

Core A-level mathematics

reproduced from the QCA's 'Subject criteria for Mathematics' document

Background knowledge:

- (a) The arithmetic of integers (including HCFs and LCMs), of fractions, and of real numbers.
- (b) The laws of indices for positive integer exponents.
- (c) Solution of problems involving ratio and proportion (including similar triangles, and links between length, area and volume of similar figures).
- (d) Elementary algebra (including multiplying out brackets, factorising quadratics with integer coefficients - to include $a^2 - b^2$ - and solution of simultaneous linear equations by eliminating a variable).
- (e) Changing the subject of a simple formula or equation.
- (f) The equation $y = mx + c$ for a straight line; gradient and intercept.
- (g) The distance between two points in 2D with given co-ordinates.
- (h) Solution of triangles using trigonometry, including the sine and cosine rules.
- (i) Volume of cone and sphere.
- (j) The following properties of a circle:
 - (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the perpendicularity of radius and tangent.

Knowledge, Understanding and Skills

Proof

A level specifications in Mathematics should require:

- (a) Construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language.

(b) Correct understanding and use of mathematical language and grammar in respect of terms such as ‘equals’, ‘identically equals’, ‘therefore’, ‘because’, ‘implies’, ‘is implied by’, ‘necessary’, ‘sufficient’, and notation such as \Leftarrow , \Rightarrow and \Leftrightarrow .

(c) Methods of proof, including proof by contradiction and disproof by counter-example.

These requirements should pervade the core content material set out below.

The A-level core

Core content material for A level examinations in Mathematics is listed below.

Algebra and functions

(a) Laws of indices for all rational exponents.

(b) Use and manipulation of surds.

(c) Quadratic functions and their graphs. The discriminant of a quadratic function. Completing the square. Solution of quadratic equations.

(d) Simultaneous equations: analytical solution by substitution, e.g. of one linear and one quadratic equation.

(e) Solution of linear and quadratic inequalities.

(f) Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, and factorisation; use of the Factor Theorem; simplification of rational expressions including factorising and cancelling, and algebraic division.

(g) Definition of a function. Domain and range of functions. Composition of functions. Inverse functions.

(h) Graphs of functions and their inverses; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.

(i) The modulus function.

(j) Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$ and combinations of these transformations.

(k) Rational functions. Partial fractions (denominators not more complicated than repeated linear terms).

(l) The Remainder Theorem.

Co-ordinate geometry in the (x, y) plane

(a) Equation of a straight line in the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$. Conditions for two straight lines to be parallel or perpendicular to each other.

(b) Co-ordinate geometry of the circle. Equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$.

(c) Cartesian and parametric equations of curves and conversion between the two forms.

Sequences and series

(a) Sequences, including those given by a formula for the n th term, and those generated by a simple recurrence relation of the form $x_{n+1} = f(x_n)$.

(b) Arithmetic series, including the formula for the sum of the first n natural numbers.

(c) The sum of a finite geometric series; the sum to infinity of a convergent geometric series.

(d) Binomial expansion of $(1 + x)^n$ for positive integer n . The notations $n!$ and $\binom{n}{r}$.

(e) Binomial series for any rational n .

Trigonometry

(a) Radian measure. Arc length, area of sector.

(b) Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.

(c) Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.

(d) Knowledge and use of $\tan \theta = \sin \theta / \cos \theta$, and $\sin^2 \theta + \cos^2 \theta = 1$ and its equivalents.

(e) Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a \cos \theta + b \sin \theta$ in the equivalent forms of $r \cos(\theta \pm \alpha)$ or $r \sin(\theta \pm \alpha)$.

(f) Solution of simple trigonometric equations in a given interval.

Exponentials and logarithms

(a) The function e^x and its graph.

(b) Exponential growth and decay.

(c) The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x ; laws of logarithms:

$$\ln_a x + \ln_a y = \ln_a xy; \quad \ln_a x - \ln_a y = \ln_a \frac{x}{y}; \quad k \ln_a x = \ln_a x^k.$$

(d) The solution of equations of the form $a^x = b$.

Differentiation

(a) The derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.

(b) Differentiation of x^n , e^x , and their sums and differences; differentiation of $\sin x$, $\cos x$, $\tan x$ and their sums and differences.

(c) Applications of differentiation to gradients, tangents and normals, maxima and minima and stationary points, increasing and decreasing functions.

(d) Differentiation using the product rule, the quotient rule, the chain rule and by the use of $\frac{dy}{dx} = \left(\frac{dx}{dy}\right)^{-1}$.

(e) Differentiation of simple functions defined implicitly or parametrically.

(f) Formation of simple differential equations.

Integration

(a) Indefinite integration as the reverse of differentiation.

- (b) Integration of x^n , e^x , $1/x$; integration of $\sin x$, $\cos x$.
- (c) Evaluation of definite integrals. Interpretation of the definite integral as the area under a curve.
- (d) Evaluation of volume of revolution.
- (e) Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain and product rules respectively.
- (f) Simple cases of integration using partial fractions.
- (g) Analytical solution of simple first order differential equations with separable variables.

Numerical methods

- (a) Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.
- (b) Approximate solution of equations using simple iterative methods.
- (c) Numerical integration of functions (e.g. using trapezium rule).

Vectors

- (a) Vectors in two and three dimensions.
- (b) Magnitude of a vector.
- (c) Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.
- (d) Position vectors. The distance between two points. Vector equations of lines.
- (e) The scalar product. Its use for calculating the angle between two lines.