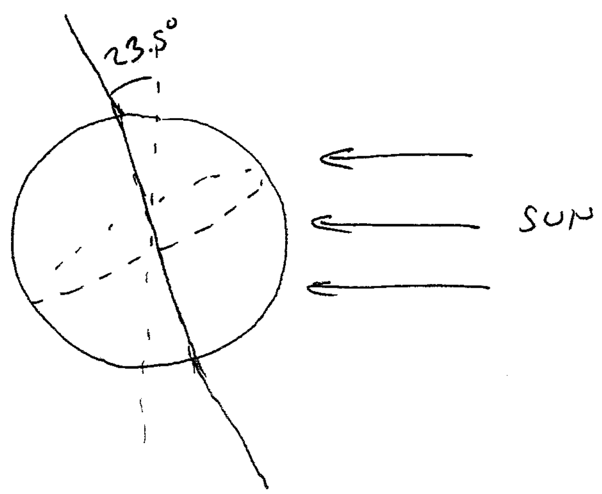
HORIZONTAL MOTION IN THE ATMOSPHERE

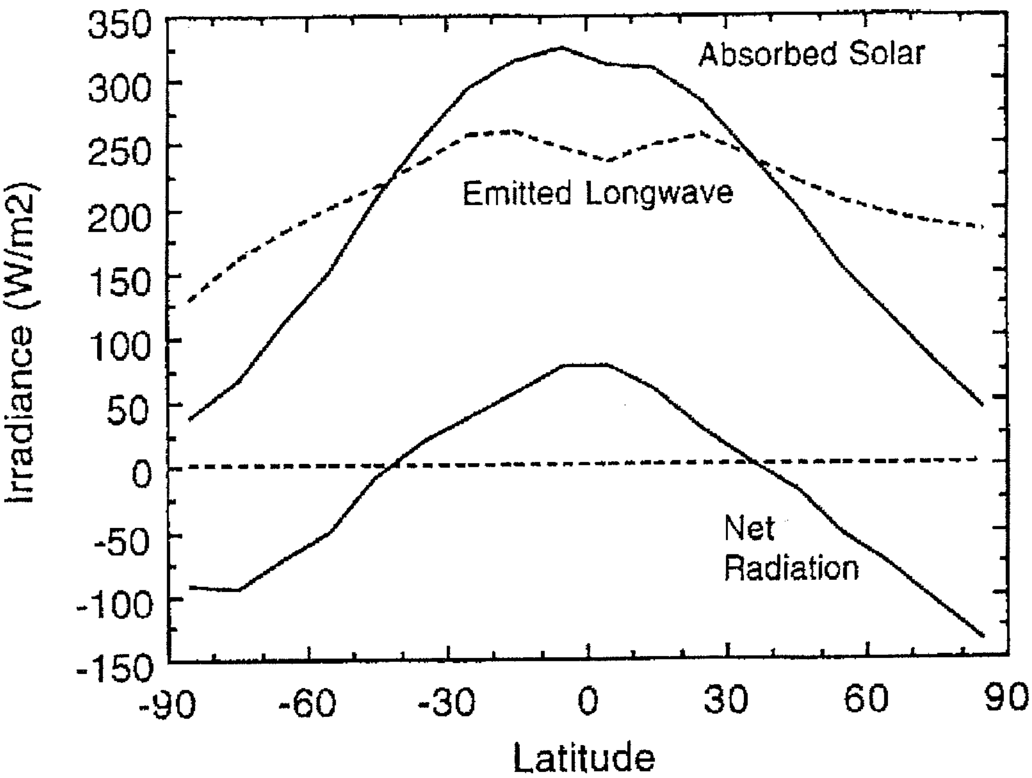
In the last three lectures, we have studied the vertical structure of the atmosphere. By considering the forces on parcels of air, and the behavior of air and water vapor, we were able to explain many features of the structure of the atmosphere.

In this lecture we will start to look at the horizontal structure and dynamics of the atmosphere. This will include the origins and behavior of weather systems which determine winds and rain.

Energy balance and driving

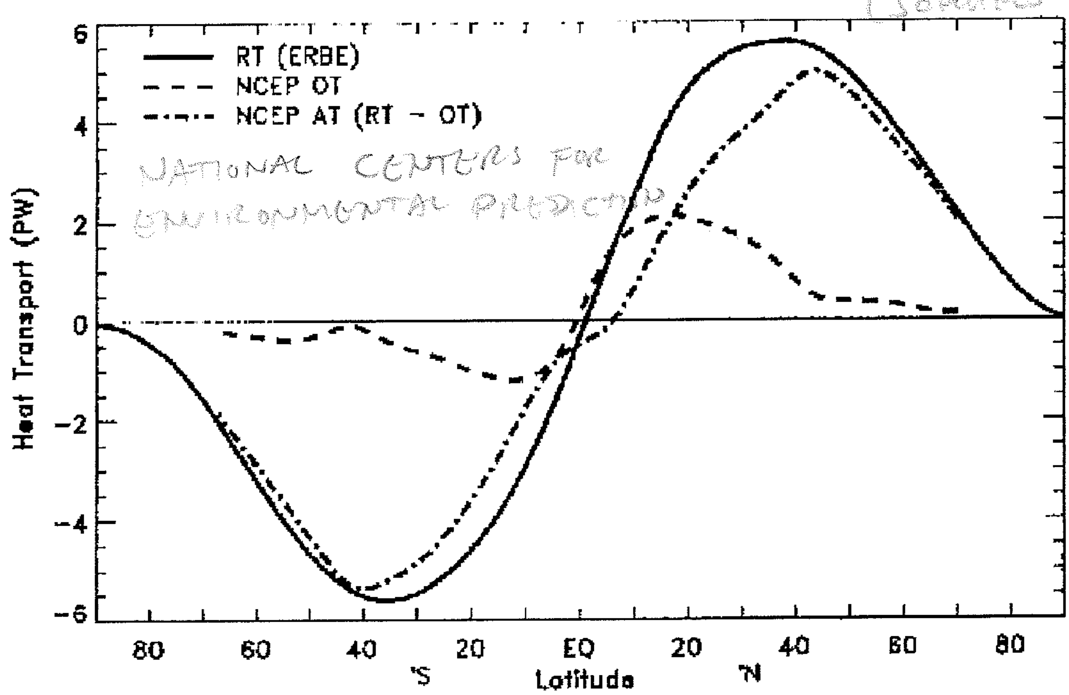


Incoming flux of radiation greater near the equator than the poles



Annual-mean absorbed solar flux, outgoing longwave radiation (OLR), and net radiation averaged around latitude circles.

Earth Radiation Budget Experiment (satellites launched '80s)



Total meridonal transport implied by ERBE measurements (RT) and the ocean transport (OT) and implied atmospheric transport (AT) from NCEP reanalysis. [Trenberth and Caron, 2001]

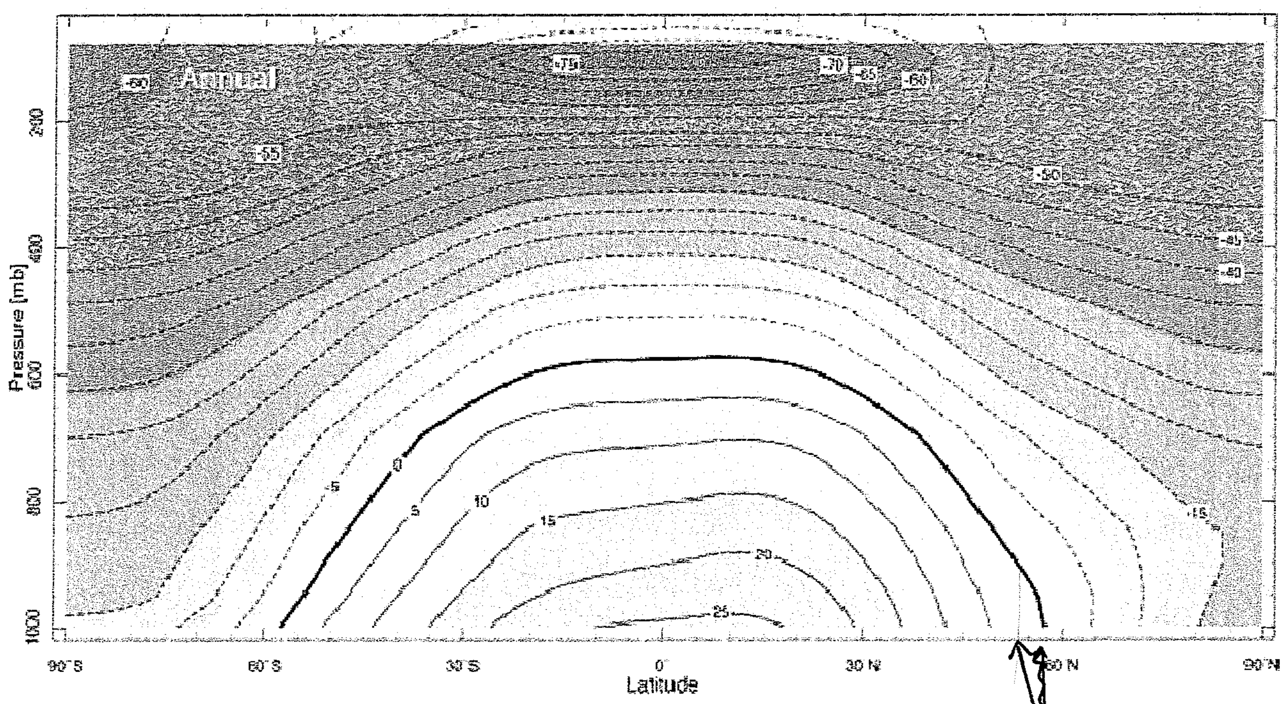
- Higher flux of solar radiation means equatorial atmosphere warmer than the poles

- This imbalance drives atmospheric and oceanic currents, which transport heat from the hot tropics to higher latitudes

- Based on satellite measurements of ingoing and outgoing radiation, the implied energy flux in each hemisphere is $\sim 6 \times 10^{15}$ W (6 PW)

[For comparison, in 2008 the average world power consumption was around 1.5×10^{13} W (15 TW)]

Zonal-Average Temperature (°C)

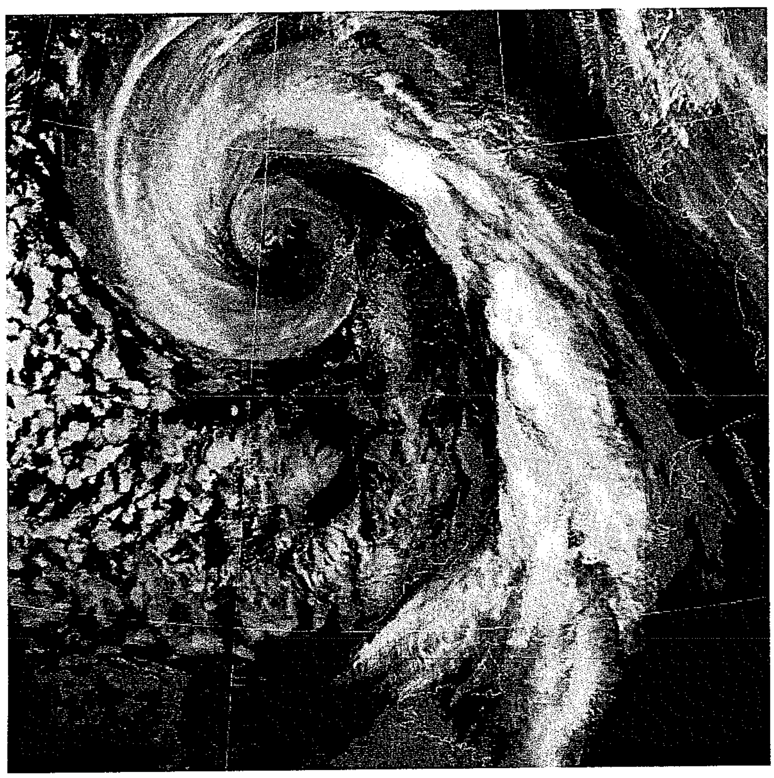


York ~~???~~
53° 58'

This region of steep temperature gradients is the origin of much of the weather which affects the UK. (POLAR FRONT)

The boundary between warm, moist air from the south and cold polar air is unstable (baroclinic instability) so kinks and leads to formation of low pressure regions: Mid-Latitude cyclones.

We will look in more detail at this formation process later in the course.

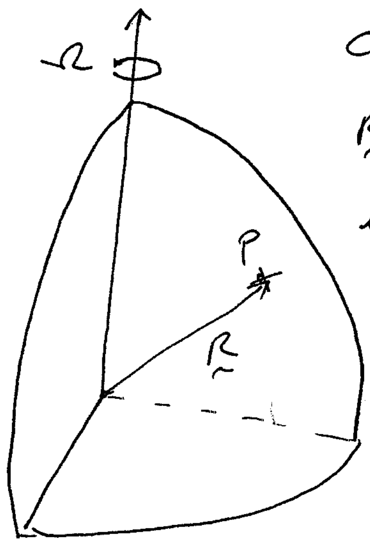


For now we'll try to explain why these cyclones have the structure they have.

Firstly, why do they rotate, forming spiral patterns?

Coriolis Force

The deflection of moving air parcels is a consequence of the rotation of the Earth. From our viewpoint moving objects appear to be subjected to forces, but really this is just because we are in a non-inertial frame of reference.



Consider a point P at a location $\underset{\sim}{R}$. From the point of view of someone in an inertial frame, P is rotating

$$\frac{dR}{dt}_{inertial} = \underset{\sim}{\Omega} \times \underset{\sim}{R}$$

Wind velocity $\underset{\sim}{V} = \frac{dR}{dt}_{Earth}$ as seen by observer on Earth

$$\Rightarrow \frac{dR}{dt}_{inertial} = \frac{dR}{dt}_{Earth} + \underset{\sim}{\Omega} \times \underset{\sim}{R}$$

Any vector quantity will appear to change over time because our coordinates are rotating

$$\frac{d}{dt}_{inertial} \underset{\sim}{V} = \left[\frac{d}{dt}_{Earth} + \underset{\sim}{\Omega} \times \right] \underset{\sim}{V}$$

$$\Rightarrow \frac{d^2R}{dt^2}_{inertial} = \left[\frac{d}{dt}_{Earth} + \underset{\sim}{\Omega} \times \right] \left(\frac{dR}{dt}_{Earth} + \underset{\sim}{\Omega} \times \underset{\sim}{R} \right)$$

$$= \frac{d^2R}{dt^2}_{Earth} + \Omega \times \frac{dR}{dt}_{Earth} + \underbrace{\frac{d}{dt}_{Earth} (\underset{\sim}{\Omega} \times \underset{\sim}{R})}_{= \Omega \times \frac{dR}{dt}_{Earth} \text{ if } \Omega \text{ const}} + \underset{\sim}{\Omega} \times \underset{\sim}{\Omega} \times \underset{\sim}{R}$$

$$\Rightarrow \left.\frac{d^2 R}{dt^2}\right|_{\text{inertial}} = \left.\frac{d^2 R}{dt^2}\right|_{\text{Earth}} + \underbrace{\Omega \times V + \Omega \times V}_{\text{CORIOLIS}} + \underbrace{\Omega \times \Omega \times R}_{\text{CENTRIPETAL}}$$

~~acceleration as seen by an observer on the Earth is the inertial acceleration with extra apparent forces~~

$$\cancel{a_{\text{Earth}} = a_{\text{inert}} =}$$

The acceleration as seen by someone standing on the Earth will therefore be

$$a_{\text{Earth}} = a_{\text{inertial}} - 2\,\Omega \times v - \Omega \times \Omega \times R$$

apparent forces caused by being in a rotating frame of reference

NOTE: Coriolis always $\perp$ to direction of movement

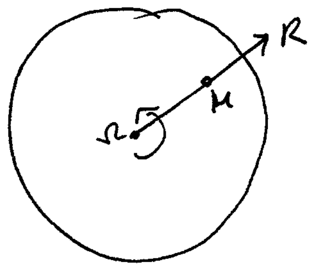
$\rightarrow$ does no work ($F \cdot V = 0$)

deflects motion to left or right

$\perp$ to Ω

An alternative explanation:

Consider a rotating carousel



A mass m at a radius R has an angular momentum

$$L = m v_\perp R = m \Omega R^2$$

If we now push this mass to a different radius, r has changed and so L has changed

$\therefore$ a torque must be exerted to keep Ω constant

$$\tau = F_C R = \frac{dL}{dt} = \frac{d}{dt}(m\Omega R^2) = 2m\Omega R \frac{dR}{dt}$$

hence sideways force we need to apply is

$$F_C = \tau/R = 2m\Omega V_R$$

which is the coriolis force

(acceleration $2\Omega V_R$)

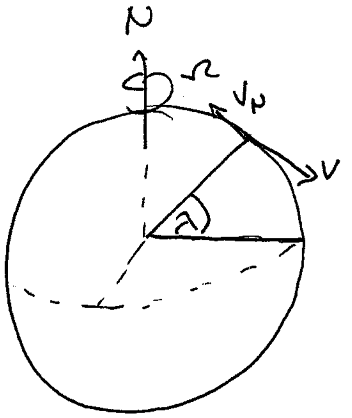
$\Updownarrow$

$2\Omega \times V_R$

Can also picture by drawing a straight line on a rotating wheel

$\Rightarrow$ a mass moving radially feels a force tangentially.

This is what happens when a mass moves north-south on the Earth:



Moving north-south takes you further or nearer to the rotation axis

λ = latitude

'radial' velocity is $V_R = -V_N \sin\lambda$

V_N: velocity northwards

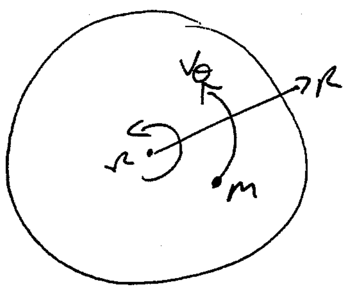
To move a mass north (i.e. inwards) we would have to apply a torque in ~~the~~ a westerly direction to keep it in a straight line (from our point of view). Therefore, without this torque a mass moving north will be deflected East (i.e. to the right).

- At the equator, $\lambda = 0 \rightarrow V_R = 0$ and the Coriolis force is zero
- At the poles, $\lambda = \pm\pi$ and the coriolis force is a maximum.

4.9

The coriolis force doesn't only affect motion north-south (or radially on a carousel)

East-west (around the carousel) motion also results in an apparent force.



Consider a mass m on a carousel. From the point of view of someone sitting on the carousel it is moving around at a velocity V_θ.

From outside (in an inertial frame), the ~~velocity~~ speed of the mass around the centre is

$$V_\theta' = V_\theta + \Omega R$$

and so the centripetal force is

$$F = \frac{m V_\theta'^2}{R} = \frac{m}{R}(V_\theta + \Omega R)^2 = \frac{m V_\theta^2}{R} + 2\frac{m}{R}V_\theta \Omega R + \frac{m}{R}\Omega^2 R$$

$$= \underbrace{\frac{m V_\theta^2}{R}}_{\text{centripetal}} + \underbrace{2m\Omega V_\theta}_{\text{Coriolis}} + \underbrace{\frac{m}{\cancel{R}}\Omega^2 R}_{\text{centripetal to stop slide}}$$

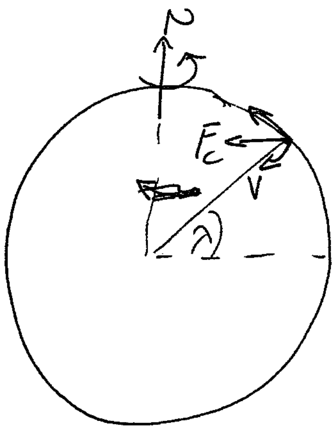
(balance gravity)

$m(2\,\Omega \times v)$

$m(\Omega \times \Omega \times R)$

This extra force $F = 2m\Omega V_\theta$ is now in the radial direction, caused by motion around the axis.

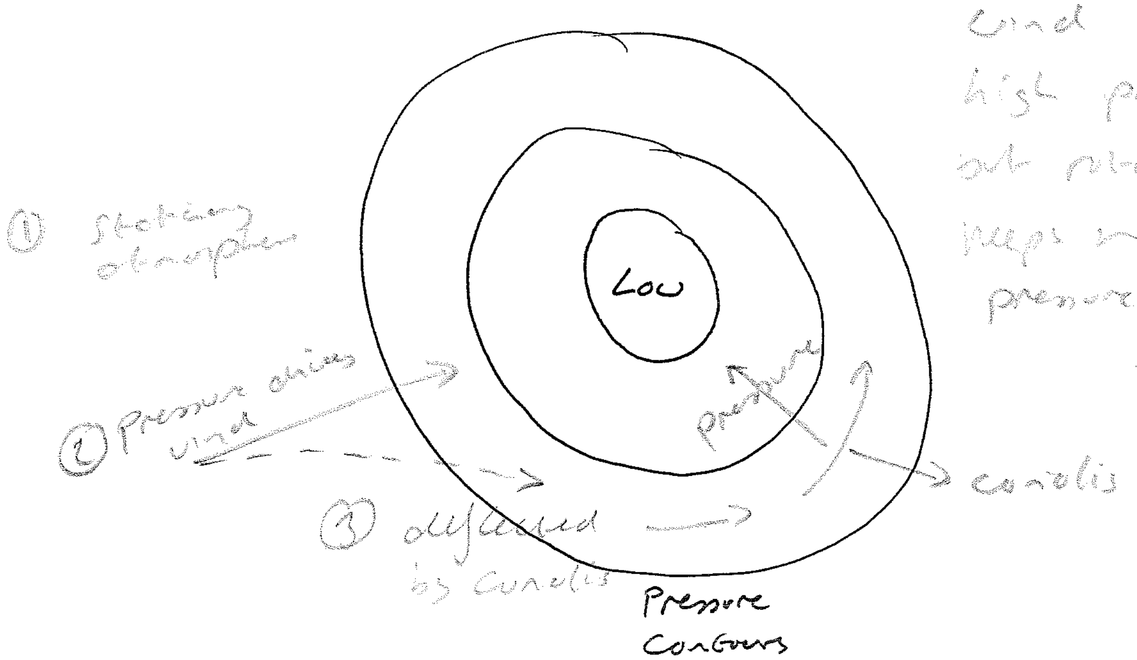
On the Earth:



Moving west (against the Earth's rotation) reduces the centripetal force outwards and so there is a net force inwards. The horizontal component of this

$$F_{North} = 2m\Omega V_{West} \sin\lambda$$

Like North-South motion, East-west motion also results in an apparent force to the right in the Northern hemisphere, and to the left in the southern hemisphere.



Wind does move from high pressure to low, but rotation of the Earth keeps moving the low pressure around

How big is the coriolis force?

acceleration magnitude $= 2\Omega V \sin\lambda$

where λ is the latitude

Ω is in radians per second : 2π per 24 hours

$\Rightarrow \underline{\Omega \simeq 7.3 \times 10^{-5} \text{ rad/s}}$

take $45°$ latitude $\rightarrow \sin\lambda = \frac{1}{\sqrt{2}}$

$\Rightarrow a_c = \underbrace{1.03 \times 10^{-4}}_{f} V$ f - coriolis parameter

So for a wind speed of 10mph = 4.5 m/s

$a_c = 4.6 \times 10^{-4} \text{ m/s}^2$

$\therefore$ needs a long time to have any effect.

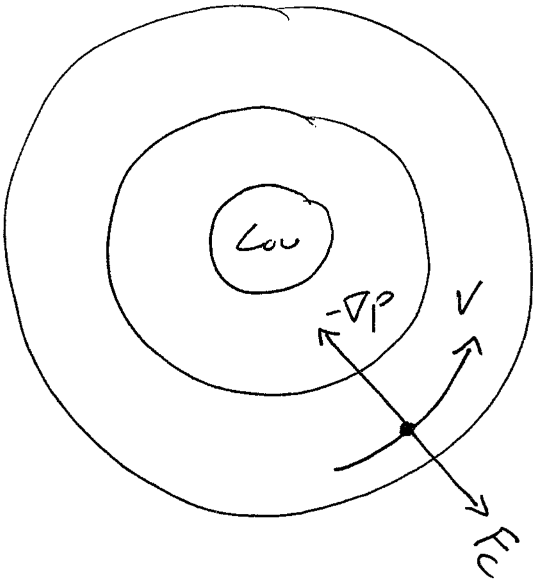
WATER IN A SINK

Does the coriolis force have any significant effect on the draining water from a sink?

$\rightarrow$ Problem question this week

Hint: Very little effect at all.

Can we use the Coriolis force to estimate wind speeds around a low or high-pressure region?



pressure gradient ∇P provides a force inwards from high to low pressure.

Low pressure system rotates anticlockwise (in Northern hemisphere) so force is to the right outwards.

This is called the <u>GEOSTROPHIC APPROXIMATION</u>.

Pressure P - N/m^2 so ∇P gradient of P is N/m^3

hence ~~of~~ the acceleration is $\frac{1}{\rho}\nabla P$

$\frac{1}{kg/m^3}$ $\quad$ $N/m^3 \rightarrow N/kg$ = acceleration

balance forces

$$\Rightarrow \frac{1}{\rho}\nabla P = f V_g$$

V_g - geostrophic wind

$$V_g = \frac{\nabla P}{f\rho}$$

Magnitude of geostrophic wind

millibar

$$\nabla P \sim 1\,\text{mb in } 100\,\text{km}$$

$$\frac{100\,\text{Pa}}{10^5\,\text{m}} \sim 10^{-3}\ \text{N/m}^3$$

$\rho \simeq 1.3\ \text{kg/m}^3$ atmospheric density at sea level

$f \simeq 1 \times 10^{-4}\ \text{s}^{-1}$

$$\Rightarrow \quad V_g \simeq \frac{10^{-3}}{10^{-4} \times 1.3} \simeq 8\ \text{m/s} \quad \text{or} \quad 18\ \text{mph}$$

In practice this approximation only applies for large-scale systems (so centrifugal force negligible) away from the surface (so friction with the ground neglected).

→ Next lecture: Other approximations and sources of wind.

SUMMARY

- The flux of solar radiation is greater at the equator than the poles

- This imbalance drives atmospheric and oceanic flows which transport around 6 PW of heat in each hemisphere.

- The POLAR FRONT is where hot moist air from the tropics meets cold polar air ~ 60° latitude.

- Unstable to kinks which develop into low pressure and high pressure systems (cyclones and anti-cyclones)

- The CORIOLIS FORCE in a rotating frame leads to an apparent acceleration

$$\underline{a}_c = -2\underline{\Omega}\times\underline{v}$$

- In the northern hemisphere moving masses are deflected to the right, in the southern hemisphere to the left.

- Low pressure regions therefore rotate anticlockwise in the northern hemisphere, whilst high-pressure (anticyclones) rotate clockwise.

- You should be able to derive the GEOSTROPHIC VELOCITY V_g