Coherent presentations arising from Garside families

Alen Đurić
(joint work with Pierre-Louis Curien and Yves Guiraud)

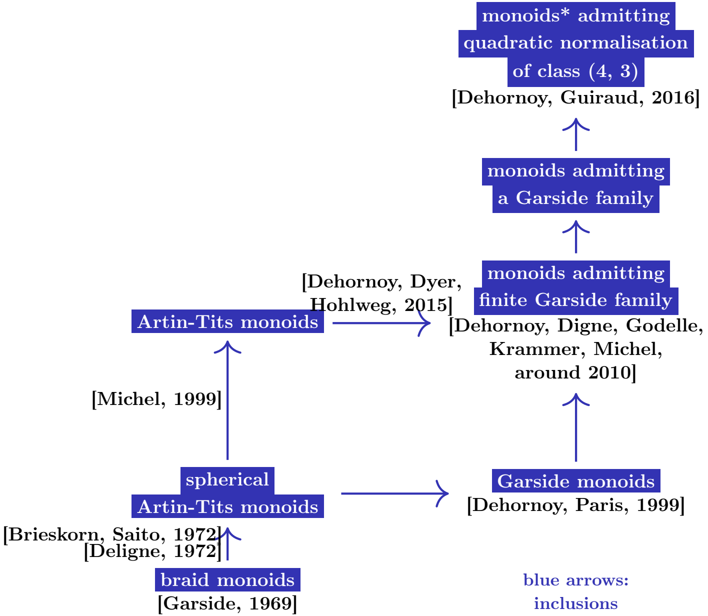
Université Paris Cité, CNRS, Inria, IRIF
https://www.irif.fr/∼djuric/

York Semigroup, 8 June 2022

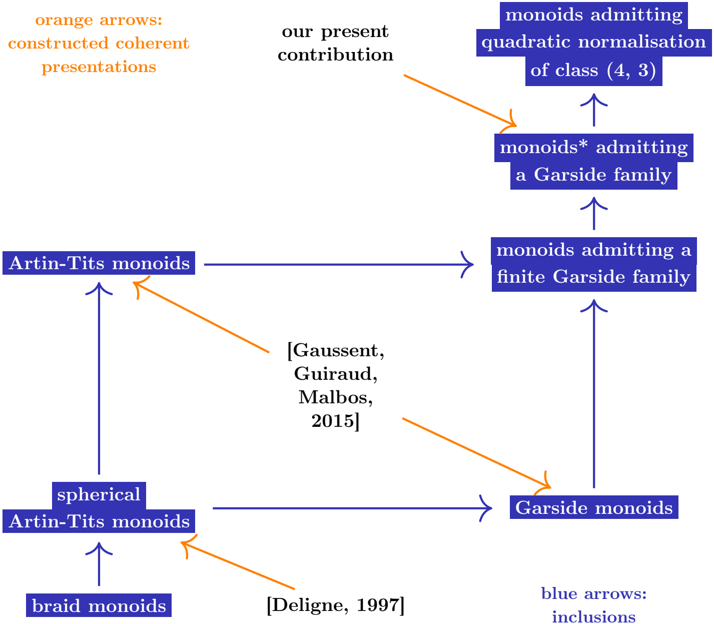
This program has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 754362.

Contents

Generalisations of greedy normal form



Overview of coherent presentations



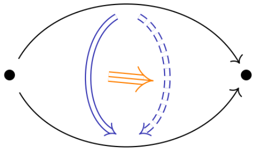
Polygraphs

- Introduced by Street (1976, 1987), and by Burroni (1993)

Terminology

- n-category: strict n-category
- (n,p)-category: its k-cells are invertible for all $k > p$
- k-sphere: a pair of parallel k-cells in an n-category $\mathscr{C}$
- acyclic cellular extension of $\mathscr{C}$: a set Γ of n-spheres of $\mathscr{C}$ s.t. all the n-spheres of $\mathscr{C}/\Gamma$ are of the form (f, f)

- 1-polygraph (X_0, X_1): a directed graph



- {generating k-cells}$= X_k$

- 2-polygraph: a triple $X = (X_0, X_1, X_2)$ s.t. (X_0, X_1) is a 1-polygraph, and X_2 is a cellular extension of X_1^*, the free category generated by (X_0, X_1)

Coherent presentations

$(3,1)$-polygraph: a quadruple $X = (X_0, X_1, X_2, X_3)$, where (X_0, X_1, X_2) is a 2-polygraph and X_3 is a cellular extension of $X_2^\top$, the free $(2,1)$-category over (X_0, X_1, X_2)

Let $\mathscr{C}$ be a category

- presentation of $\mathscr{C}$: a 2-polygraph (X_0, X_1, X_2) s.t. $\mathscr{C}$ is isomorphic to $\overline{X} := X_1^*/X_2$, the category presented by X
- extended presentation of $\mathscr{C}$: a $(3,1)$-polygraph X s.t. $\mathscr{C} \cong \overline{X}$

Definition

A coherent presentation of $\mathscr{C}$ is an extended presentation (X_0, X_1, X_2, X_3) of $\mathscr{C}$ s.t. X_3 is an acyclic cellular extension of $X_2^\top$.

Motivation

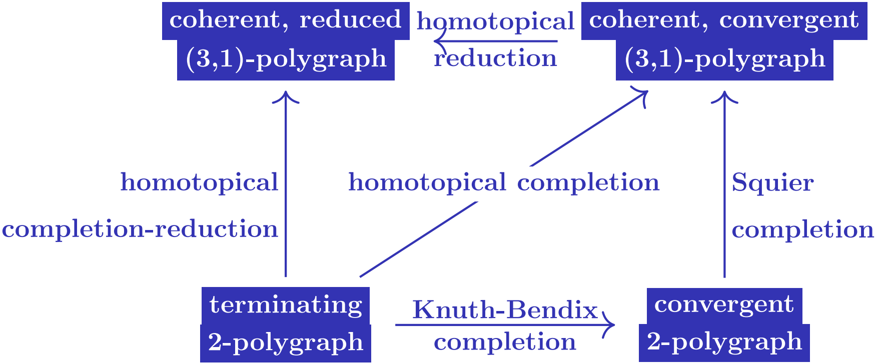
Coherent presentations are closely related to

- weak actions of monoids on categories, investigated by Deligne for spherical Artin-Tits monoids
- cofibrant approximations in the canonical model structure on 2-categories, given by Lack

  See theorem

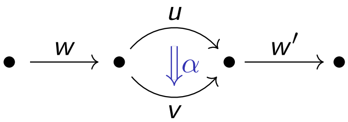
- polygraphic resolutions of monoids, defined by Métayer, from which abelian resolutions can be deduced

Homotopical completion-reduction



Rewriting

Rewriting step of a 2-polygraph X: a 2-cell of the free category X_2^* which contains a single generating 2-cell of X



where $\alpha \in X_2$, and w and w' are 1-cells of X_2^*

Let u and v be 1-cells of X_2^* (also called words if X_0 is a singleton)

- u rewrites to v: there is a finite rewriting sequence with source u and target v
- u is irreducible: there is no rewriting step whose source is u
- a normal form of u, denoted by $\widehat{u}$: the irreducible 1-cell to which u rewrites, if it is unique

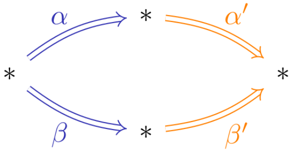
Rewriting properties of polygraphs

Let X be a 2-polygraph

Branching in X: a pair $\{\alpha, \beta\}$ of rewriting sequences having the same source

X is said to be

- confluent if every branching $\{\alpha, \beta\}$ can be completed to sequences having the same target

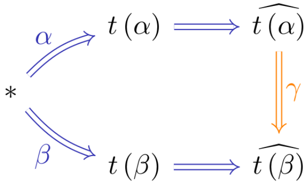


- terminating if it has no infinite rewriting sequence
- convergent if it is both confluent and terminating

Termination order on X: a well-founded order $\leq$ on parallel 1-cells of X_2^*, respecting the 0-composition, s.t. $s(\alpha) > t(\alpha)$ holds for all $\alpha \in X_2$

Knuth-Bendix completion

Critical branching: minimal nontrivial overlap of two rewriting steps



Theorem (Knuth, Bendix, 1970)

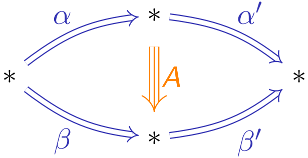
Every Knuth-Bendix completion of a 2-polygraph X equipped with a total termination order is a convergent presentation of the category $\overline{X}$.

Remark about Knuth-Bendix

There is an alternative to requiring a termination order at the beginning: orient the newly added generating 2-cells "by hand", and verify after each addition in an ad hoc manner whether a terminating presentation is maintained.

Squier completion

A family of generating confluences of convergent 2-polygraph X: a cellular extension of $X_2^{\top}$ having, for each critical branching $\{\alpha, \beta\}$ of X, exactly one 3-cell A



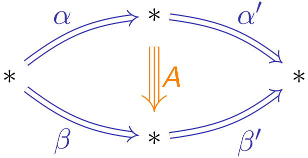
A Squier completion of a convergent 2-polygraph X: a $(3,1)$-polygraph s.t. its generating 3-cells form a family of generating confluences of X

Theorem (Squier, 1994)

Let X be a convergent 2-polygraph. Every family of generating confluences of X is an acyclic cellular extension of $X_2^{\top}$.

Squier completion

A family of generating confluences of convergent 2-polygraph X: a cellular extension of $X_2^\top$ having, for each critical branching $\{\alpha, \beta\}$ of X, exactly one 3-cell A



A homotopical completion of a terminating 2-polygraph X: a Squier completion of a Knuth-Bendix completion of X

Corollary

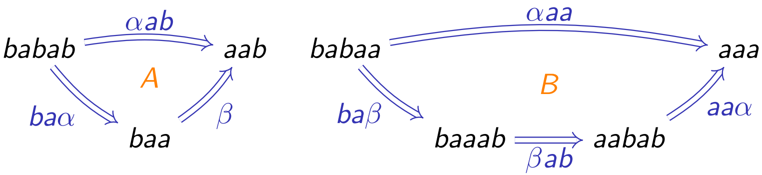
Let X be a terminating presentation of a category $\mathscr{C}$. Every homotopical completion of X is a coherent convergent presentation of $\mathscr{C}$.

Example of homotopical completion

The Klein bottle monoid:

$$\left\langle a, b \,\middle|\, bab \overset{\alpha}{\Rightarrow} a \right\rangle^{+}$$

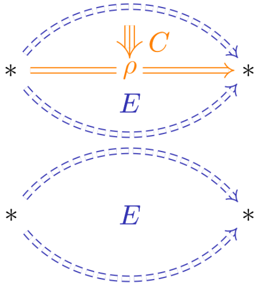
1. Termination order: compare lengths then apply lexicographic order, e.g. $b < aa < ab$
2. Exactly one critical branching: $\{\alpha ab, ba\alpha\}$
3. The Knuth-Bendix completion adds $\beta : baa \Rightarrow aab$
4. The Squier completion adds the generating 3-cell A



5. β causes only one new critical branching: $\{\alpha aa, ba\beta\}$
6. The generating 3-cell B is added

Homotopical reduction

- Systematic way of recursively removing some redundant and collapsible generating cells
- Analogous to collapsing scheme (now aka Morse matching) introduced by Brown (1989)



Theorem

Let X be a terminating 2-polygraph presenting a category $\mathscr{C}$. Then, every homotopical completion-reduction of X is a coherent presentation of $\mathscr{C}$.

Example

A homotopical reduction of the $(3,1)$-polygraph constructed on the previous slide is the presentation $\left(a, b \,\middle|\, bab \overset{\alpha}{\Rightarrow} a \,\middle|\, \emptyset \right)$.

Garside's presentation of Artin-Tits monoids

- Introduced by Deligne (1997) for spherical Artin-Tits monoids, and by Michel (1999) for general Artin-Tits monoids

Let $B^+(W)$ be an Artin-Tits monoid associated to a Coxeter group W See definition

Graphical notation for $u, v, w \in B^+(W)$

- $u \frown v$: $\|uv\| = \|u\| + \|v\|$ holds in W
- $u \frown v \frown w$: $u \frown v$, $v \frown w$ and $\|uvw\| = \|u\| + \|v\| + \|w\|$

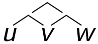
Definition

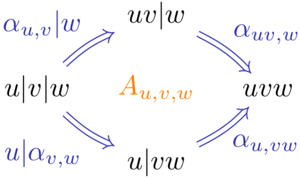
Garside's presentation of $B^+(W)$ is a 2-polygraph $\mathrm{Gar}_2(W)$ having:

- a single generating 0-cell,
- elements of $W \setminus \{1\}$ as generating 1-cells,
- and a generating 2-cell $\alpha_{u,v} : u|v \Rightarrow uv$ for all $u, v \in W \setminus \{1\}$ such that $u \frown v$.

Garside's coherent presentation of Artin-Tits monoids

$\mathrm{Gar}_3(W)$: the extended presentation of $B^+(W)$ obtained by adjoining to $\mathrm{Gar}_2(W)$ a 3-cell $A_{u,v,w}$ for all $u, v, w \in W \setminus \{1\}$

s.t. 



Theorem (Gaussent, Guiraud, Malbos, 2015)

For every Coxeter group W, the Artin-Tits monoid $B^+(W)$ admits $\mathrm{Gar}_3(W)$ *as a coherent presentation.*

- adapts in a straightforward way to Garside monoids See definition

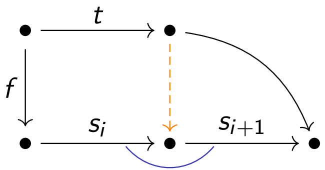
Idea of the proof: homotopical completion-reduction of Garside's presentation of $B^+(W)$

Definition of Garside family

Let S be a subfamily of a left-cancellative monoid M

- Greedy decomposition: an S-word $s_1|\cdots|s_q$ is said to be S-normal if for all $i < q$

$$\forall t \in S, \forall f \in M, (t \preceq fs_is_{i+1} \implies t \preceq fs_i)$$



- Garside family in M: a subfamily S such that every element of M admits an S-normal decomposition

See examples

Properties of Garside families

Let M be a left-cancellative monoid having no nontrivial invertible element, and S a Garside family in M

- S is closed under right divisor and right-mcm
- Normalisation map $N^S : S^* \to S^*$ assigns to each $w \in S^* \setminus \{1\}$ the strict S-normal decomposition of the evaluation of w, and $N^S(1) = 1$
- N^S is left-weighted, i.e. for all $s, t \in S$, the element s is a left divisor in M of the leftmost letter of $N^S(s|t)$
- Rewriting rules $s|t \Rightarrow N^S(s|t)$, for all $s, t \in S \setminus \{1\}$ with $s|t$ not S-normal, yield a convergent presentation of M

Proposition about $\mathrm{Gar}_2(S)$ (Dehornoy, Guiraud, 2016)

Let M be a left-cancellative monoid containing no nontrivial invertible element, and $S \subseteq M$ a Garside family s.t. $1 \in S$. Then M admits, as a presentation, the 2-polygraph $\mathrm{Gar}_2(S)$, with $u \frown v$ denoting $uv \in S$.

Garside's presentation of M, with respect to S: $\mathrm{Gar}_2(S)$

Direction of our contribution

Motivation

1. One can use Proposition about $\mathsf{Gar}_2(S)$ to interpret $\mathsf{Gar}_2(W)$ as a special case of $\mathsf{Gar}_2(S)$
2. Although certain steps of the proof by Gaussent, Guiraud and Malbos do rely on the arithmetic properties of Artin-Tits monoids, the general structure of the proof mostly relies on the specific "shape" of the relations involved

Challenges

- Attaining termination
- Computing homotopical completion

Notation formally redefined

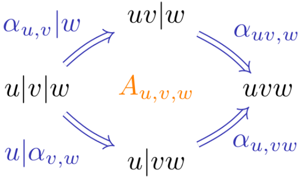
Let M be a monoid generated by a set S containing 1

2-polygraph $\mathrm{Gar}_2(S)$

- a single generating 0-cell
- elements of $S \setminus \{1\}$ as generating 1-cells
- and a generating 2-cell $\alpha_{u,v} : u|v \Rightarrow uv$ for all $u, v \in S \setminus \{1\}$ such that $u \frown v$ (meaning $uv \in S$)

(3, 1)-polygraph $\mathrm{Gar}_3(S)$

- $\mathrm{Gar}_2(S)$
- and 3-cell $A_{u,v,w}$ for all $u, v, w \in S \setminus \{1\}$ s.t. $u \frown v \frown w$ (meaning $u \frown v$, $v \frown w$ and $uvw \in S$)



Main statement

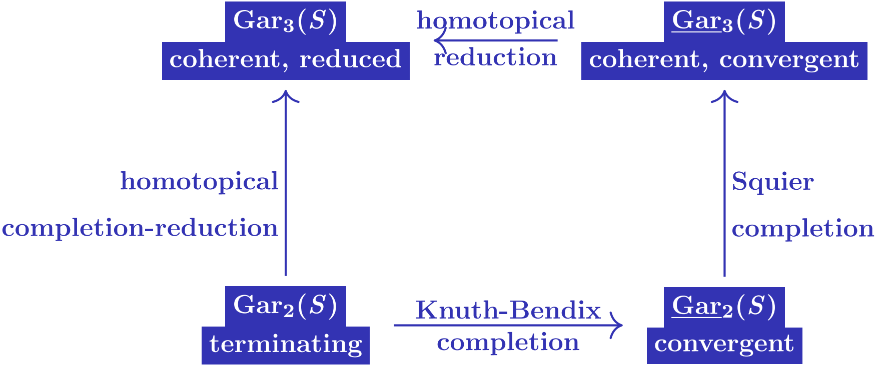
Definition

Given a subfamily S of a monoid M, we say that M is S-right-noetherian if there exists no $g \in S$ admitting an infinite sequence $(h_n)_{n=1}^{\infty}$ in $S \cap \mathrm{Div}(g)$ such that for every n there exists a non-invertible f_n in S satisfying $h_n f_n = h_{n+1}$.

Theorem

Assume that M is a left-cancellative monoid containing no nontrivial invertible element, and that it is S-right-noetherian for a Garside family containing 1. If M admits right-mcms, then M admits the $(3,1)$-polygraph $\mathrm{Gar}_3(S)$ as a coherent presentation.

Overview of proof



Attaining termination

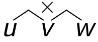
Let M be a monoid generated by a set S containing 1

2-polygraph $\underline{\mathrm{Gar}}_2(S) := \mathrm{Gar}_2(S)$ + generating 2-cells β

- a single generating 0-cell
- elements of $S \setminus \{1\}$ as generating 1-cells
- generating 2-cells

$$\alpha_{u,v} : u|v \Rightarrow uv, \qquad u, v \in S \setminus \{1\},$$

$$\beta_{u,v,w} : u|vw \Rightarrow uv|w, \qquad u, v, w \in S \setminus \{1\},$$



Proposition about $\underline{\mathrm{Gar}}_2(S)$

Assume that M is a left-cancellative monoid containing no nontrivial invertible element, and that it is S-right-noetherian for a Garside family S containing 1. Then the 2-polygraph $\underline{\mathrm{Gar}}_2(S)$ is terminating.

See sketch of proof

Squier completion of $\underline{\mathrm{Gar}}_2(S)$

$\underline{\mathrm{Gar}}_3(S) := \underline{\mathrm{Gar}}_2(S)$ + nine families of generating 3-cells

See diagrams

Proposition about $\underline{\mathrm{Gar}}_3(S)$

Let M be a left-cancellative monoid admitting right-mcms, and S a subfamily of M closed under right-mcm and right divisor. Assume that the 2-polygraph $\underline{\mathrm{Gar}}_2(S)$ is a terminating presentation of M. Then M admits, as a coherent convergent presentation, the $(3,1)$-polygraph $\underline{\mathrm{Gar}}_3(S)$.

Idea of proof

- Relying on Proposition about $\mathrm{Gar}_2(S)$, mimic the proof by Gaussent, Guiraud and Malbos
- Make new justifications as demanded by a more general setting

See sketch of proof

Corollaries and reduction

Theorem

Assume that M is a left-cancellative monoid containing no nontrivial invertible element, and that it is S-right-noetherian for a Garside family S containing 1.

1. *The 2-polygraph $\underline{\mathrm{Gar}}_2(S)$ is a convergent presentation of M.*
2. *If M admits right-mcms, then M admits the $(3,1)$-polygraph $\underline{\mathrm{Gar}}_3(S)$ as a coherent convergent presentation*

Proposition (Gaussent, Guiraud, Malbos, 2015)

The $(3,1)$-polygraph $\mathrm{Gar}_3(S)$ can be obtained as a homotopical reduction of $\underline{\mathrm{Gar}}_3(S)$.

Corollary

Let M be a left-cancellative noetherian monoid containing no nontrivial invertible element, and $S \subseteq M$ a Garside family containing 1. Then M admits the $(3,1)$-polygraph $\mathrm{Gar}_3(S)$ as a coherent presentation.

Applications

Free abelian monoid $\mathbb{N}^{(I)}$ over an infinite basis

- not of finite type, hence neither Artin-Tits nor Garside
- Garside family

$$S_I = \left\{ g \in \mathbb{N}^{(I)} \,\middle|\, \forall k \in I, g(k) \in \{0, 1\} \right\}$$

- conditions of Theorem: product on $\mathbb{N}^{(I)}$ is based on the addition of integers
- $u \frown v$: u and v have disjoint supports

Monoid B_∞^+ of all positive braids on infinitely many strands indexed by positive integers

- not of finite type, hence neither Artin-Tits nor Garside
- Garside family

$$S_\infty = \bigcup_{n \geq 1} \mathrm{Div}(\Delta_n)$$

- conditions of Theorem: preserved from braid monoids
- $u \frown v$: uv is a simple braid

Applications, continued

Dual braid monoid B_n^{+*}

- generators: $a_{i,j}$ with $1 \leq i < j \leq n$
- relations: $a_{i,j}a_{i',j'} = a_{i',j'}a_{i,j}$ for $[i,j]$ and $[i',j']$ disjoint or nested; $a_{i,j}a_{j,k} = a_{j,k}a_{i,k} = a_{i,k}a_{i,j}$ for $1 \leq i < j < k \leq n$
- Garside monoid: (B_n^{+*}, Δ_n^*) with $\Delta_n^* = a_{1,2} \cdots a_{n-1,n}$
- further homotopical reduction after Theorem for B_4^{+*}

Artin-Tits monoid of type $\widetilde{A}_2$

- presented by

$$\langle \sigma_1, \sigma_2, \sigma_3 | \sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2, \sigma_2\sigma_3\sigma_2 = \sigma_3\sigma_2\sigma_3, \sigma_3\sigma_1\sigma_3 = \sigma_1\sigma_3\sigma_1 \rangle^+ \tag{1}$$

- Garside family: sixteen right divisors of the elements $\sigma_3\sigma_1\sigma_2\sigma_1$, $\sigma_1\sigma_2\sigma_3\sigma_2$, and $\sigma_2\sigma_3\sigma_1\sigma_3$ (Dehornoy, Dyer, Hohlweg, 2015)
- Theorem and further homotopical reduction: (1) is coherent

Further directions

- Prove that a monoid M having a Garside family S admits the following polygraphic resolution

$$\mathrm{Gar}\,(S) = \left\{ \gamma_{u_0 \cdots u_n} \;\middle|\; u_0 \cdots u_n \in S \setminus \{1\}\,,\ u_0 \frown \cdots \frown u_n \right\}$$

  where $\gamma_{u_0 \cdots u_n}$ denotes n-cube
  - challenge: determine boundary maps
- Extend our results to a wider class of monoids, guided by the plactic monoids See perspective
- Generalise application of our results from B_4^{+*} to general dual braid monoids
  - challenges: describe those pairs of elements of $\mathrm{Div}\,(B_n^{+*}) \setminus \{1\}$ whose product is in $\mathrm{Div}\,(B_n^{+*})$; formalise the additional heuristic reduction
  - way: study noncrossing partitions (e.g. the paper by Bessis, Digne and Michel)
- Extend application of our results from $\widetilde{A}_2$ to $\widetilde{A}_n$, relying on the paper by Dehornoy, Dyer and Hohlweg (2015)
  - challenges: as in the previous direction

Thank you!

Motivation for coherent presentations

Let X be a $(3,1)$-polygraph

- $\widetilde{X} = X_2^\top / X_3$: the $(2,1)$-category presented by X

Theorem (Gaussent, Guiraud, Malbos, 2015)

Let X be an extended presentation of a category $\mathscr{C}$. TFAE:

- *X is a coherent presentation of $\mathscr{C}$;*
- *$\widetilde{X}$ is a cofibrant approximation of $\mathscr{C}$ (viewed as a 2-category);*
- *for every 2-category $\mathscr{D}$, the category of pseudofunctors from $\mathscr{C}$ to $\mathscr{D}$ and the category of 2-functors from $\widetilde{X}$ to $\mathscr{D}$ are equivalent, and this equivalence is natural in $\mathscr{D}$.*

Back to main

Coxeter groups and Artin-Tits monoids

Coxeter group: group W presented by

$$\left\langle S \text{ finite} \,\middle|\, \left\{ s^2 = 1, sts\cdots = tst\cdots \,\middle|\, s,t \in S \right\} \right\rangle$$

Spherical Artin-Tits monoid corresponding to a finite Coxeter group W:

$$B^+(W) = \langle S \text{ finite} \,|\, \{ sts\cdots = tst\cdots \,|\, s,t \in S \} \rangle^+$$

Examples

The permutation group S_n, e.g. $S_3 = \left\langle s,t \,\middle|\, s^2 = t^2 = 1, tst = sts \right\rangle$

The braid monoid $B_n^+ = B^+(S_n)$, e.g. $B_3 = \langle s,t \,|\, tst = sts \rangle^+$

Properties of Artin-Tits monoids

- cancellative
- contain no nontrivial invertible element
- admit conditional right-lcms
- noetherian

Back to main

Garside monoids

Definition

A Garside monoid is a pair (M, Δ) such that the following conditions hold:

- M is a cancellative monoid;
- there is a map $\lambda : M \to \mathbb{N}$ such that $\lambda(fg) \geq \lambda(f) + \lambda(g)$ and $\lambda(f) = 0 \implies f = 1$;
- every two elements have a left-gcd and a right-gcd and a left-lcm and a right-lcm;
- $\Delta \in M$, called the Garside element, is such that the left and the right divisors of Δ coincide, and they generate M;
- the family of all divisors of Δ is finite.

Theorem (Gaussent, Guiraud, Malbos, 2015)

Every Garside monoid M admits $\mathrm{Gar}_3(M)$, with $u \frown v$ denoting $uv \in \mathrm{Div}(\Delta)$, as a coherent presentation.

Back to main

Examples of Garside families

- Coxeter group W is a Garside family in Artin-Tits monoid $B^+(W)$
- Every Artin-Tits monoid $B^+(W)$ admits a finite Garside family (Dehornoy, Dyer, Hohlweg 2015)
- If $B^+(W)$ is spherical, a finite Garside family is given by W
- In the particular case of a braid monoid, the family of all simple braids is a Garside family
- Every Garside monoid (M, Δ) has a finite Garside family given by $\mathrm{Div}(\Delta)$
- The monoid B^+_∞ of all positive braids on infinitely many strands indexed by positive integers admits

$$S_\infty = \bigcup_{n \geq 1} \mathrm{Div}(\Delta_n)$$

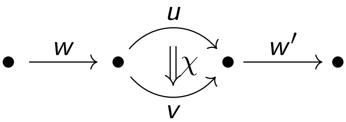
as a Garside family

Back to main

Sketch of proof

Notation

- h (w): the leftmost letter of word w
- χ-step for a generating 2-cell χ:



- χ_i: a χ-step where w has length $i-1$
- for an infinite sequence of positive integers $u = i_1|i_2|\cdots$, we write χ_u for the path $\cdots \circ \chi_{i_2} \circ \chi_{i_1}$

Suppose there is an infinite rewriting path

- $\beta_{i_1|i_2|\ldots}$: an infinite rewriting path of β-steps having source u of minimal length
- position 1 occurs infinitely many times in $i_1|i_2|\cdots$
- $i_{c_1}|i_{c_2}|\cdots$: constant subsequence of $i_1|i_2|\cdots$ taking all the members whose value is 1
- $u^{(n)}$: the nth word in $\beta_{i_1|i_2|\ldots}$, i.e. the source of β_{i_n}

Sketch of proof, continued

Consider the leftmost letter

- $$\mathsf{h}\left(u^{(n+1)}\right) = \begin{cases} \mathsf{h}\left(u^{(n)}\right) & \text{if } i_{n+1} \neq 1 \\ \mathsf{h}\left(u^{(n)}\right) f_n \text{ for some } f_n \in S & \text{if } i_{n+1} = 1 \end{cases} \quad (2)$$

  g : the leftmost letter of the S-normal form of u
- normalisation map N^S left-weighted: $\mathsf{h}\left(u^{(n)}\right)$ left divides g for all n

Consider the sequence in $S \cap \mathsf{Div}(g)$

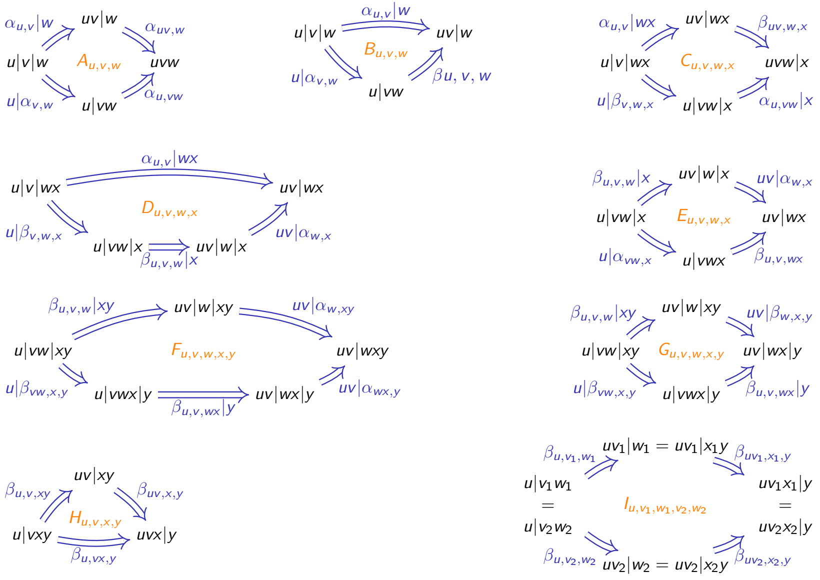
$$\left(\mathsf{h}\left(u^{(c_n)}\right)\right)_{n=1}^{\infty} \quad (3)$$

- by (2), we have $\mathsf{h}\left(u^{(c_{n+1})}\right) = \mathsf{h}\left(u^{(c_n)}\right) f_{c_n}$
- existence of the sequence (3) contradicts the fact that M is S-right-noetherian

Conclusion: $\underline{\mathsf{Gar}}_2(S)$ is terminating

Back to main

$\underline{\text{Gar}}_3(S) := \underline{\text{Gar}}_2(S)$ + generating 3-cells



Back to main

See sketch of proof

Squier completion of $\underline{\mathrm{Gar}}_2(S)$

$\underline{\mathrm{Gar}}_3(S)$:= $\underline{\mathrm{Gar}}_2(S)$ + nine families of generating 3-cells

See diagrams

Proposition about $\underline{\mathrm{Gar}}_3(S)$

Let M be a left-cancellative monoid admitting right-mcms, and S a subfamily of M closed under right-mcm and right divisor. Assume that the 2-polygraph $\underline{\mathrm{Gar}}_2(S)$ is a terminating presentation of M. Then M admits, as a coherent convergent presentation, the $(3,1)$-polygraph $\underline{\mathrm{Gar}}_3(S)$.

Sketch of proof

Discussion: length of the intersection of the sources of 2-cells forming a critical branching

Length-one case: exactly as proved by Gaussent, Guiraud and Malbos; all critical branchings confluent; the generating 3-cells A, ..., G added

Squier completion of $\underline{\mathrm{Gar}}_2(S)$

$\underline{\mathrm{Gar}}_3(S) := \underline{\mathrm{Gar}}_2(S)$ + nine families of generating 3-cells

See diagrams

Proposition about $\underline{\mathrm{Gar}}_3(S)$

Let M be a left-cancellative monoid admitting right-mcms, and S a subfamily of M closed under right-mcm and right divisor. Assume that the 2-polygraph $\underline{\mathrm{Gar}}_2(S)$ is a terminating presentation of M. Then M admits, as a coherent convergent presentation, the $(3,1)$-polygraph $\underline{\mathrm{Gar}}_3(S)$.

Sketch of proof

Discussion: length of the intersection of the sources of 2-cells forming a critical branching

Length-two case: new justifications needed

the only way for this case to occur: $u \; v_1 \; w_1$ and $u \; v_2 \; w_2$ s.t. $v_1 w_1 = v_2 w_2$

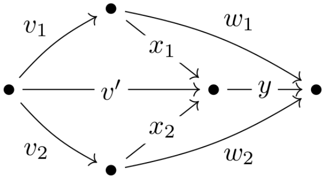
Sketch of proof, continued

$$uv_1|w_1 \overset{\beta_{u,v_1,w_1}}{\Longleftarrow} u|v_1w_1 = u|v_2w_2 \overset{\beta_{u,v_2,w_2}}{\Longrightarrow} uv_2|w_2$$

Assumptions of the theorem yield

- $\exists v' \in S$: a right-mcm of v_1 and v_2
- $\exists! x_k \in S$: the right complement of v_k in v'
- $\exists! y \in S$: the right complement of v' in $v_k w_k$
- $w_k = x_k y$

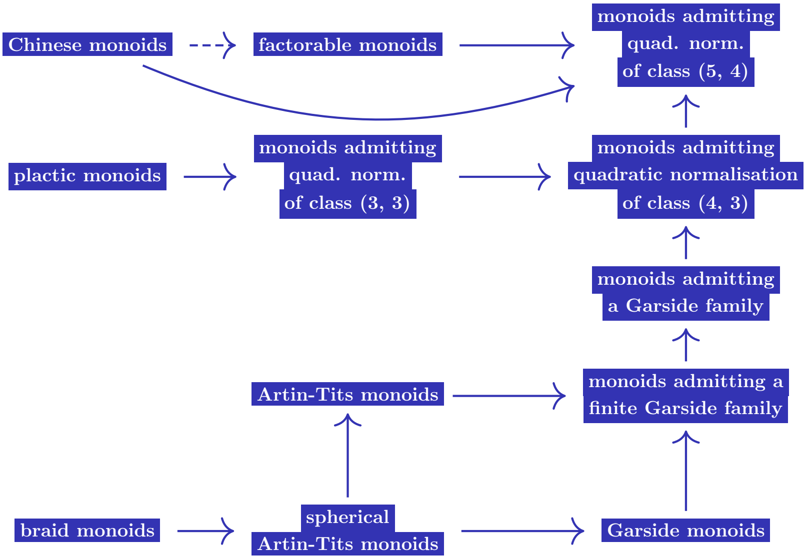
Verify that all the generating 1-cells in the definitions of the generating 3-cells of $\underline{\text{Gar}}_3(S)$ are, indeed, elements of $S \setminus \{1\}$



- $y \neq 1$
- x_1 and x_2 are not both equal to 1
- $x_k = 1$ yields H with $x := x_k$
- $x_1 \neq 1$ and $x_2 \neq 1$ yield I

Back to main

Perspective



Back to main