ENV. PHYS. **WATER IN THE ATMOSPHERE**

Last lecture we studied the change in pressure and temperature as a parcel of air moves upwards through the atmosphere.

Why did we find that this gave a gradient $\frac{dT}{dh}$ which was too steep: -10 K/km rather than the observed -6 K/km?

Many simplifications used, but main effect missing was Water; we assumed the air was dry.

Moist air

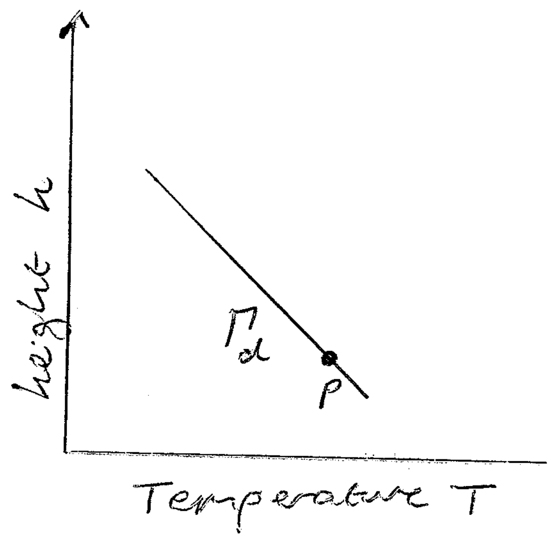
What if the air contains a small amount of moisture?

Water molecules in gas phase (water vapour) behave just like any other gas. It is always quite a minor component so does not affect the adiabatic expansion of the air.

$\Rightarrow$ Not much. $\frac{dT}{dh} = -\Gamma \simeq -\Gamma_d$ the same as dry air

Saturated air

Imagine an atmosphere which is mostly dry and follows the adiabatic lapse rate. A parcel of air at point P is then saturated with air and moved upwards a small amount.



As it expands ⟶ cools adiabatically

cold air can "hold" less water than warm air
[we'll see shortly that the air doesn't hold anything]

→ water vapor condenses into liquid

→ this releases energy (latent heat) which heats the air above what it would be if dry.

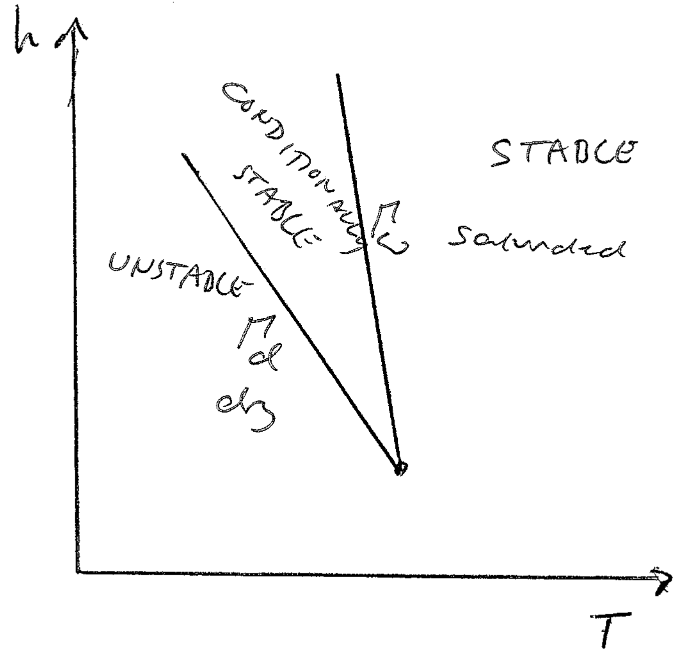
If a parcel of air is saturated but surrounded by dry air, when it rises it will end up even hotter than the surroundings

⇒ Saturated air is unstable at the dry adiabatic lapse rate

⇒ The saturated lapse rate Γ_w must be less than the dry lapse rate Γ_d

$$\Gamma_w < \Gamma_d$$

Because $\Gamma_w < \Gamma_d$, the stability of the atmosphere depends on the moisture content



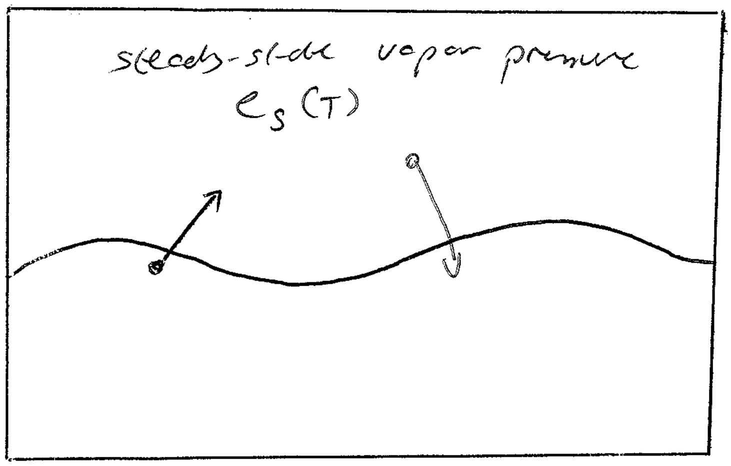
Water has a large <u>latent heat of evaporation</u>, i.e. it takes a lot of energy to evaporate water, and when it condenses again this energy goes into heat.

⇒ Convection of water vapour has a large impact on the structure of the troposphere, transporting energy upwards and so reducing the lapse rate.

Rising air will follow the dry lapse rate until it saturates. At this point when it rises further water must condense out, and so it follows the moist lapse rate.

To see how this works, we need to look at how liquids evaporate.

Imagine a box, half-filled with water:



Molecules in the liquid with enough energy will leave the liquid and become a gas.
Meanwhile, molecules of water vapour can condense again and join the liquid.

This process happens even if the liquid is not at boiling temperature.

Eventually the number of molecules evaporating matches the number condensing, and we reach steady state. The resulting pressure of the water vapor above the surface is called the
<u>Saturation vapor pressure.</u>

<u>IMPORTANT</u>: The Saturation Vapor pressure does <u>not</u> depend on the total pressure, so is not affected by presence or absence of air.

The pressure of the water vapor in the box would be the same with or without air in the box. The total pressure of air + water vapor would be different.

PARTIAL PRESSURE

When we derived the ideal gas law by considering molecules bouncing off walls of a box, interactions between the particles were not important [they do make a small difference, but not significant for our purposes]. This means that we can consider each component of a mix of gasses independently, then just add up the pressures

$$P = P_{N_2} + P_{O_2} + P_{H_2O} + \cdots$$

The pressure due to each component individually is called its Partial Pressure e.g. the "Partial Pressure of water vapor" is the pressure due to the water molecules alone, and does not depend on the other gasses.

The partial pressure of water vapor can be calculated using Thermodynamics or Statistical mechanics, and is given by the Clausius - Clapeyron relation (c. 1834).

A good approximation is the August - Roche - Magnus or just Magnus formula for the saturated vapor pressure of water e_s :

$$e_s = C \exp\left[\frac{AT}{B+T}\right] \quad \text{pascals}$$

with $A = 17.625$ with T in <u>Celsius</u>.

$B = 243.04\ °C$

$C = 610.94$ Pa

This is accurate to about 0.4% for T between -40°C and +50°C.

<u>Relative Humidity</u>

The relative humidity is how much water vapor is in the air relative to the maximum (saturated) amount. This can be defined in terms of the partial pressure e of water vapor:

$$RH = 100 \frac{e}{e_s(T)}$$

The relative humidity is therefore 100% when the partial pressure of water vapor is saturated.

Because the saturated value depends on temperature, 100% humidity on a cold day doesn't feel nearly as sticky as 100% humidity on a hot day.

DEW

In the early morning or evening the ground is cold because it has radiated heat away. When the ground becomes colder than the air above it, water can condense out, forming dew.

What temperature does the ground need to be for this to happen?

Depends on the air temperature and relative humidity

- Air close to the ground cools
- the saturated vapor pressure $e_s \sim e^{AT/B} \rightarrow$ decreases with ~~pr~~ temperature
- eventually e_s will fall so that $e_s = e$ and the air is saturated.
- If the temperature falls further, $e_s < e$, and so some water vapor must condense to reduce e.

Therefore, dew will start to form when the air becomes saturated. This temperature is called the <u>Dew Point</u>. T_d

If $e(T)$ is the partial vapor pressure in the air at temperature T, then we can write

$$e_s(T_d) = e(T) \qquad e(T) = \frac{RH}{100} \cdot e_s(T)$$

$$\Rightarrow \cancel{C} e^{\left(\frac{AT_d}{B+T_d}\right)} = \cancel{C} \frac{RH}{100} e^{\left(\frac{AT}{B+T}\right)}$$

Take log of both sides

$$\frac{AT_d}{B+T_d} = \ln\left(\frac{RH}{100}\right) + \frac{AT}{B+T} \qquad AT_d = \left[\ln\left(\frac{RH}{100}\right) + \frac{AT}{B+T}\right](B+T_d)$$

$$\left[A - \ln\left(\frac{RH}{100}\right) - \frac{AT}{B+T}\right]T_d = B\left[\ln\left(\frac{RH}{100}\right) + \frac{AT}{B+T}\right]$$

$$\Rightarrow T_d = \frac{B\left[\ln\left(\frac{RH}{100}\right) + \frac{AT}{B+T}\right]}{A - \ln\left(\frac{RH}{100}\right) - \frac{AT}{B+T}}$$

This expression $T_d = \dfrac{B\left[\ln\left(\frac{RH}{100}\right) + \frac{AT}{B+T}\right]}{A - \ln\left(\frac{RH}{100}\right) - \frac{AT}{B+T}}$ is

quite accurate, but a little too cumbersome most of the time. It would be nice to have a simpler approximation we can use most of the time. To do this, we can make the following assumptions:

- high humidity $RH \gtrsim 50\%$
- $T \sim 0°C$ (273 K)

Taylor expand $\ln\left(\frac{RH}{100}\right)$ around $RH = 100$

$$\rightarrow \ln\left(\frac{RH}{100}\right) \simeq \ln(1) + \underbrace{\frac{d}{dx}(\ln x)}_{\frac{1}{x} = 1}\left(\frac{RH}{100} - 1\right) = \frac{1}{100}(RH - 100)$$

Since T is small, we simplify the denominator

$$A - \ln\left(\frac{RH}{100}\right) - \frac{AT}{B+T} \sim A \qquad B+T \sim B$$

$$T_d \simeq \frac{B}{A}\left[\frac{1}{100}(RH-100) + \frac{AT}{B}\right] = \underbrace{\frac{B}{100A}}_{\cancel{7}.25}(RH-100) + T$$

$$\simeq T - \frac{1}{7}(100 - RH)$$

better fit to data is ~ 5, rather than 7.

A reasonable approximation which will allow us to ~~approximate~~ calculate the dew point is

$$\boxed{T_d \simeq T - \left(\frac{100 - RH}{5}\right)}$$

where RH is the relative humidity (in %) and this is valid for $T \sim 0°C$, and $RH \gtrsim 50\%$ i.e. moist air only.

Hence if the air is at 5°C and 80% relative humidity, dew will form if the ground temperature falls below

$$T_d \approx 5 - \left(\frac{100-80}{5}\right) \simeq \underline{\underline{1°C.}}$$

[Full expression gives $T_d = 1.8°C$]

Cloud Formation

This same equation for the dew point can also be used to study clouds.

- Warm air rises, taking water vapor with it
- As the air rises the temperature falls.
- T falls so e_s falls.
- Eventually $e_s < e$ and so water vapor will start to condense

NOTE: needs some nucleation sites to start e.g. smoke, dust, salt etc. In practice always present.

$\Rightarrow$ Clouds will start to form once a rising parcel of moist air reaches its dew point.

The same principle as before applies: Once the temperature of a parcel of air reaches its dew point (saturation), any further fall in temperature will cause water vapor to condense into droplets.

What height will clouds form? need to know:

T_0 - temperature close to the ground

Γ_d - (dry) adiabatic lapse rate

At a height z, the temperature is therefore

$$T \approx T_0 - \Gamma_d z$$

clouds form when this reaches the Dew point $T \approx T_d$

$$\Rightarrow \cancel{T_0} - \Gamma_d z \approx \cancel{T_0} - \left(\frac{100 - RH}{5}\right)$$

$$\Rightarrow z \approx \frac{1}{5\Gamma_d}(100 - RH) \qquad \Gamma_d \approx 10\ \text{K/km}$$

$$\approx \frac{1}{50}(100 - RH)\ \text{km}$$

$$\approx 20(100 - RH)\ \text{m}$$

e.g. 80% humidity at ground level $\underline{z \approx 400\text{m}}$

A small correction to the above estimate is to modify the coefficient

$$\boxed{z \simeq \left(20 + \frac{T_0}{5}\right)(100 - RH)\ \text{meters}} \qquad T_0 \text{ in } {}^\circ\text{C}$$

This is accurate to within $\pm 15\%$ for $50 \leq RH \leq 100\%$

and $0^\circ < T_0 < 30^\circ\text{C}$

Moist air can be lifted to this height and form clouds by:

- Convection, if the air is unstable ($\Gamma > \Gamma_d$)

  This forms <u>cumulus</u> clouds. Cumulo<u>nimbus</u> are when this produces rain

- Wind driving air over mountains

- Warm air meeting cold air (fronts)

  Warm air lifted up, cools & forms <u>stratus</u> ("layer") clouds

* Next lecture we'll start to look at wind and some of these other mechanisms.

SUMMARY

- Water has a big impact on the structure of our atmosphere
- Unsaturated water vapor behaves much like dry air, and changes temperature at the same lapse rate Γ_d
- When saturated air cools, water vapor condenses, giving up its latent heat. The lapse rate for saturated air is therefore less than for dry air $\Gamma_w < \Gamma_d$.
- The saturated partial pressure of water is approximately given by the Magnus formula [NOT expected to know formula]
- Relative Humidity is the amount of water in the air ~~relative~~ as a percentage of the saturated amount.

The Dew point T_d is the temperature a parcel of air must be cooled to, to become saturated.

It is approximately given by $T_d \simeq T - \left(\frac{100 - RH}{5}\right)$

By combining the dry lapse rate and dew point, the height of the cloud base can be estimated

[You should be able to derive this]

$z \simeq 20\,(100 - RH)$ metres