WIND

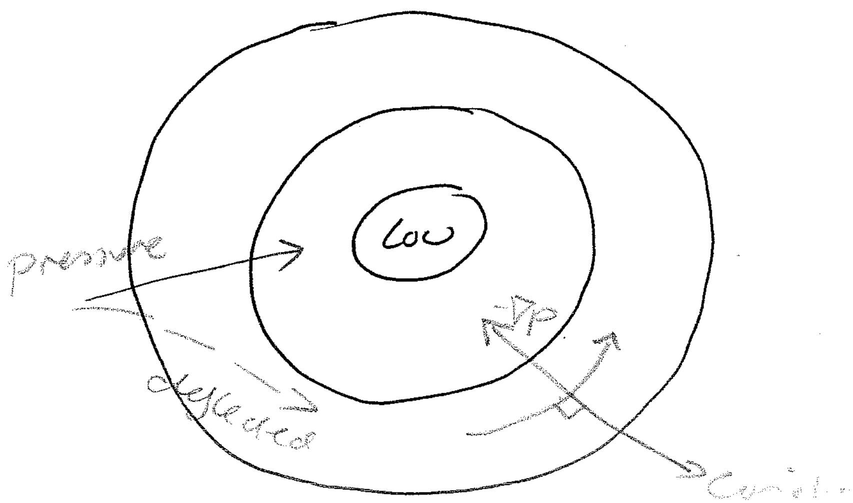
Last lecture we introduced the Coriolis force due to the rotation of the Earth. As observers in a non-inertial frame of reference, moving objects appear to be deflected to the right in the northern hemisphere, and to the left in the southern hemisphere

The magnitude of the Coriolis force is

$$F_c = 2m\Omega v \sin\lambda$$

λ - Latitude
Ω - Rotation of the Earth (radians/sec)
v - Velocity of the object

This is why weather systems rotate

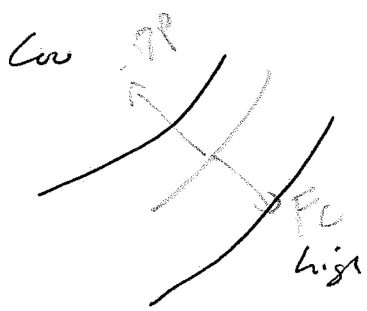


$\Rightarrow$ In the northern hemisphere, wind rotates anticlockwise around lows, clockwise around highs.

Another way to remember this is that in the Northern hemisphere:

If your back is to the wind, the low pressure is to your left.

Geostrophic approximation



balance pressure force against Coriolis

$$\frac{1}{\rho}\nabla P = \underbrace{2\Omega \sin\theta}_{f\text{ - Coriolis parameter}} V_g$$

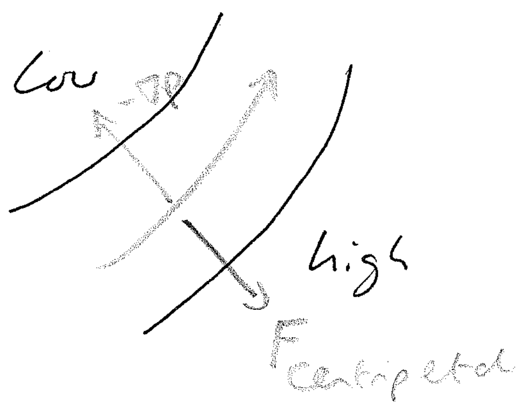
$$\Rightarrow \boxed{V_g = \frac{\nabla P}{f\rho}}$$ Geostrophic wind speed

This is not the whole story, and there are several other effects which can be important:

1. Centrifugal forces on fast spinning air
2. Friction between the air and the ground
3. Friction between layers of air
   e.g. air near ground and air above

Cyclostrophic approximation

If a weather system is rotating quickly enough, the centripetal acceleration can become more important than the Coriolis acceleration. This happens is systems like Tornadoes and dust-devils.

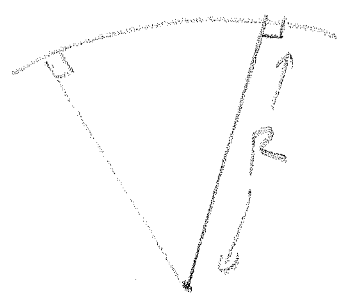


Now balance the pressure gradient and the centripetal acceleration

$$\frac{1}{\rho}\nabla p = \frac{V^2}{R}$$

R - radius of curvature

$$V^2 = \frac{R}{\rho}\underbrace{\nabla p}_{\frac{\partial p}{\partial R}}$$



$$\Rightarrow \quad V = \sqrt{\frac{R}{\rho}\frac{\partial p}{\partial R}}$$

- Wind speed increases with distance from centre of low
- Can only happen in low pressure systems
- Can be either clockwise or anticlockwise

When is the cyclostrophic approximation valid? Depends on the relative importance of the Coriolis force and centrifugal force.

$$\frac{1}{\rho}\nabla P = fV + \frac{V^2}{R}$$

Coriolis acceleration

Centri~~petal~~fugal acceleration

Take the ratio

$$\frac{\text{Centripetal acceleration}}{\text{Coriolis acceleration}} = \frac{V^2/R}{fV}$$

$$= \frac{V}{fR}$$

This ratio $\boxed{Ro = \frac{V}{fR}}$ is the ROSSBY NUMBER

e.g.

(1) Tornado $V \simeq 100$ mph, 45 m/s

$R \simeq 100$ m

$f = 2\Omega \sin\lambda \simeq 10^{-4}$ rad/s

S.S

$\Rightarrow R_o \sim 4 \times 10^3$ for a tornado

cyclostrophic approximation good for tornadoes

② Mid-latitude cyclone (low pressure region)

$\sim 1500 - 5000$ km diameter

wind speed < 10 mph $(4.5\ m/s)$

taking $R = 1000$ km $(10^6 m)$

$$\Rightarrow R_o = \frac{4.5}{10^{-4} \cdot 10^6} = 4.5 \times 10^{-2}$$

$\Rightarrow$ Geostrophic approximation generally good for mid-latitude cyclones.

③ Hurricane

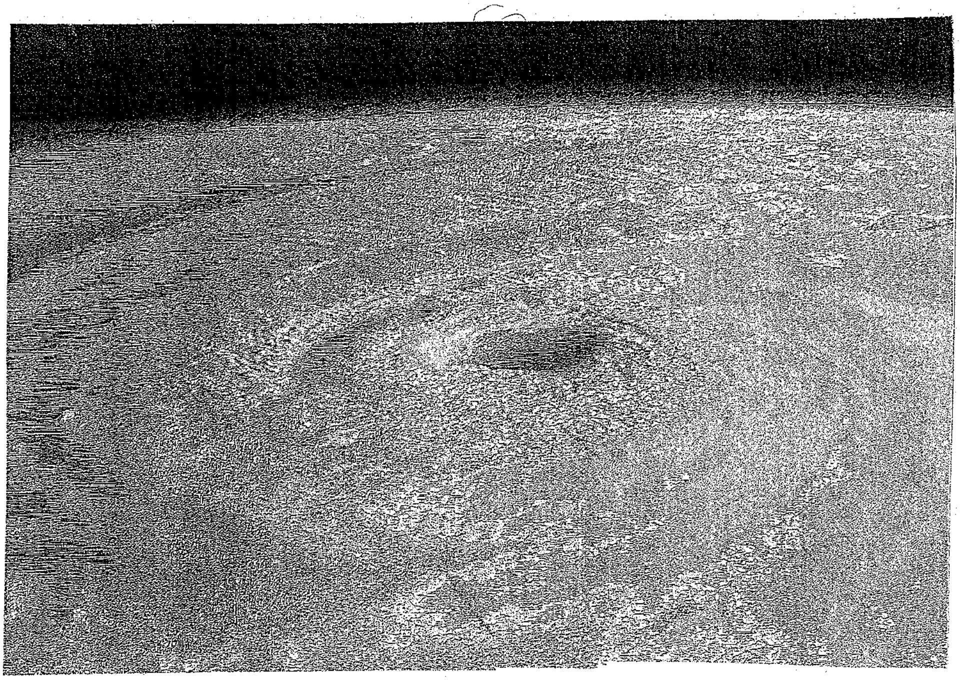
$R \sim 500\,km$

$V \sim 70\,mph \quad (30\,m/s)$

$\Rightarrow R_o \sim 0.6$

Centrifugal and Coriolis forces comparable

Large variation with radius



Hurricane Isabel (2003) seen from ISS

Hurricane eye (region of falling air) is typically ~ ~~[illegible]~~ 25 km in diameter, and peak wind speeds occur around the eye wall

$$R_o \simeq \frac{30}{10^{-4} \cdot 2.5\times10^{4}} \Rightarrow R_o \sim 12$$

towards the middle of a hurricane $R_o > 1$

Moving outwards away from the eye wall,
the wind speed drops and R increases

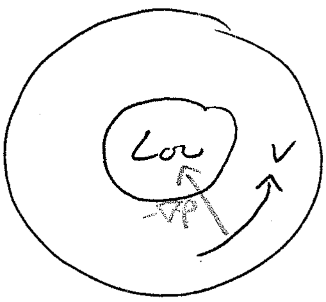
$\Rightarrow$ Rossby number Ro falls

from >1 near eye to <1 at the edges

Can't use geostrophic or cyclostrophic approximation
to describe hurricanes

Need to include both effects

Gradient Wind



$$\underbrace{\frac{1}{\rho}\nabla P}_{\frac{1}{\rho}\frac{\partial P}{\partial R}} - Vf = \frac{V^2}{R}$$

Rearrange

$$\frac{V^2}{R} + Vf - \frac{1}{\rho}\nabla P = 0$$

Quadratic solution:

$$V = \frac{-f \pm \sqrt{f^2 + \frac{4}{R\rho}\nabla P}}{2/R}$$

$$\boxed{V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} + \frac{R}{\rho}\frac{\partial P}{\partial R}}}$$

Gradient wind

[Should be able to derive, not remember]

Around a low pressure system (cyclone)

$\frac{\partial P}{\partial R} > 0$ (pressure increasing with radius)

So either

- Negative root: $$V = -\frac{fR}{2} - \underbrace{\sqrt{\frac{f^2R^2}{4} + \frac{R}{\rho}\frac{\partial P}{\partial R}}}_{> \frac{fR}{2}}$$

$\Rightarrow V < -fR$

i.e. $\frac{V}{fR} < -1$ i.e. cyclostrophic

Rossby number

- Positive root

$$V = -\frac{fR}{2} + \sqrt{\frac{f^2R^2}{4} + \frac{R}{\rho}\frac{\partial P}{\partial R}}$$

Provides a good approximation in many atmospheric phenomena with curved flow.

Curvature reduces the wind velocity below the geostrophic approximation

$$\frac{1}{\rho}\nabla P = Vf + \frac{V^2}{R}$$

$$V = \underbrace{+\frac{\nabla P}{f\rho}}_{\text{geostrophic}} - \frac{V^2}{R} \leftarrow \text{as radius gets smaller this becomes important.}$$

Around high pressure regions (anti-cyclones):
here V is negative (clockwise rotation)
So $|V|$ is <u>greater</u> than geostrophic approximation.

<u>Note</u>: Since $\frac{\partial P}{\partial R} < 0$, the term inside the square root can go negative

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} + \underbrace{\frac{R}{\rho}\frac{\partial P}{\partial R}}_{\text{negative}}}$$

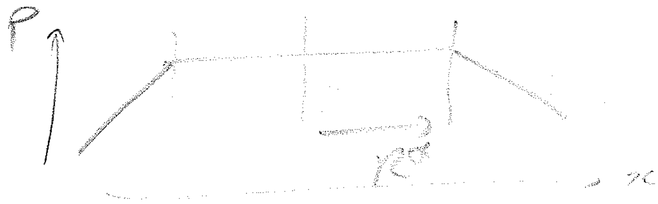
A complex velocity is unphysical, so to avoid this,

$$\frac{f^2R^2}{4} > \frac{R}{\rho}\frac{\partial P}{\partial R} \qquad \frac{fR}{4} = \underbrace{\frac{1}{f\rho}\left|\frac{\partial P}{\partial R}\right|}_{V_g \text{ geostrophic}}$$

$$fR > 4V_g \quad \text{or} \quad R > \frac{4V_g}{f}$$

→ Inside $R^* \sim \frac{4}{f}V_g$, the equation doesn't have real solutions. In this region other effects must be important [friction]

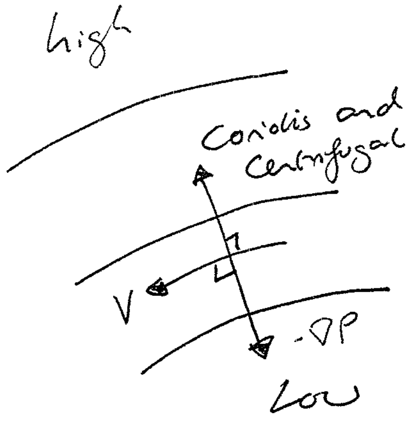
⇒ Usually find little wind inside high pressure regions. and small pressure gradients



FRICTION

In addition to pressure gradients, Coriolis and centrifugal forces, moving air also experiences friction with the ground, ocean, and air above.

If there was no friction, air could continue to follow isobars (lines of constant pressure)



Forces $\perp$ to $\underset{\sim}{V}$

$\Rightarrow \underset{\sim}{F} \cdot \underset{\sim}{V} = 0$

So no work done

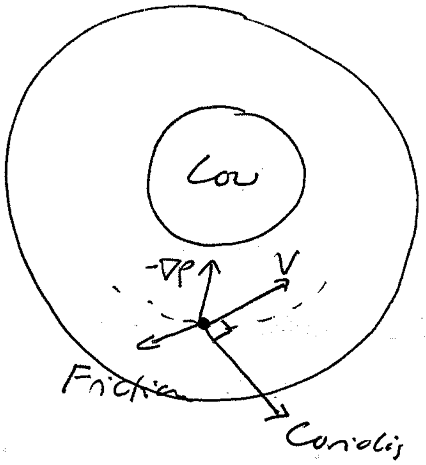
If the wind experiences friction with the ground, it will experience a friction force opposite to its velocity.

Assuming the force $\propto V^2$

$$F = -kV^2$$

If this becomes the dominant effect,

$$kV^2 = \frac{1}{\rho}\left|\frac{\partial P}{\partial R}\right| \quad \text{ANTITRIPTIC wind}$$



More generally, friction can lead to air spiralling inwards towards low pressure

NOTE

Wind also feels a friction force due to higher or lower wind speeds higher up (wind shear).

$\Rightarrow$ Can reverse direction due to this friction

Next lecture.

SUMMARY

- Rossby number is the ratio of centrifugal to Coriolis forces $R_o = \frac{V}{fR}$

- When $R_o >> 1$, the Coriolis force can be neglected → Cyclostrophic velocity

  This is the case for tornadoes, dust devils etc.

$$V^2 \simeq \frac{R}{\rho}|\nabla p|$$

- When $R_o << 1$, the centrifugal force can be neglected → Geostrophic velocity

$$V_g \simeq \frac{|\nabla p|}{f\rho}$$

  This is usually the case for mid-latitude cyclones and "everyday" weather.

- In hurricanes, R_o goes from >1 near the middle to $<<1$ at the edge

  → need to include both

  GRADIENT WIND

- Curvature lowers velocity around lows relative to geostrophic, raises velocity around highs

- Within a radius $R^* \sim \frac{4}{f}V_g$ of a high-pressure region, gradient wind doesn't have real solution. In this region, friction is important and winds tend to be small.