PG Dip in High Intensity Psychological Interventions

Analyses for qualitative data

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Analyses for qualitative data

Also called nominal, categorical.

Only two categories: dichotomous, attribute, quantal, binary.

Methods:

- Chi-squared test for association
- Fisher's exact test
- Risk ratio, relative risk, rate ratio
- Odds ratio

Contingency tables

Cross tabulation of two categorical variables:

Acceptance of HIV test grouped by marital status

	Acceptance of HIV test		
Marital status	Accepted	Rejected	Total
Married	71	415	486
Living w. partner	41	181	222
Single	15	35	50
Div./wid./sep.	7	23	30
Total	134	654	788

Meadows J, Jenkinson S, Catalan J. (1994) Who chooses to have the HIV antibody test in the antenatal clinic? *Midwifery* **10**, 44-48.

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This kind of cross-tabulation of frequencies is also called a **contingency table** or **cross classification**.

Called 4 by 2 table or 4×2 table.

In general, $r \times c$ table.

Contingency tables

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Want to test the null hypothesis that there is no relationship or association between the two variables.

If the sample is large, we can do this by a chi-squared test.

If the sample is small, we must use Fisher's exact test.

The chi-squared test for association

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Null hypothesis: no association between the two variables.

Alternative hypothesis: an association of some type.

The chi-squared test for association

Acceptance of HIV test grouped by marital status

Marital status	Acceptance of HIV test: Accepted	Rejected	Total
Married	82.6		486
Living w. partner			222
Single			50
Div./wid./sep.			30
Total	134	654	788

Proportion who accepted = 134/788

Out of 486 married, expect 486 × 134/788 = 82.6 to accept if the null hypothesis were true.

The chi-squared test for association

Acceptance of HIV test grouped by marital status

Marital status	Acceptance of HIV test: Accepted	Rejected	Total
Married	82.6		486
Living w. partner			222
Single			50
Div./wid./sep.			30
Total	134	654	788

Proportion who refused = 654/788

Out of 486 married, expect 486 × 654/788 = 403.4 to refuse if the null hypothesis were true.

The chi-squared test for association

Acceptance of HIV test grouped by marital status

Marital status	Acceptance of HIV test: Accepted	Rejected	Total
Married	82.6	403.4	486
Living w. partner			222
Single			50
Div./wid./sep.			30
Total	134	654	788

Proportion who refused = 654/788

Out of 486 married, expect 486 × 654/788 = 403.4 to refuse if the null hypothesis were true.

Note that 82.6 + 403.4 = 486.

The chi-squared test for association

Acceptance of HIV test grouped by marital status

Marital status	Acceptance of HIV test: Accepted	Rejected	Total
Married	82.6	403.4	486
Living w. partner	37.8	184.2	222
Single			50
Div./wid./sep.			30
Total	134	654	788

Out of 222 living with partner, expect 222 × 134/788 = 37.8 to accept if the null hypothesis were true.

Out of 222 living with partner, expect 222 × 654/788 = 184.2 to refuse if the null hypothesis were true.

Note that 37.8 + 184.2 = 222.

The chi-squared test for association

Acceptance of HIV test grouped by marital status

Marital status	Acceptance of HIV test: Accepted	Rejected	Total
Married	82.6	403.4	486
Living w. partner	37.8	184.2	222
Single	8.5	41.5	50
Div./wid./sep.	5.1	24.9	30
Total	134	654	788

Note that 82.6 + 37.8 + 8.5 + 5.1 = 134,

403.4 + 184.2 + 41.5 + 24.9 = 654.

Observed and expected frequencies have the same row and column totals.

The chi-squared test for association

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Marital status	Acceptance of HIV test: Accepted	Rejected	Total
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Expected frequency if null hypothesis true =

$$\frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

The chi-squared test for association

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Single	15	8.5	35	41.5	50
Div./wid./sep.	7	5.1	23	24.9	30
Total	134		654		788

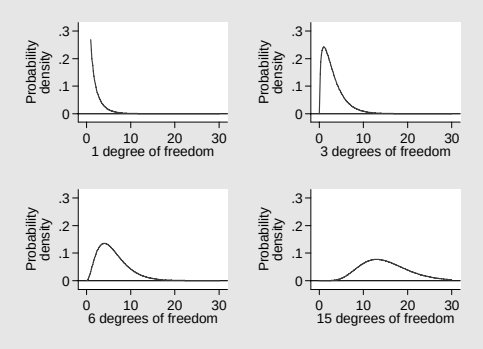
Compare the observed and expected frequencies.

Add (observed – expected)^2/expected for all cells = 9.15.

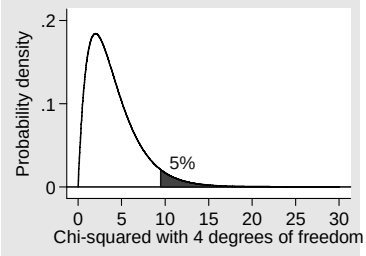
If null hypothesis true and samples are large enough, this is an observation from a chi squared distribution, often written χ^2.

The Chi-squared distribution

Family of distributions, one parameter, called the **degrees of freedom**.



Percentage points of the Chi-squared Distribution



Percentage points of the Chi-squared Distribution

Degrees of freedom	Probability that the tabulated value is exceeded			
	10% 0.10	5% 0.05	1% 0.01	0.1% 0.001
1	2.71	3.84	6.63	10.83
2	4.61	5.99	9.21	13.82
3	6.25	7.81	11.34	16.27
4	7.78	9.49	13.28	18.47
5	9.24	11.07	15.09	20.52
6	10.64	12.59	16.81	22.46
7	12.02	14.07	18.48	24.32
8	13.36	15.51	20.09	26.13
9	14.68	16.92	21.67	27.88
10	15.99	18.31	23.21	29.59
.	.	.	.	.
.	.	.	.	.

The chi-squared test for association

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Total	134		654		788

For a contingency table, the degrees of freedom are given by:

(number of rows – 1) × (number of columns – 1).

We have (4 – 1) × (2 – 1) = 3 degrees of freedom.

$\chi^2 = 9.15$, 3 d.f., $P<0.05$. Using a computer, $P = 0.027 = 0.03$.

The chi-squared test for association

The test statistic follows the Chi-squared Distribution provided the expected values are large enough.

This is a large sample test.

The smaller the expected values become, the more dubious will be the test.

The conventional criterion for the test to be valid is this: the chi-squared test is valid if at least 80% of the expected frequencies exceed 5 and all the expected frequencies exceed 1.

Also known as the **Pearson chi-squared test**.

Fisher's exact test

Also called the **Fisher-Irwin exact test**.

Works for any sample size.

Used to be used only for small samples in 2 by 2 tables, because of computing problems.

Calculate the probability of every possible table with the given row and column totals.

Sum the probabilities for all the tables as or less probable than the observed.

Fisher's exact test

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$\chi^2 = 9.15$, 3 d.f., P = 0.027.

Fishers' exact test: P = 0.029.

Risk ratio

Wound healing by type of bandage

Bandage	Healed	Did not heal	Total
Elastic	35 53.8%	30 46.2%	65 100%
Inelastic	19 28.4%	48 71.6%	67 100%
Total	54	78	132

Want an estimate of the size of the treatment effect.

Difference between proportions: 0.538 – 0.284 = 0.254
or 53.8% – 28.4% = 25.4 percentage points.

Proportion who heal is called the **risk** of healing for that population.

Risk ratio = 53.8/28.4 = 1.89.

Also called **relative risk**, **rate ratio**, **RR**.

Risk ratio

Wound healing by type of bandage

Bandage	Healed	Did not heal	Total
Elastic	35 53.8%	30 46.2%	65 100%
Inelastic	19 28.4%	48 71.6%	67 100%
Total	54	78	132

Risk ratio, RR = 53.8/28.4 = 1.89

We can find a 95% confidence interval = 1.22 to 2.95.

RR is not in the middle of its confidence interval.

Test of significance is the usual chi-squared test.

Odds

	Healed	Did not heal	Total
Elastic	35 53.8%	30 46.2%	65 100%

Risk of healing = 35/65 = 0.538

Odds of healing = 35/30 = 1.17

Risk = number experiencing event divided by number who could.

Odds = number experiencing event divided by number who did not experience event.

Risk: for every person treated, 0.538 people heal,
for every 100 people treated, 53.8 people heal.

Odds: for every person who does not heal, 1.17 people heal,
for every 100 people who do not heal, 117 people heal.

Odds ratio

Wound healing by type of bandage

Bandage	Healed	Did not heal	Total
Elastic	35	30	65
Inelastic	19	48	67
Total	54	78	132

Odds of healing given elastic bandages: 35/30 = 1.17.

Odds of healing given inelastic bandages: 19/48 = 0.40.

Odds ratio = (35/30)/(19/48) = 1.17/0.40 = 2.95.

For every person who does not heal, 2.95 times as many will heal with elastic bandages as will heal with inelastic bandages.

Odds ratio

Wound healing by type of bandage

Bandage	Healed	Did not heal	Total
Elastic	35	30	65
Inelastic	19	48	67
Total	54	78	132

Odds ratio, OR = 2.95

We can find a 95% confidence interval = 1.43 to 6.06.

OR is not in the middle of its confidence interval.

Test of significance is the usual chi-squared test.

Odds ratio

Wound healing by type of bandage

Bandage	Healed	Did not heal	Total
Elastic	35	30	65
Inelastic	19	48	67
Total	54	78	132

Odds ratio for healing: OR = (35/30)/(19/48) = 2.95.

Doesn't matter which way round we do it.

Odds ratio for treatment: OR = (35/19)/(30/48) = 2.95.

Both OR = (35×48)/(30 ×19).

Ratio of cross products.

Odds ratio

Wound healing by type of bandage

Bandage	Did not heal	Healed	Total
Elastic	30	35	65
Inelastic	48	19	67
Total	78	54	132

Switching the rows or columns inverts the odds ratio.

Odds ratio for not healing given elastic bandage:
OR = (30/35)/(48/19) = 0.339 = 1/2.95.

There are only two possible odds ratios.