Toroidal confinement devices

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Last time...

- Power balance in a steady-state reactor gives the condition $nT\tau_E > 3 \times 10^{21}\mathrm{m}^{-3}$ keV s
    - Optimal $n\tau_E$ is at around $T = 10\mathrm{keV}$
    - Limits on magnet strength give order of magnitude density $n \sim 10^{20}\mathrm{m}^{-3}$
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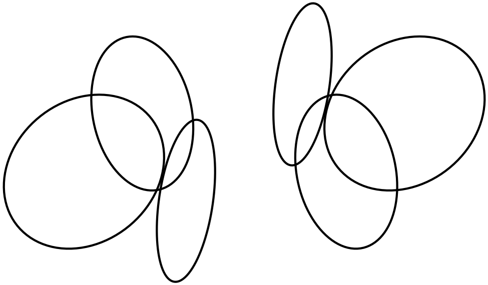
  - Limits on magnet strength give order of magnitude density $n \sim 10^{20}\mathrm{m}^{-3}$
  - Hence the energy confinement time needs to be $\tau_E \sim$ seconds
- Tandem magnetic mirror machines
  - Conservation of magnetic moment μ and energy leads to charged particles bouncing off high-field regions
  - Particles with a high enough $v_{||}$ are lost from the ends
  - Collisions in a plasma lead to diffusion in velocity through lots of small deflections
  - This leads to particle and energy loss from the ends of the machine

As we saw last time, the big problem with mirror machines is the ends. The obvious solution is to get rid of the ends!



Arrange the coils into a circle

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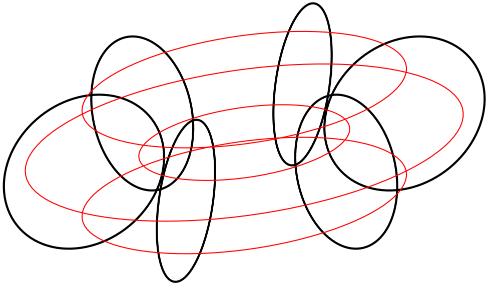
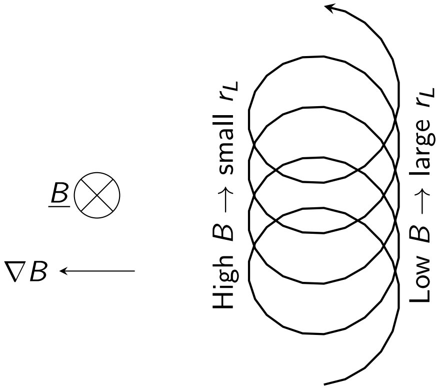


Arrange the coils into a circle which creates closed field-lines, thus solving the end-losses... What do particle orbits look like in this machine?

Drifts in a torus

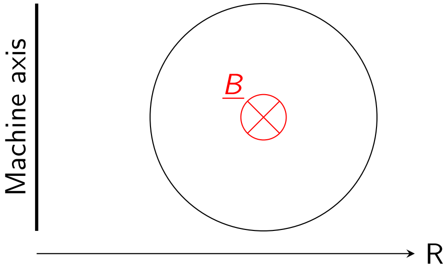
- In this new toroidal configuration, the magnetic field is stronger on the inside than on the outside
- In a non-uniform magnetic field, the Larmor radius changes around the orbit, producing a drift perpendicular to $\underline{B}$ and ∇B



So what does this do to our toroidal device?

- The drift velocity is given by

$$\underline{v}_D = \frac{mv_\perp^2}{2qB}\frac{\underline{B}\times\nabla B}{B^2}$$



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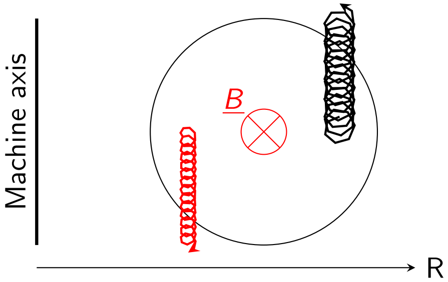

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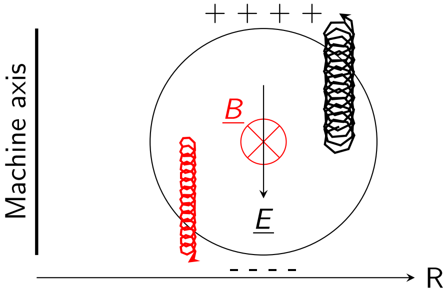

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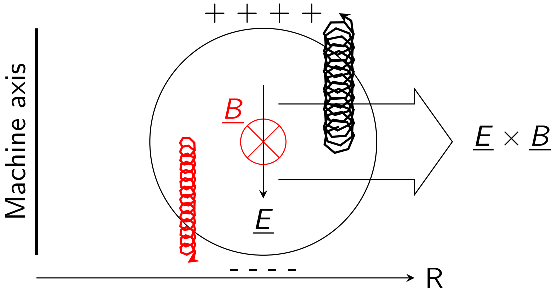
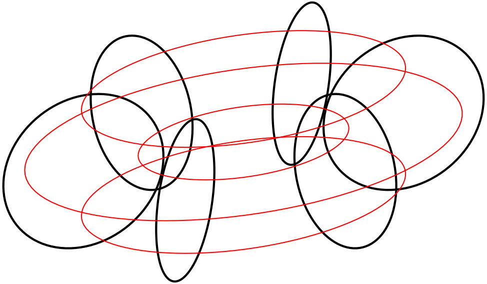


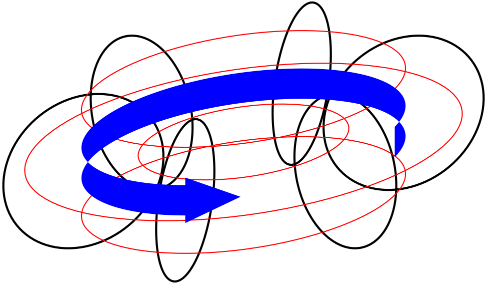
- Since this depends on the charge q, it is in opposite directions for electrons and ions
- Charge separation creates a vertical electric field
- $\underline{E}\times\underline{B}$ drift carries plasma radially outwards

Tokamak design



- In the late 1950s Soviet physicists Igor Yevgenyevich Tamm and Andrei Sakharov came up with the tokamak design ("toroidal'naya kamera v magnitnykh katushkakh" - "toroidal chamber in magnetic coils")

Tokamak design



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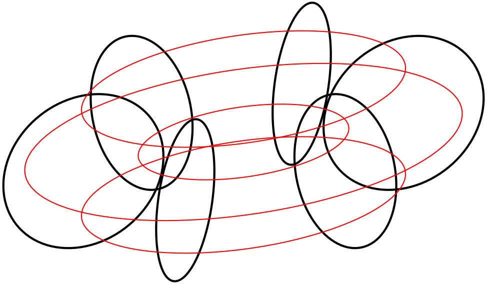
- Main modification is to pass a current through the plasma
- In 1968 Soviet scientists announced a record temperature in an MCF device (1000 eV)
- Now the primary focus of MCF research

Magnetic fields in a tokamak

The magnetic field in a tokamak is a combination of

- The **toroidal** magnetic field produced by external coils



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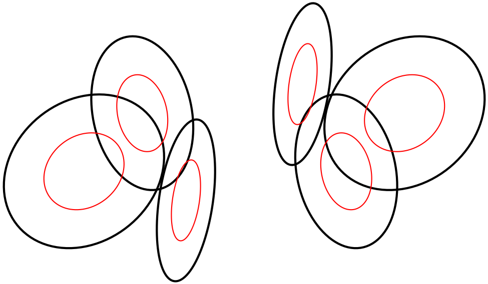
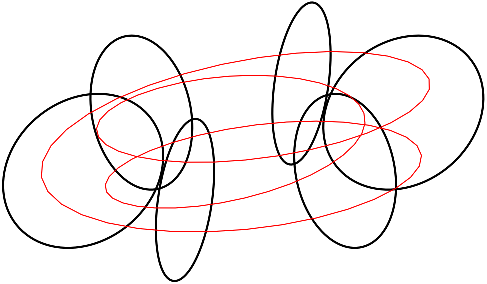


Magnetic fields in a tokamak

The magnetic field in a tokamak is a combination of

- The **toroidal** magnetic field produced by external coils
- The **poloidal** field due to the plasma current
- The combination of these fields produces a helix



- Every time the magnetic field goes once around toroidally (the long way), it is shifted by an angle ι by the poloidal field
- This angle is called the **rotational transform**

Tokamak design

- The combination of the "toroidal" field from the coils, and the "poloidal" field from the plasma current produces a helical magnetic field
- The higher the toroidal current, the higher the rotational transform ι

Tokamak design

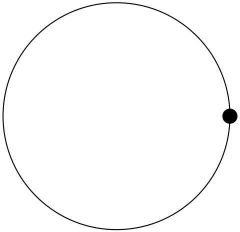
- The combination of the "toroidal" field from the coils, and the "poloidal" field from the plasma current produces a helical magnetic field
- The higher the toroidal current, the higher the rotational transform ι
- This current is a source of free energy in a tokamak, and having too much current can lead to instabilities
- The number of times a field-line goes around toroidally for one poloidal turn is called the **safety factor**

$$\boxed{q = \frac{2\pi}{\iota}}$$

- For reasons we'll go into later, this needs to be above 1 i.e. the rotational transform shouldn't get above 2π.

Flux surfaces

- Each time a field-line goes around the tokamak, it is shifted poloidally by an angle of ι
- If $q = 2\pi/\iota$ is a rational number n/m, then the field-line will join onto itself after n toroidal turns and m poloidal turns
- If however q is irrational, then the field-line will trace out an entire surface, coming arbitrarily close to any point.



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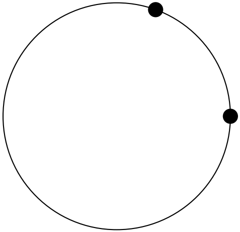

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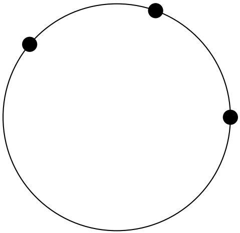

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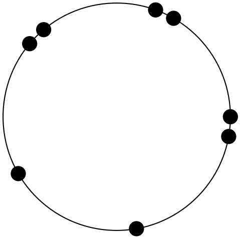

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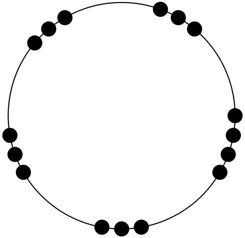
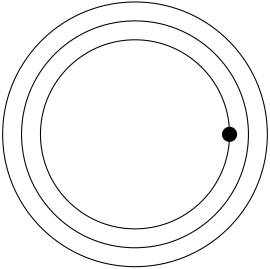


Field-lines lie on surfaces, called **flux surface**.

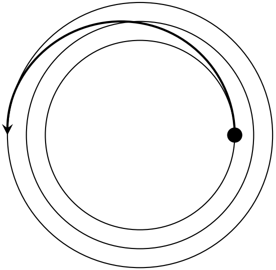
Consider an ion on a flux surface



Particle orbits in a tokamak

Consider an ion on a flux surface

- As an ion travels along a field-line, it's drifting upwards due to ∇B (and curvature)
- Parallel motion along helical field moves the ion poloidally



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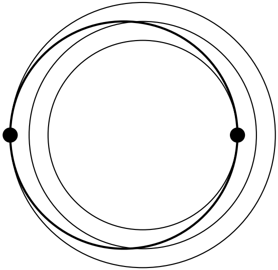
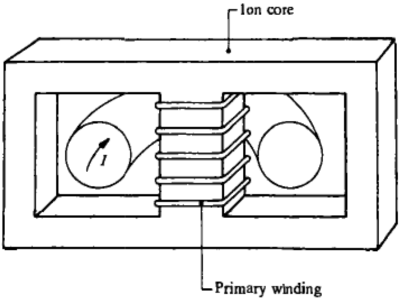


- Ions are always drifting upwards, but don't go anywhere!
- $\Rightarrow$ Rotational transform is vital for toroidal confinement

Toroidal current

How is this toroidal current produced?

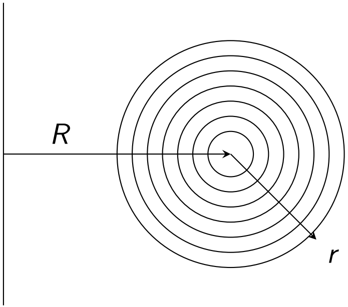
- Induction, using the plasma as the secondary. Current in primary winding is varied, inducing a toroidal electric field



- This produces a toroidal electric field which drives a current
- This relies on changing the current in the primary winding, so cannot be used in steady-state
- RF waves and particle beams can be used for continuous current drive

Tokamak confinement

So how big do we need to build this machine?



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Tokamak confinement

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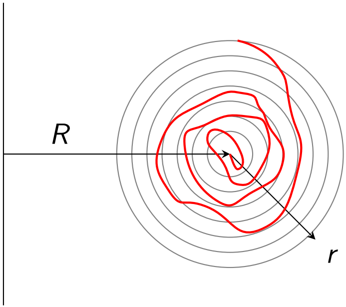
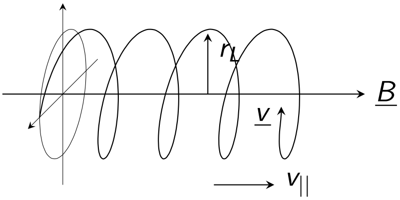


R = Major radius, r = minor radius

We need to know how long it will take a particle (and more importantly energy) to get from the center to the edge of the machine

From last time: Need $\tau_E \sim$ seconds to achieve ignition

Particles in a magnetic field follow a helix



The position of the particle $\underline{x}$ is

$$\underline{x} = \underline{X} + \frac{1}{\Omega}\underline{b} \times \underline{v}$$

where $\underline{X}$ is the guiding centre, and $\Omega = qB/m$ is the gyro-frequency. Small changes in velocity due to collisions therefore lead to a change in position

$$\delta X = -\frac{1}{\Omega}\underline{b} \times \delta\underline{v}$$

Collisional transport

This is a key difference from collisions in a gas: changes in velocity lead to a shift in the orbit position

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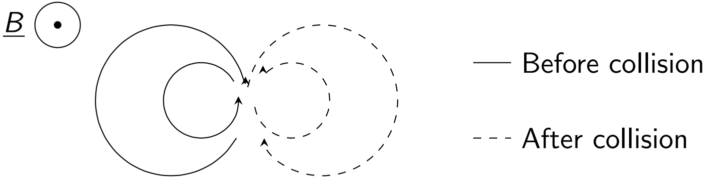
Here imagine for simplicity a large deflection collision:

- Two unlike particles in a magnetic field



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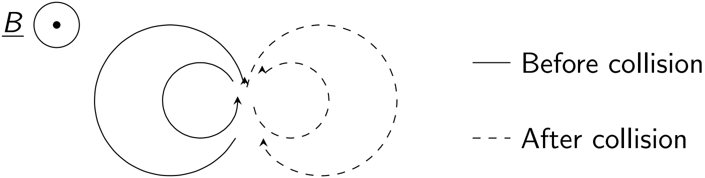
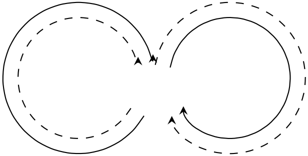


- Two like particles



Just switch position $\Rightarrow$ No net particle diffusion

Collisional transport

- Last time we found that by colliding with ions, the velocity of an electron changes on a timescale

$$\tau_{ei} = 3.44 \times 10^{11} \left(\frac{1m^{-3}}{n_e}\right) \left(\frac{T_e}{1eV}\right)^{3/2} \frac{1}{Z_i \ln \Lambda}$$

and so the position of an electron changes by $v_\perp / \Omega = r_L$.

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- Treat this as a random walk, step size r_L and timescale τ_{ei}
- After N steps, we can expect a particle to have moved

$$\boxed{d \sim \sqrt{N} r_L \simeq \sqrt{\frac{t}{\tau}} r_L}$$

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At 10keV, $n = 10^{20}\text{m}^{-3}$ and $B = 1\text{T}$, $\tau_{ei} = 1.7 \times 10^{-4}\text{s}$ and $r_{Le} \simeq 2.4 \times 10^{-4}\text{m}$. After 1 second, a typical electron will have moved a distance

$$d_e \sim \sqrt{\frac{1}{1.7 \times 10^{-4}}} \cdot 2.4 \times 10^{-4} = 1.8 \times 10^{-2} m$$

Collisional transport

What about ion transport?

- Ions colliding with ions doesn't lead to any particle transport as the total change in velocity is zero (conservation of momentum)
- Since ions are more massive than electrons, it takes more collisions to change the momentum so $\tau_{ie} = \frac{m_i}{m_e}\tau_{ei}$
- The ion Larmor radius is also larger than the electron'

$$\frac{r_{Li}}{r_{Le}} = \frac{v_{\perp i}}{v_{\perp e}}\frac{m_i}{m_e} = \sqrt{\frac{m_i}{m_e}} \sim 61.0$$

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- This means that the transport of ions

$$d_i = \sqrt{\frac{t}{\tau_{ie}}} r_{Li} = \sqrt{\frac{t}{\tau_{ei}}} r_{Le} = d_e$$

  i.e. the transport of particles is the same for electrons and ions. This is called **ambipolarity** and is due to the conservation of momentum

Heat transport

- Transport of electrons and ions only depends on the electron-ion collision rate τ_{ei}, and is ambipolar i.e. transport of electrons and ions is the same
- This is not the case for heat transport, where collisions between the same species can transfer energy
- Since the ion Larmor radius is much larger than the electron (by factor of $\sqrt{m_i/m_e} \simeq 61$), they transport the most heat
- The collision rate for ions with ions τ_{ii} is longer than τ_{ei} because of the lower thermal velocity, but shorter than τ_{ie}:

$$\tau_{ei} < \tau_{ii} \sim \sqrt{\frac{m_i}{m_e}}\frac{1}{Z^2} < \tau_{ie} \sim \frac{m_i}{m_e}\tau_{ei}$$

- τ_{ii} therefore determines the dominant collisional transport here

Heat transport

We can do the same calculation as before, that heat takes a step r_{Li} at a rate $1/\tau_{ii}$. Hence the distance this energy will be transported in a time t is

$$d_E \sim \sqrt{\frac{t}{\tau_{ii}}} \cdot r_{Li}$$

which for $T = 10\text{keV}$, $n = 10^{20}\text{m}^{-3}$ and $B = 1\text{T}$ gives

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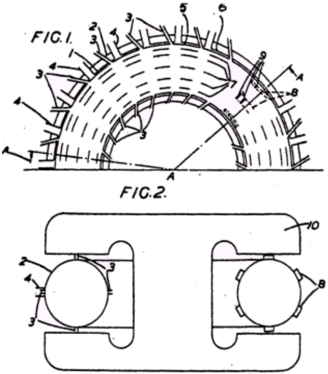
This is fantastic! It implies that we can make a machine with a minor radius r of a few $\times$10cm which will ignite

Toroidal machines

This was one source of early optimism about MCF, and lead to an early design for a reactor: G. P. Thomson and M. Blackman, British Patent No. 817 681, 1946[1] [2]



- Major radius R=1.3m
- Minor radius r = 0.3m
- Plasma current 0.5MA, created by 3 GHz RF wave

[1] Plasma Physics and Controlled Fusion, Volume 38(5),pp. 643-656 (1996)
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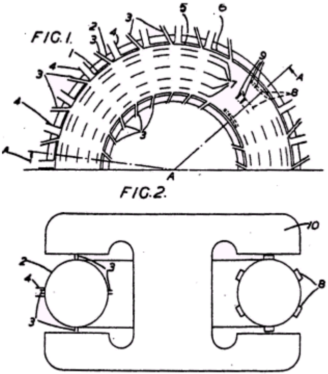

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So why don't we have tabletop fusion reactors?

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Toroidal machines

- The transport mechanism we've looked at is called **classical** transport, and it is pretty small
- Unfortunately, there are many other heat transport mechanisms in MCF devices
  - The variation in B field leads to particle bouncing / trapping (like in the tandem mirror). This leads to **neoclassical** transport $\rightarrow$ **Next lecture**

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  - Turbulence: like all fluids, plasma can be turbulent. This is usually (but not always) the dominant heat transport mechanism
  - Large-scale instabilities involving re-configuring the plasma
- A large part of this course is about these different mechanisms and how they can be controlled to achieve ignition