Collisional transport

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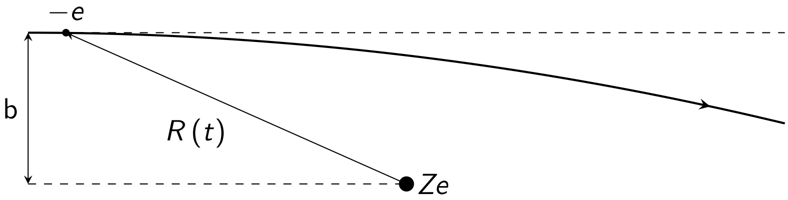
Confinement

- At fusion temperatures we need to confine charged particles
- In "open" systems like mirrors, particles with a particular range of velocities will escape
- Toroidal configurations like tokamaks solve this end loss problem[1] but particles and energy still leak out

- Instabilities and turbulence are the primary reason, and will be covered later in the course
- Even without turbulence, collisions between particles lead to transport and other effects
- Over the next 3 lectures we'll look at collisions and particle orbits in toroidal machines

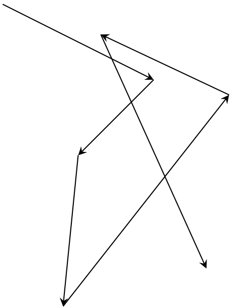
[1] Mostly. See stellarator lecture later.



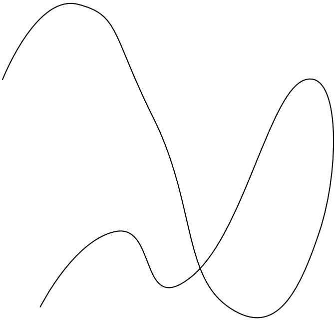
- Charged particles (e.g. electron and ion) pass each other with impact parameter b
- Electrostatic forces between them changes their velocity
- Most of these "collisions" occur between particles separated by $\sim \lambda_D$, the Debye length, since charges are shielded for $b \gg \lambda_D$
- The ratio $\Lambda = \lambda_D / b_{min}$, where b_{min} is the impact parameter which would give a "large" deflection is typically very large, so $\ln \Lambda \sim 10 \rightarrow 20$. Therefore collisions are small angle.

Collision times

Because most of the deflection of a particle in a plasma is due to lots of small collisions, the path of a particle looks quite different to in a gas



Molecule path in a gas



Path in a plasma

How to define a collision time when particles are always colliding?

Collision times

In a plasma, the **collision time** is defined as the average time it takes a particle to be deflected by 90^o.

$$\tau_{ei} = \frac{12\pi^{3/2}}{2^{1/2}} \frac{m_e^{1/2} T_e^{3/2} \epsilon_0^2}{n_i Z^2 e^4 \ln \Lambda}$$

$$\tau_{ei} = 3.44 \times 10^{11} \left(\frac{1m^{-3}}{n_e}\right) \left(\frac{T_e}{1eV}\right)^{3/2} \frac{1}{Z_i \ln \Lambda}$$

Often written as a **collision rate**

$$\nu_{ei} \equiv 1/\tau_{ei}$$

Putting in $T_e = 10\text{keV}$, and a density of $n_e = 10^{20}\text{m}^{-3}$ gives $\tau_{ei} = 1.7 \times 10^{-4}\text{s}$.

Plasma resistivity

One consequence of collisions is that plasmas have an electrical resistance.

- Consider an electron moving along a magnetic field. It experiences a force due to an applied electric field

$$\mathbf{F} = -e\mathbf{E}$$

- Collisions with ions deflect the electrons, so on a timescale τ_{ei} electrons lose their momentum. This can be written as a force

$$\mathbf{F} = -m_e\left(\mathbf{V}_e - \mathbf{V}_i\right)\nu_{ei}$$

- In steady state these forces balance (neglecting other effects)

$$\begin{aligned} -e\mathbf{E} \quad &- \quad m_e\left(\mathbf{V}_e - \mathbf{V}_i\right)\nu_{ei} = 0 \\ \mathbf{E} \quad &= \quad \underbrace{\frac{m_e\nu_{ei}}{e^2 n}}_{\eta}\underbrace{en\left(\mathbf{V}_i - \mathbf{V}_e\right)}_{\mathbf{J}} \end{aligned}$$

Plasma resistivity

The plasma resistivity is therefore approximately:

$$\eta = \frac{m_e \nu_{ei}}{e^2 n} \simeq 10^{-3} Z_i T_e^{-3/2} \quad \text{with } T_e \text{ in eV}$$

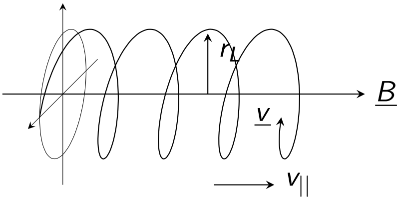
So at $T_e = 10\text{keV}$, $\eta \simeq 10^{-9} \quad \Omega\text{m}$
For reference, copper has a resistivity of around $1.7 \times 10^{-8} \Omega\text{m}$.

- In a tokamak, plasma current requires external drivers to be sustained (except the bootstrap current. See later)

- Resistivity also leads to **Ohmic heating** ηJ^2
- Since $\eta \sim T_e^{-3/2}$ whilst Bremsstrahlung radiation loss goes like $\sim T^{1/2}$, this sets a maximum temperature achievable with Ohmic heating. Usually this limit is around 3keV, below the required fusion temperatures $\Rightarrow$ Need extra **auxilliary** heating.

Collisions in magnetic fields

Particles in a magnetic field follow a helix



The position of the particle $\underline{x}$ is

$$\underline{x} = \underline{X} + \frac{1}{\Omega}\underline{b} \times \underline{v}$$

where $\underline{X}$ is the guiding centre, and $\Omega = qB/m$ is the gyro-frequency. Small changes in velocity due to collisions therefore lead to a change in position

$$\delta X = -\frac{1}{\Omega}\underline{b} \times \delta\underline{v}$$

Collisions in magnetic fields

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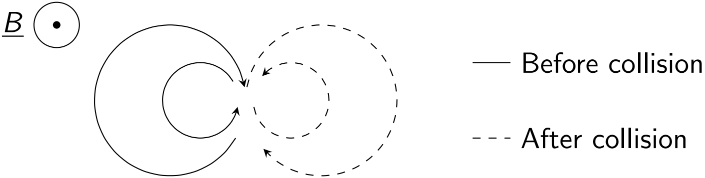
Here imagine for simplicity a large deflection collision:

- Two unlike particles in a magnetic field



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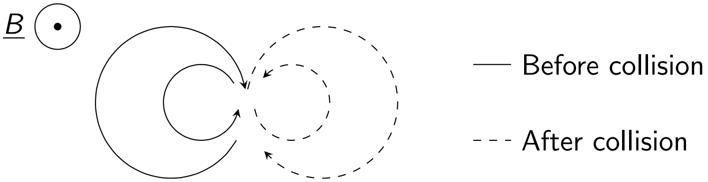

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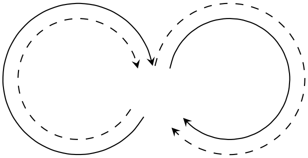

Just switch position $\Rightarrow$ No net particle diffusion

Collisional transport

- The velocity of an electron changes on a timescale

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- Treat this as a random walk, step size r_L and timescale τ_{ei}
- After N steps, we can expect a particle to have moved

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At 10keV, $n = 10^{20}\text{m}^{-3}$ and $B = 1\text{T}$, $\tau_{ei} = 1.7 \times 10^{-4}\text{s}$ and $r_{Le} \simeq 2.4 \times 10^{-4}\text{m}$. After 1 second, a typical electron will have moved a distance

$$d_e \sim \sqrt{\frac{1}{1.7 \times 10^{-4}}} \cdot 2.4 \times 10^{-4} = 1.8 \times 10^{-2} m$$

Collisional transport

What about ion transport?

- Ions colliding with ions doesn't lead to any particle transport as the total change in velocity is zero (conservation of momentum)
- Since ions are more massive than electrons, it takes more collisions to change the momentum so $\tau_{ie} = \frac{m_i}{m_e}\tau_{ei}$
- The ion Larmor radius is also larger than the electron'

$$\frac{r_{Li}}{r_{Le}} = \frac{v_{\perp i}}{v_{\perp e}}\frac{m_i}{m_e} = \sqrt{\frac{m_i}{m_e}} \sim 61.0$$

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- This means that the transport of ions

$$d_i = \sqrt{\frac{t}{\tau_{ie}}} r_{Li} = \sqrt{\frac{t}{\tau_{ei}}} r_{Le} = d_e$$

i.e. the transport of particles is the same for electrons and ions. This is called **ambipolarity** and is due to the conservation of momentum

Heat transport

- Transport of electrons and ions only depends on the electron-ion collision rate τ_{ei}, and is ambipolar i.e. transport of electrons and ions is the same
- This is not the case for heat transport, where collisions between the same species can transfer energy
- Since the ion Larmor radius is much larger than the electron (by factor of $\sqrt{m_i/m_e} \simeq 61$), they transport the most heat
- The collision rate for ions with ions τ_{ii} is longer than τ_{ei} because of the lower thermal velocity, but shorter than τ_{ie}:

$$\tau_{ei} < \tau_{ii} \sim \sqrt{\frac{m_i}{m_e}}\frac{1}{Z^2} < \tau_{ie} \sim \frac{m_i}{m_e}\tau_{ei}$$

- τ_{ii} therefore determines the dominant collisional transport here

Heat transport

We can do the same calculation as before, that heat takes a step r_{Li} at a rate $1/\tau_{ii}$. Hence the distance this energy will be transported in a time t is

$$d_E \sim \sqrt{\frac{t}{\tau_{ii}}} \cdot r_{Li}$$

which for $T = 10\text{keV}$, $n = 10^{20}\text{m}^{-3}$ and $B = 1\text{T}$ gives

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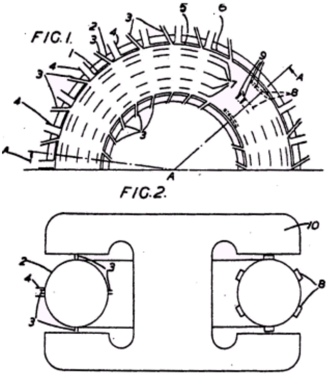
This is fantastic! It implies that we can make a machine with a minor radius r of a few $\times 10\text{cm}$ which will ignite

Toroidal machines

This was one source of early optimism about MCF, and lead to an early design for a reactor: G. P. Thomson and M. Blackman, British Patent No. 817 681, 1946[2] [3]



- Major radius R=1.3m
- Minor radius r = 0.3m
- Plasma current 0.5MA, created by 3 GHz RF wave

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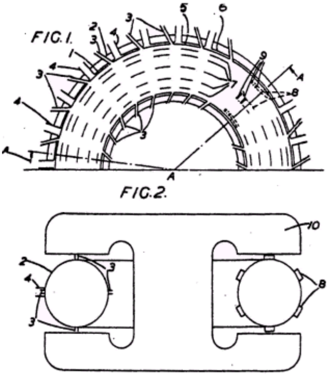

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So why don't we have tabletop fusion reactors?

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Toroidal machines

- The transport mechanism we've looked at is called **classical** transport, and it is pretty small
- Unfortunately, there are many other heat transport mechanisms in MCF devices
  - The variation in B field leads to particle bouncing / trapping (like in the tandem mirror). This leads to **neoclassical** transport $\rightarrow$ **Next lecture**

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  - Turbulence: like all fluids, plasma can be turbulent. This is usually (but not always) the dominant heat transport mechanism
  - Large-scale instabilities involving re-configuring the plasma
- A large part of this course is about these different mechanisms and how they can be controlled to achieve ignition