Mechanics 1

Introduction to Classical Mechanics

Principles of Force & Motion

(or….Swings and Roundabouts)

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Why study mechanics?

(isn't it all out of date – quantum, relativity?)

- YES and NO!
- Quantum mechanics describes the very small – nuclei, atoms, …..
- Relativity deals with the very fast
- In the 'classical limit' i.e. everyday examples, classical mechanics gives an excellent description
- Classical mechanics is more intuitive, more in line with everyday experience and so is simpler to understand
  - Don't need relativity to explain the trajectory of a tennis ball
- Many of the concepts you meet in this course are equally important to modern physics
- Good foundation!

Course Outline

- Kinematics
- Projectiles
- Circular motion
- Newton's laws
- Conservation laws – energy and momentum
- Angular motion
- Conservation laws – angular momentum

Course Textbook

- University Physics with Modern Physics, Young and Freedman, 11th edition
- Chapters 1-10 covered in this course
- Read sections 1.1 – 1.6
- Rest of chapter 1 and some of chapter 2 covered in this lecture
- After lecture make sure you can do the example questions covered in the textbook and understand the summary sections.

Lecture 1

- Tools
  - Problem solving techniques
  - Recap on maths (vectors)

- Kinematics in 1D

Problem Solving

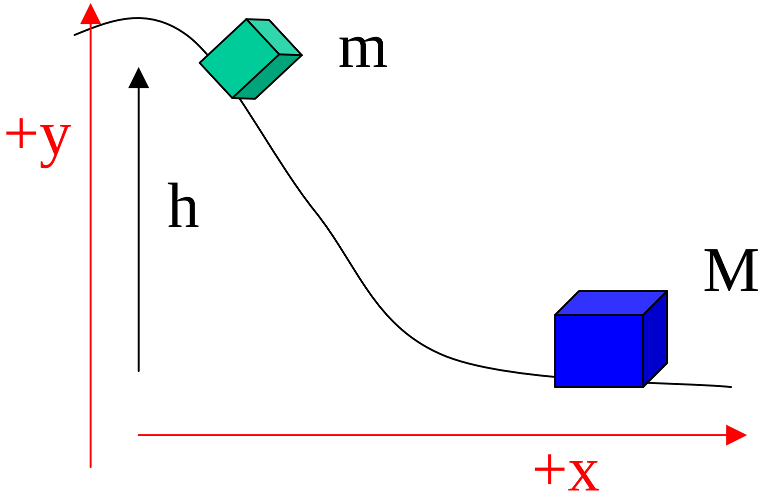
1. Draw a sketch
2. Define a sensible notation
3. Identify principles
4. Appeal to mathematics
5. Look where you are going
6. Check it makes sense – why?

Example

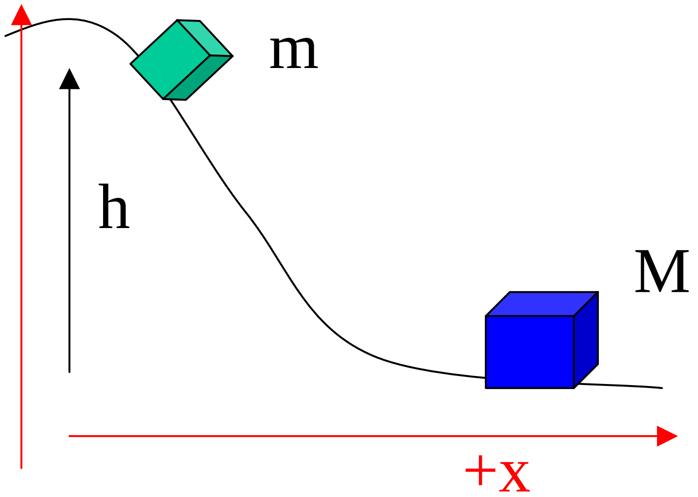
A mass m slides down a frictionless slope and collides elastically with mass M. Mass m is initially a height of 20 cm up the slope.

If m = 1 kg and M = 2 kg, what is the final velocity of m?

- draw a sketch
- define notation



- Identify principles
  1. energy conservation during initial motion **and** in collision process
  2. momentum conservation in collision process



- Appeal to maths
  1. Energy conservation in initial motion

$$\boldsymbol{mgh = \tfrac{1}{2}\, mu^2}$$

  2. energy conservation in collision process

$$\boldsymbol{\tfrac{1}{2}\, mu^2 = \tfrac{1}{2}\, mv^2 + \tfrac{1}{2}\, MV^2}$$

  3. momentum conservation in collision process

$$\boldsymbol{mu = mv + MV}$$

- Look where you're going!
  - Have 3 equations, 3 unknowns
  - Want v
- Be aware of alternative solutions
  - E.g. +/- solutions to square roots
- Keep numbers until last
  - Lose valuable info

- Solve eqn. 1. for u:

$$u = \sqrt{2gh} \qquad 4.$$

(want +ve square root)

- Rearrange eqn. 2.

$$V^2 = \frac{m\left(u^2 - v^2\right)}{M} \qquad 5.$$

- Rearrange eqn. 3. and square

$$V^2 = \frac{m^2(u-v)^2}{M^2} \qquad 6.$$

- Equate eqns. 5 and 6:

$$\frac{m^2(u-v)^2}{M^2} = \frac{m(u^2-v^2)}{M} = \frac{m(u-v)(u+v)}{M}$$

- Simplify $$\frac{m(u-v)}{M} = u+v$$

- Rearrange $$v = \frac{m-M}{m+M}.u$$

- Use eqn. 4. and solve:

$$v = \frac{m-M}{m+M}.\sqrt{2gh}$$

$= -2/3 \text{ ms}^{-1}$

Finally…..

1. Check if answer makes sense
    - Does the number seem reasonable?
    - Is the direction (sign) correct?
2. Check the units are correct
    - Units on both sides of final equation should be the same
    - See 'Physics of Matter' lectures
3. Appeal to special cases
    - Does the answer make sense if you take a variable to be very large or very small
    - E.g. when $M \gg m$, v tends to $-u$ …..sensible!

Maths Toolkit

Vectors and scalars

Vector properties

Vector manipulation

Vectors and Scalars

- *Scalar*: physical quantity with signed magnitude

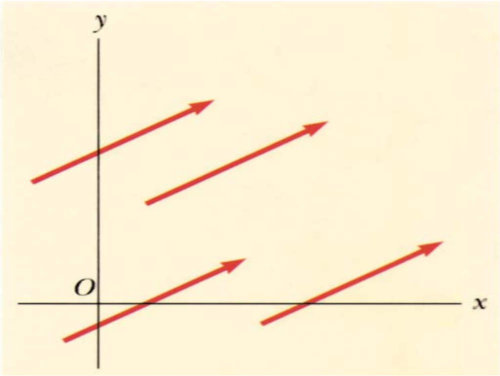
- Examples:
  - Distance
  - Speed
  - Energy

- *Vector:* physical quantity with magnitude and direction

- Examples:
  - Displacement
  - Velocity
  - Acceleration

Properties of Vectors

- Notation – **A** A $\vec{A}$
- Vectors are equal if they have the same magnitude and direction – see figure
- Magnitude of a vector is $|\vec{A}|$ or A
- Can represent vectors as arrows, matrices or in component form



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\underline{A} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\mathbf{k}$

Make sure you know and understand all material in sections 1.7-1.10 in textbook

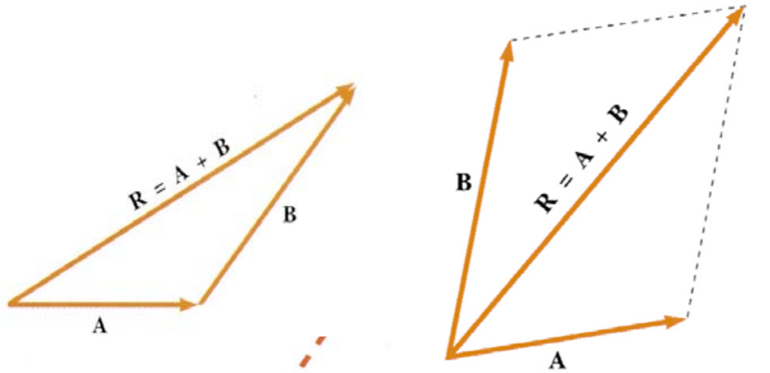
Vector Manipulation

- Addition

  $\underline{R} = \underline{A} + \underline{B} = \underline{B} + \underline{A}$

  $R_x = A_x + B_x$

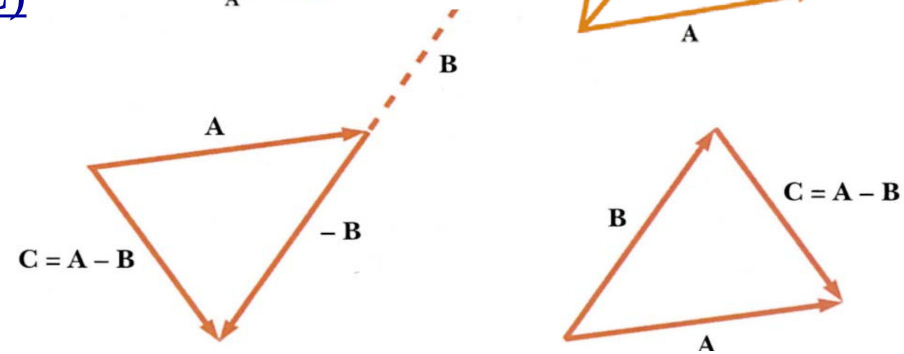
  $\underline{R} = (\underline{A}+\underline{B})+\underline{C} = \underline{A}+(\underline{B}+\underline{C})$



- Subtraction

  $\underline{C} = \underline{A} - \underline{B} = \underline{A} + (-\underline{B})$

  $\underline{A} + (-\underline{A}) = 0$



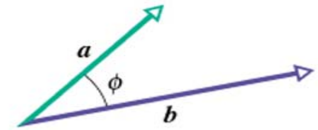
- Multiplication by a scalar

  $\underline{B} = s\underline{A}$

  $\underline{B}$ will be parallel to $\underline{A}$ if s is +ve
  and antiparallel if s is -ve

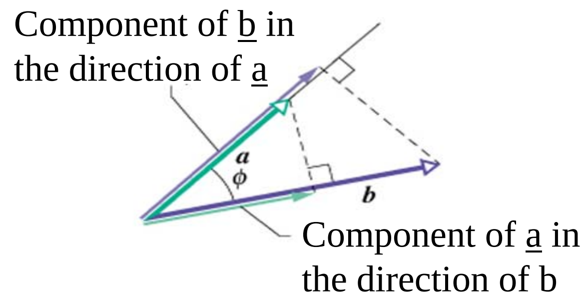
More vector manipulation

- Dot product   A.B = scalar



(a)

$$\underline{\boldsymbol{A}} \cdot \underline{\boldsymbol{B}} = |\underline{\boldsymbol{A}}||\underline{\boldsymbol{B}}|\cos\phi$$

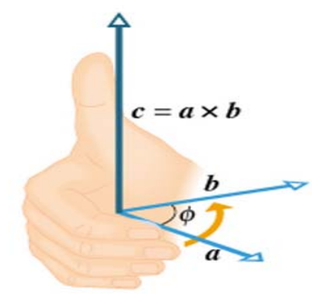


(b)

The dot product or scalar product is the projection of A onto B times B

- Vector product   A x B = vector

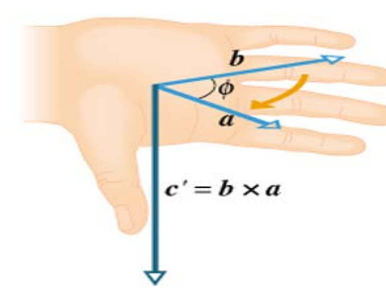
$$|\underline{\boldsymbol{A}} \times \underline{\boldsymbol{B}}| = |\underline{\boldsymbol{A}}||\underline{\boldsymbol{B}}|\sin\phi$$



(a)

The cross product is always perpendicular to both A and B, according to the right hand screw rule

$$\underline{\boldsymbol{A}} \times \underline{\boldsymbol{B}} = -\underline{\boldsymbol{B}} \times \underline{\boldsymbol{A}}$$

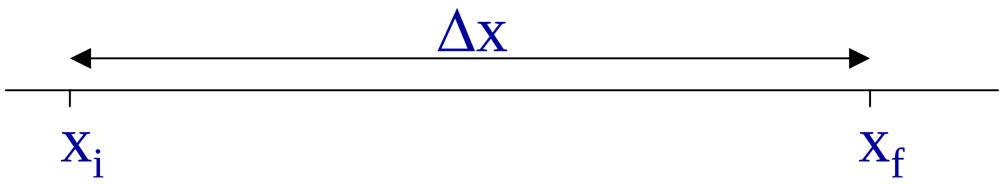


(b)

Kinematics in 1D

(Young & Freedmann – Chapter 2)

Consider motion in a straight line, which we take to be the x-axis. At time t_i a car is at position x_i, and at position x_f at a later time t_f.



The change in the position is the displacement:

$$\Delta x = x_f - x_i$$

Definition

Velocity is the rate of change of position.

The average velocity is defined as:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Both displacement and average velocity can be positive or negative

The average speed is defined as:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{s}{t}$$

Since total distance and total time are always positive, average speed is always positive

The average velocity is the slope or gradient of the straight line joining the initial and final points

The average velocity depends on the time interval - see graph.

As we make the time interval shorter and shorter, the average velocity approaches the velocity at a particular instant, t. So can define the instantaneous velocity as

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The instantaneous velocity is the gradient of the line tangent to the x versus t curve

Relative velocity

- A frame of reference is an extended object whose parts are at rest relative to each other
- Consider a boat in river. If the water is moving at a velocity v_{wb} relative to the river banks and a kayak is paddling at velocity v_{kw} relative to the water, then the velocity of the kayak relative to the river banks is:

$$v_{kb} = v_{kw} + v_{wb}$$

Only true when $v << c$ – see Relativity next term

Acceleration

Acceleration is rate of change of velocity

The average acceleration is defined as:

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

The instantaneous acceleration is defined as:

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The instantaneous acceleration is the gradient of the line tangent to the v versus t curve

Equations of motion

Motion with constant acceleration

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\Delta x = v_0 t + \tfrac{1}{2} a t^2$$

$$\Delta x = v_{av} t = \tfrac{1}{2}\left(v_0 + v\right)t$$

Example

An astronaut is shipwrecked on an unknown planet. To find the free fall acceleration (acceleration due to gravity) he performs two experiments. He wanders until he finds a cliff, drops a stone over the cliff edge and notes that it takes 2 seconds to hit the ground. A similar stone, thrown 1m in the air over the cliff edge, takes 2.34 seconds to reach the ground. What is g(planet)?

Summary

- Displacement: $\Delta \mathbf{x} = \mathbf{x_f} - \mathbf{x_i}$
- Average velocity: $v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$
- Instantaneous velocity: $v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
- Relative velocity: $\mathrm{v}_{kb} = \mathrm{v}_{kw} + \mathrm{v}_{wb}$
- Average acceleration $a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$
- Instantaneous acceleration: $a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

For next lecture:

- Vectors, especially
  - manipulation in component form
  - dot product
- Equations of motion
- Differentiation and integration