Stellarators

Dr Ben Dudson

Department of Physics, University of York
Heslington, York YO10 5DD, UK

6th February 2014
Toroidal devices need a rotational transform so that the $\nabla B$ drift doesn’t lead to charge separation.

In tokamaks, this is provided by a toroidal current, some of which needs to be driven using transformer action, radio waves or neutral beams.

Particle trapping (neoclassical effects) lead to the bootstrap current, which can provide much of the toroidal current.

The distribution of toroidal current has important effects on plasma performance and stability.

In this lecture we’ll look at other ways to produce a rotational transform without a toroidal current.
It can seem counter-intuitive that you could create a toroidal machine with rotational transform without having a plasma current.

- As an example, consider the first such design
- Proposed in 1951 by Lyman Spitzer, Jr\(^1\) \(^2\)
- Called a *stellarator* since it was inspired by the sun

---

\(^1\)Project Matterhorn. Declassified, renamed PPPL in 1961
If we follow a field-line around this figure of eight
If we follow a field-line around this figure of eight, field-lines are shifted by an angle of $4\alpha$ each time they go around the machine. This shift is the rotational transform. As in a tokamak, this leads to the formation of flux surfaces.
If we follow a field-line around this figure of eight, field-lines are shifted by an angle of $\alpha$ each time they go around the machine. This shift is known as the rotational transform. As in a tokamak, this leads to the formation of flux surfaces.
Stellarators

If we follow a field-line around this figure of eight

\[ x \times x \alpha x \alpha x \alpha \]

In the figure of eight design, field-lines are shifted by an angle of \( 4\alpha \) each time they go around the machine. This shift is the rotational transform. As in a tokamak, this leads to the formation of flux surfaces.

Dr Ben Dudson
Magnetic Confinement Fusion (4 of 23)
If we follow a field-line around this figure of eight, field-lines are shifted by an angle of $\alpha$ each time they go around the machine. This shift is the rotational transform. As in a tokamak, this leads to the formation of flux surfaces.
In the figure of eight design, field-lines are shifted by an angle of $4\alpha$ each time they go around the machine. This shift is the **rotational transform**.

As in a tokamak, this leads to the formation of flux surfaces.
Producing rotational transform

- The first designs like the racetrack demonstrate the principle, but did not have very good confinement: they turned out to be unstable with a finite plasma pressure.

- They are complicated to build, and the fields from neighbouring coils tended to interfere with each other.
The first designs like the racetrack demonstrate the principle, but did not have very good confinement: they turned out to be unstable with a finite plasma pressure. They are complicated to build, and the fields from neighbouring coils tended to interfere with each other. There are three ways to get a rotational transform in toroidal devices:

- A toroidal current, either driven externally or by the Bootstrap current (i.e. tokamak-like)
- A deformation in 3D (torsion) of the magnetic axis, as in the racetrack machine
- Non-circular deformation of the magnetic surfaces in resonance with magnetic field lines
Helical Axis Stellarators

To produce a twist in the magnetic axis, draw a helical path, then position the toroidal field coils perpendicular to this path. This produces a configuration called a Helical Axis stellarator, or Heliac.

Figure: Heliac configuration produced by displacing toroidal field coils\(^3\)

\(^3\)A.H. Boozer, Phys. Plasmas 5, 1647 (1998)
Rather than twisting the magnetic axis, so-called classical stellarators deform the flux surface shape by adding helical coils with currents in alternating directions (so field cancels out on axis).

\[ l = 3 \] stellarator with \( 2l = 6 \) helical windings

\(^{a}\)R.L. Miller, R.A. Krakowski LANL report LA-8978-MS (1981)
Heliotrons

Following declassification of fusion research in 1958, stellarator research started in Japan. An alternative design to the classical stellarator was developed\(^4\) which uses only half the number of coils, all carrying current in the same direction.

Heliotrons are easier to build since fewer coils, and the forces between them are reduced.

The Large Helical Device (LHD) is of this design.

The helical coils in classical stellarators, heliotrons and variants are hard to build, because they are interlocking, and inside the toroidal coils.

Plot the currents in these coils as a function of $\theta$ and $\phi$.

Figure: R.L. Miller, R.A. Krakowski LANL report LA-8978-MS (1981)
Modular coils

- The helical coils in classical stellarators, heliotrons and variants are hard to build, because they are interlocking, and inside the toroidal coils
- Plot the currents in these coils as a function of $\theta$ and $\phi$
- Replace with an essentially equivalent set of modular coils

Figure: R.L. Miller, R.A. Krakowski LANL report LA-8978-MS (1981)
Modular coils were a big breakthrough in stellarator design.

- Coils can be independently built and then assembled. Design is more difficult, but result is more practical for large reactors.

Figure: R.L. Miller, R.A. Krakowski, LANL report LA-8978-MS (1981)
Modular coils

Modular coils were a big breakthrough in stellarator design
- Coils can be independently built and then assembled. Design is more difficult, but result is more practical for large reactors

Figure: Wendelstein 7-X under construction, 2010
Modular coils

Modular coils were a big breakthrough in stellarator design

- Coils can be independently built and then assembled. Design is more difficult, but result is more practical for large reactors

Figure: Wendelstein 7-X nearly finished, October 2013
Modular coils were a big breakthrough in stellarator design

- Coils can be independently built and then assembled. Design is more difficult, but result is more practical for large reactors.

- More importantly, using modular coils allows stellarators to be designed "plasma first", rather than "coil first" \(^5\). We can design the plasma equilibrium based on physics considerations, and then design a set of coils to produce the required field.

- Stellarators designed this way are called **advanced stellarators**, the first example of which was Wendelstein 7-AS.

- Performing and optimising these designs has only become possible with advances in computing power.

---

Particle orbits in stellarators

- Following a field-line around a tokamak the magnetic field strength is approximately sinusoidal.

![Diagram showing the magnetic field strength over a field line.](image-url)
Particle orbits in stellarators

- Following a field-line around a tokamak the magnetic field strength is approximately sinusoidal.
- In a classical stellarator, there is another harmonic.

\[ |B| \]

Stellarator

along field line
Particle orbits in stellarators

- Following a field-line around a tokamak the magnetic field strength is approximately sinusoidal.
- In a classical stellarator, there is another harmonic.

As in a tokamak, particles can be passing or trapped due to toroidicity.
Particle orbits in stellarators

- Following a field-line around a tokamak the magnetic field strength is approximately sinusoidal.
- In a classical stellarator, there is another harmonic.

As in a tokamak, particles can be passing or trapped due to toroidicity.
- There are also particles which get trapped in local minima.
These particles trapped in local minima are confined to regions on the upper or lower half of the flux surface⇒ their vertical $\nabla B$ drift doesn’t cancel out, and they drift straight out of the machine. Called super-banana or direct loss orbits \(^6\)

This drift is different for electrons and ions, and so leads to electric fields.

The same process can happen in tokamaks due to toroidal ripple produced by a limited number of toroidal field coils

This direct loss lead to pretty poor performance in the 1960s and interest moved to tokamaks

Particle orbits in a tokamak revisited

So why do particle orbits stay near flux surfaces in tokamaks?

\[ \text{Consider the toroidal angular momentum of a particle in a tokamak:} \]

\[ Rv_\phi \frac{d}{dt} (Rv_\phi) = qR(v \times B) \phi \]

Using the expressions for an axisymmetric poloidal field:

\[ B_\theta = \nabla \psi \times \nabla \phi \]

\[ v \times (\nabla \psi \times \nabla \phi) = \nabla \psi (v \cdot \nabla \phi) - \nabla \phi (v \cdot \nabla \psi) \]

\[ \Rightarrow m \frac{d}{dt} (Rv_\phi) = -qv \cdot \nabla \psi \]

Since \[ v \cdot \nabla \psi = \frac{\partial \psi}{\partial t}, \] this becomes

\[ \frac{dp_\phi}{dt} = 0 \]

where \[ p_\phi = mRv_\phi + q \psi. \]
So why do particle orbits stay near flux surfaces in tokamaks?

Consider the toroidal angular momentum of a particle in a tokamak $R v_\phi$

$$m \frac{d}{dt} (R v_\phi) = q R (v \times B)_\phi$$

Using the expressions for an axisymmetric poloidal field

$$B_\theta = \nabla \psi \times \nabla \phi \quad v \times (\nabla \psi \times \nabla \phi) = \nabla \psi (v \cdot \nabla \phi) - \nabla \phi (v \cdot \nabla \psi)$$

$$\Rightarrow m \frac{d}{dt} (R v_\phi) = -q v \cdot \nabla \psi$$
So why do particle orbits stay near flux surfaces in tokamaks?

Consider the toroidal angular momentum of a particle in a tokamak $Rv_\phi$

$$m \frac{d}{dt} (Rv_\phi) = qR (v \times B)_\phi$$

Using the expressions for an axisymmetric poloidal field

$$B_\theta = \nabla \psi \times \nabla \phi \quad v \times (\nabla \psi \times \nabla \phi) = \nabla \psi (v \cdot \nabla \phi) - \nabla \phi (v \cdot \nabla \psi)$$

$$\Rightarrow m \frac{d}{dt} (Rv_\phi) = -qv \cdot \nabla \psi$$

Since $v \cdot \nabla \psi = \frac{\partial \psi}{\partial t}$, this becomes

$$\frac{dp_\phi}{dt} = 0 \quad \text{where} \quad p_\phi = mRv_\phi + q\psi$$
$p_\phi = mRv_\phi + q\psi$ is called the canonical momentum, and is conserved in an axisymmetric configuration.

In lecture 3, we derived an expression for the width of a banana orbit by considering the bounce time and $\nabla B$ drift velocity.

An alternative way is to consider the change $|\delta \psi| \sim |\nabla \psi| \delta r_b$.

This gives an upper bound on the particle orbit width.
Canonical momentum

- \( p_\phi = mRv_\phi + q\psi \) is called the **canonical momentum**, and is conserved in an axisymmetric configuration.
- In lecture 3, we derived an expression for the width of a banana orbit by considering the bounce time and \( \nabla B \) drift velocity.
- An alternative way is to consider the change \( |\delta\psi| \sim |\nabla\psi| \delta r_b \).
- This gives an upper bound on the particle orbit width.

A more elegant way to derive the canonical momentum is through the particle Lagrangian. Symmetries lead to conserved quantities, and in this case axisymmetry in a tokamak leads to conservation of canonical momentum.

\[ \Rightarrow \] Toroidal symmetry leads to a limit on how far particles can deviate from flux surfaces without collisions.
Quasi-symmetric stellarators

- In classical stellarators, there is no toroidal symmetry. Hence canonical momentum is not conserved, and there is no bound on particle excursions from a flux surface.

- Quasi-symmetric stellarators\(^7\)\(^8\) aim to introduce a new symmetry angle and so a canonical momentum.

---


In classical stellarators, there is no toroidal symmetry. Hence canonical momentum is not conserved, and there is no bound on particle excursions from a flux surface.

Quasi-symmetric stellarators\(^7\)\(^8\) aim to introduce a new symmetry angle and so a canonical momentum

In these designs, \(|B|\) a function of only \(\psi\) and a linear combination \(M\theta - N\phi\) (in Boozer coordinates)

- When \(N = 0\), the field has Quasi-Axial Symmetry (QAS)
- When \(M = 0\), the field has Quasi-Poloidal Symmetry (QPS)
- When \(M = 1\) and \(N \neq 0\) then the field has Quasi-Helical Symmetry (QHS)

In this case there is a symmetry direction, and a conserved canonical momentum. This leads to a limit on the deviation of particles from flux surfaces, and the same toroidal neoclassical theory applies.


Helically Symmetric eXperiment

- Helically symmetric stellarator, constructed at Wisconsin as first test of quasi-symmetry
- Began operation in 1999
- Major radius 1.20m, minor radius 0.15m
- Confirmed reduction of direct loss orbits

Figure: HSX
http://www.hsx.wisc.edu
Low aspect-ratio, high performance ($\beta$) machines called “compact stellarators” have been designed.

At low aspect-ratio, quasi-helicity cannot be attained\(^a\)

Instead, another symmetry can be used such as quasi-axisymmetry.

Major radius 1.42m, minor radius 0.33m


**Note:** The NCSX project was mothballed in 2008.
Quasiomnigenous stellarators

A second way to reduce neoclassical transport in stellarators is to make the radial particle drift average out over a bounce orbit.


- The bounce-averaged radial drift can be minimised for a chosen population of particles (i.e. super-bananas), hence the “quasi-”
Quasiomnigeneous stellarators

A second way to reduce neoclassical transport in stellarators is to make the radial particle drift average out over a bounce orbit.

- These are called omnigenous or linked mirror designs, and need not be symmetric.
- The bounce-averaged radial drift can be minimised for a chosen population of particles (i.e. super-bananas), hence the “quasi-”
- The variation in field-strength $\nabla B$ within a flux surface is proportional to the curvature.
- Since super-banana particles are trapped in regions of low $B$, minimise the curvature in these regions, and put the curvature needed to bend plasma into a torus in regions of high $B$.

---

9L.S.Hall, B.McNamara, Phys. Fluids 18:552 (1975)
A large Quasi-Omnigenous stellarator being built at IPP Greifswald

- Straight sections with triangular cross-section and low $B$, and curved sections with crescent cross-sections and high $B$.
- Major radius 5.5m, minor radius 0.52m, magnetic field 3T
- Expected to have JET-like plasma performance, and discharge length of $\sim 30$ minutes
- Planned completion date 2014 at cost of $\sim$ $300$ million

Figure: A.H. Boozer, Phys. Plasmas 5(5):1647 (1998)
Stellarator equilibria are quite different to a tokamak:

- In some ways it is easier: currents in the plasma are usually optimised to be small, so the coil currents alone determine the equilibrium.
- The Grad-Shafranov equation cannot be used, so now the full 3D $J \times B = \nabla P$ must be solved. Two codes which do this are VMEC$^{11}$ and PIES$^{12}$

---

---

Figure: W7-X coils: M. Drevlak et al. Nucl. Fusion 45, 731 (2005)

---


Stellarator design is about optimising trade-offs

- First a plasma equilibrium must be constructed with desired aspect ratio, pressure and rotational transform
- Quasi-symmetric or quasi-omnigenous configurations minimise neoclassical transport

---

13 See e.g. work by Pavlos Xanthopoulos using GS2
Stellarator design is about optimising trade-offs

- First a plasma equilibrium must be constructed with desired aspect ratio, pressure and rotational transform
- Quasi-symmetric or quasi-omnigenous configurations minimise neoclassical transport
- The plasma must avoid large-scale instabilities
- The Pfirsch-Schüter and Bootstrap currents are calculated. These are usually minimised to have maximum control over the plasma configuration
- In the last couple of years it has become possible to start optimising anomalous (turbulent) transport\textsuperscript{13}

\textsuperscript{13}See e.g. work by Pavlos Xanthopoulos using GS2
Stellarator design is about optimising trade-offs

- First a plasma equilibrium must be constructed with desired aspect ratio, pressure and rotational transform
- Quasi-symmetric or quasi-omnigenous configurations minimise neoclassical transport
- The plasma must avoid large-scale instabilities
- The Pfirsch-Schüter and Bootstrap currents are calculated. These are usually minimised to have maximum control over the plasma configuration
- In the last couple of years it has become possible to start optimising anomalous (turbulent) transport\(^\text{13}\)
- A design of coils is produced which create the desired field
- These coils may not be practical, so modify the design
- Repeat until an optimised design is produced

\[^{13}\text{See e.g. work by Pavlos Xanthopoulos using GS2}\]
There were several problems with Stellarators

- Complicated coil configurations need to be very precise (~ 1mm), and are difficult to design and expensive to build. They must be capable of carrying Mega-Ampéres of current, and withstanding huge forces.

- Generally, stellarator configurations cannot be varied to the extent that tokamaks can. This makes experimentation more difficult.

- Achieving good particle confinement is more difficult in stellarators than tokamaks.

- So far, stellarators have not reached the same performance (pressure $\beta$ and density) as tokamaks.

- The divertor and heat-handling regions have a more complicated geometry than tokamaks, complicating the engineering.
Advantages of stellarators

- Stellarators are intrinsically steady-state, as there is no need to drive a plasma current.
- Lack of plasma current removes a large class of instabilities which we’ll see later in tokamaks.
- Because the rotational transform and position of the plasma is set by external coils and not by currents in the plasma, stellarators do not suffer violent disruptions.
- Stellarators have a much greater range of designs than tokamaks, potentially allowing greater optimisation of performance.
- With the construction of Wendelstein 7-X, new concepts such as quasi-symmetry, and the interest in 3D fields in tokamaks, there is renewed interest in stellarator research.