PE) against the actual birthweight.[3]

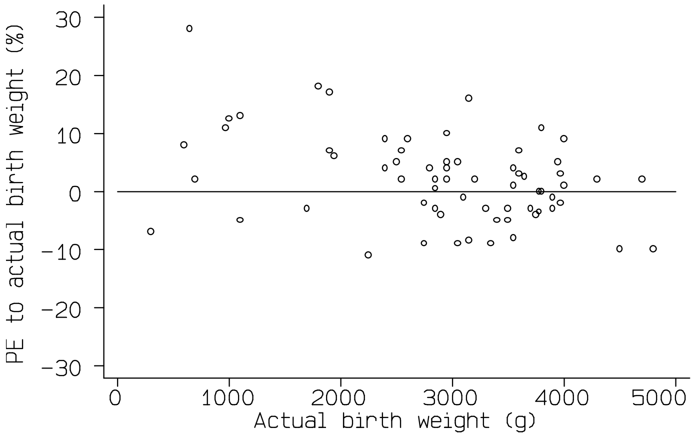

Figure 17. Regression of artificial data Y on X,with prediction limits

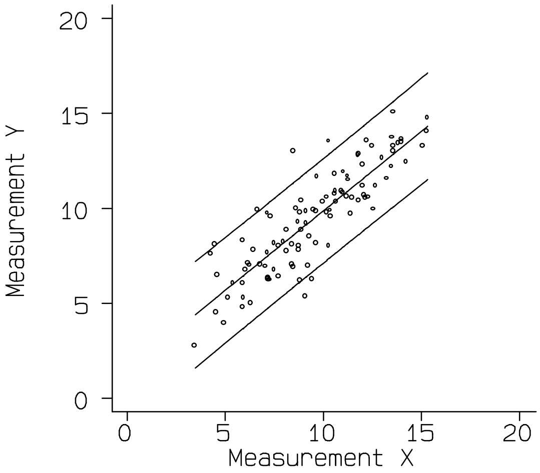

Table 1. Results of a study of subjective visual grading versus objective quantification with macroscopic morphometry and thin-section CT densitometry in pulmonary emphysema [6]

Linear Regression Results: Subjective Scores and Densitometric and Morphometric Measurements

Reader	Subjective Score and Densitometric Measurement	Subjective Score and Morphometric Measurement
1	0.350,1.059	0.629,1.365
2	−0.008,0.598	0.443,1.147
3	0.002,0.658	0.854,1.038

Note.—Data are 95% CIs for the intercepts of regression lines

Table 2. Regression equations for correlations between measured and nominal bone mineral density in hardboard plus aluminium whole body phantom [7]

		Intercept, a	Slope, b	Correlation, *r*
Hologic QDR 4500A	Legs	0.22	0.77	0.990
	Arms	0.11	0.88	0.998
	Spine	0.08	0.82	0.986
Lunar Expert	Legs	0.09	0.96	0.999
	Arms	0.05	0.99	0.999
	Spine	0.10	0.85	0.990

All intercepts (a) are significantly higher than zero and all slopes (b) are significantly lower than 1.0 ($P<0.05$)