# Performance limits

Ben Dudson

Department of Physics, University of York, Heslington, York YO10 5DD, UK

$24^{th}$ February 2014

# Previously...

In the last few lectures we've covered the basics of plasma instabilities, the factors which determine their growth, and some of the tools use in their analysis

- Growing solutions to linearised equations of motion
- Potential energy ($\delta W$) considerations. Instabilities stabilised by compression and field-line bending
- Pressure driven instabilities
  - Stability depends on sign of $\kappa \cdot \nabla p$ : **Good and bad curvature**
  - Tend to localise around $q = m/n$ to minimise field-line bending
- Current driven instabilities
  - Ideal MHD $m = 1$ **internal kink**, $m > 1$ **external kink** with $q = m/n$ surface outside the plasma
  - Resistivity allows magnetic islands to form (**tearing modes**). In MHD these are governed by $\Delta'$
  - Islands flatten the pressure profile, change the Bootstrap current: **Neoclassical Tearing Mode**
  - Critical threshold width $w_c$ due to **incomplete flattening**

# Current limits

We have already seen that there is a limit to the current in a tokamak:

- If the total current becomes high enough so that $q < 1$ outside the plasma then the whole plasma becomes kink unstable
- If $q = 1$ appears inside the plasma then the m,n = 1,1 internal kink can be unstable (e.g. sawteeth, fishbone)
- If $q = 2$ appears then 2,1 NTMs can appear (see later)
- It's for this reason that advanced tokamak scenarios keep current out of the core to keep $q$ high

# Beta limits

What we'll concentrate on here is limits to the plasma **beta**

$$\boxed{\beta \equiv \frac{\mu_0 p}{B^2}}$$

This is a useful quantity because it quantifies the efficiency of plasma confinement:

- Fusion power output increases with pressure.
- In a power plant, magnetic field costs money: $B$ is generated either in expensive toroidal field coils, or driven using expensive current-drive systems.
- To make commercial fusion power viable, $\beta$ must be maximised

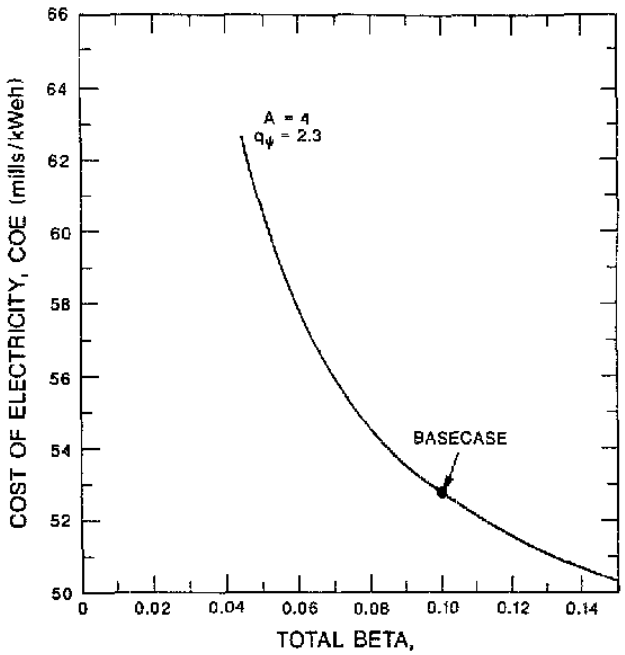
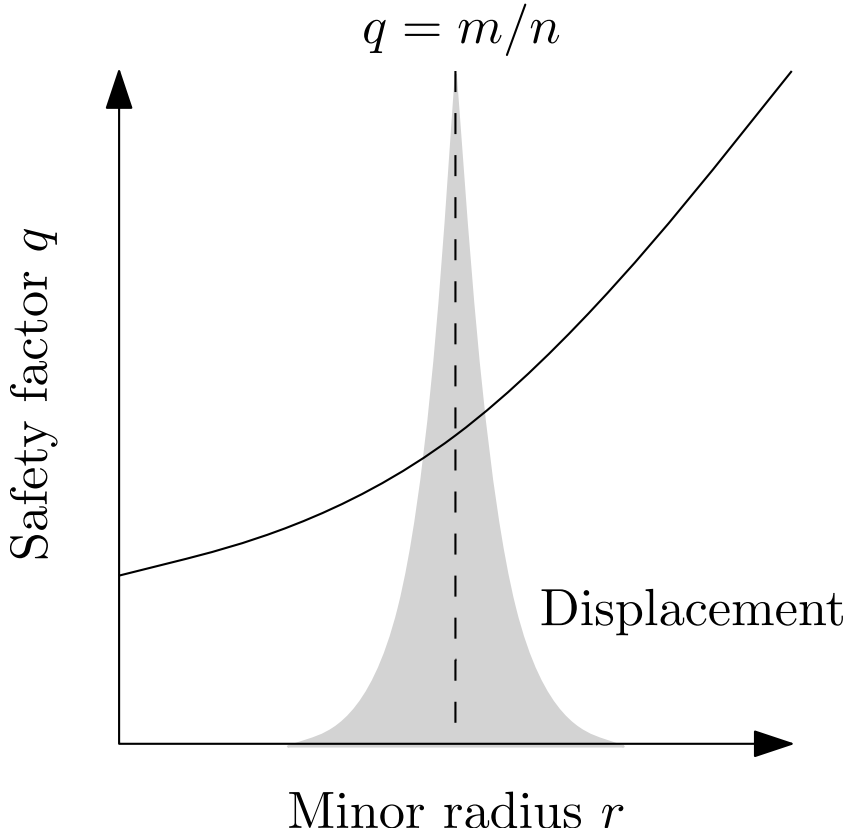



Figure :
R.A.Krakowski,J.G.Delene
*J.Fusion Energy* **7**(1), 1988

# Beta limits: Ballooning modes

In lecture 11, we looked at pressure-driven instabilities where the key terms in the Energy equation are

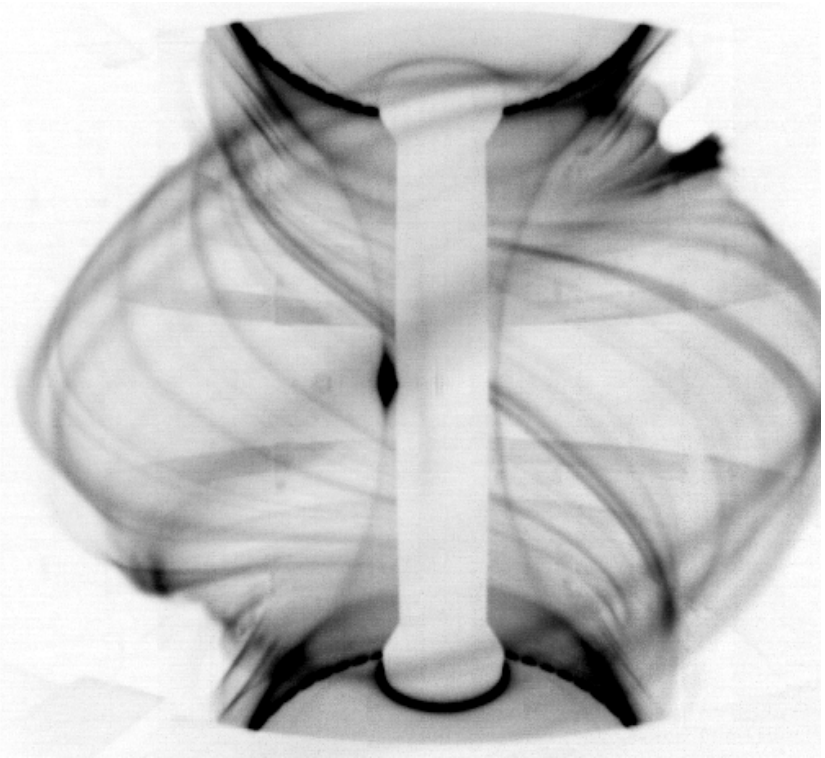
$$\delta W_p = \frac{1}{2}\int d^3x \left[\frac{|B_1|^2}{\mu_0} - 2\left(\xi\cdot\nabla p\right)\left(\underline{\kappa}\cdot\xi_\perp^*\right)\right]$$




- Interchange modes minimise field-line bending by localising around their resonant surface $q = m/n$
- Stabilised by magnetic shear (twisting of field-lines)
- The Mercier condition says that interchange modes are stable in tokamaks provided that $q > 1$.

# Beta limits: Ballooning modes

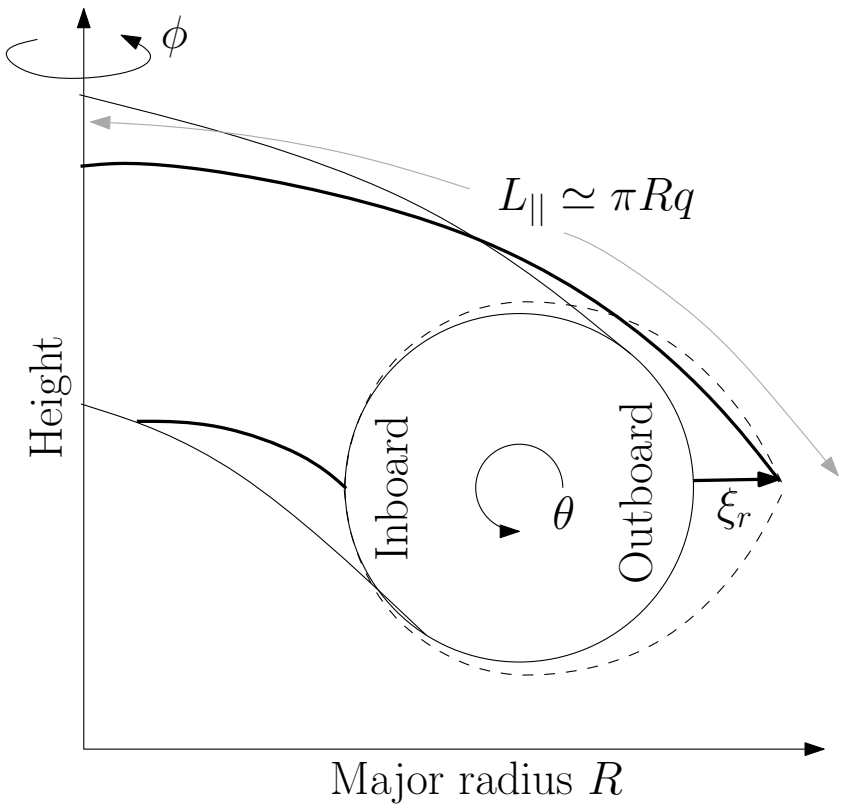
- Unfortunately this doesn't mean that tokamaks are immune from pressure-driven instabilities
- During high-performance mode (H-mode), steep gradients form close to the plasma edge.



- These collapse quasi-periodically in eruptions called **Edge Localised Modes** (ELMs).
- Leading theory is peeling-ballooning modes (Connor, Hastie, Wilson)
- Pressure-driven (ballooning) and edge current (peeling)

# Beta limits: Ballooning modes

Using the ideal MHD energy equation, we can estimate the pressure limit for a **ballooning mode**:




Field-line bending so mode is maximum on the outboard side, minimum on inboard side. Consider case when parallel bending dominates:

$$\frac{|B_1|^2}{2\mu_0} \simeq \frac{|B_0 \cdot \nabla \xi_r|^2}{2\mu_0}$$

Using the length along field-lines,

$$\frac{|B_1|^2}{2\mu_0} \sim \frac{\left|B_0 \xi_r / L_{||}\right|^2}{2\mu_0} = \frac{B_0^2}{2\mu_0} \frac{\xi_r^2}{\pi^2 q^2 R^2}$$

This gives the energy (density) needed to bend field-lines

# Beta limits: Ballooning modes

For ballooning modes to be stable, the energy available from the pressure gradient has to be less than this field-line bending i.e.

$$(\xi \cdot \nabla p)\,(\underline{\kappa} \cdot \xi_\perp^*) < \frac{B_0^2}{2\mu_0}\frac{\xi_r^2}{\pi^2 q^2 R^2}$$

Taking $\underline{\xi}$ at the outboard midplane (maximum perturbation), $\nabla p$ and $\kappa$ are both in the direction of $\underline{\xi}_r$. Therefore,

$$(\xi \cdot \nabla p)\,(\underline{\kappa} \cdot \xi_\perp^*) \simeq \xi_r^2 \kappa \frac{dp}{dr}$$

For stability then:

$$\xi_r^2 \kappa \frac{dp}{dr} < \frac{B_0^2}{2\mu_0}\frac{\xi_r^2}{\pi^2 q^2 R^2}$$

Since $\kappa \sim -1/R$, this becomes

$$-\frac{dp}{dr} < \frac{B_0^2}{2\mu_0}\frac{1}{\pi^2 q^2 R}$$

# Beta limits: Ballooning modes

We can use this to get a beta limit by setting $\frac{dp}{dr} \sim -p_0/r$ where $p_0$ is the core pressure and $r$ the minor radius.

$$\frac{\mu_0 p_0}{B_0^2} = \beta < \frac{1}{2\pi^2}\frac{r}{qR}$$

If $q \simeq 2$, $r/R = \epsilon \simeq 1/3$ this gives a limit of $\beta \sim 0.5\%$

- This is in the right ballpark for conventional tokamaks: a couple of percent is quite typical
- More important than a global *beta* limit is the effect ELMs have on divertor power loads
- A "natural" ELM on ITER is predicted to produce $20\mathrm{MW/m^2}$, and must be reduced by a factor of $\sim 20$ for acceptable component lifetimes
- Research ongoing into means of controlling these events

# Beta limits: Normalised beta and the Troyon limit

From ballooning, we have the beta limit $\beta \sim r/\left(q^2 R\right)$. Using $q = \dfrac{rB_\phi}{RB_\theta}$ and $B_\theta \sim I_\phi / r$ where $I_\phi$ is the toroidal current this becomes:

$$\beta \sim \frac{r}{R}\frac{R^2\left(I_\phi/r\right)^2}{r^2 B_\phi^2} = \left(\frac{I_\phi}{rB_\phi}\right)^2 \frac{R}{r}$$

- Experiments and more careful calculations find a linear dependence on $I_\phi/\left(rB_\phi\right)$
- The quantity $\beta_N \equiv \dfrac{\beta\left(\%\right)}{I_\phi/\left(rB_\phi\right)}$ is called the **Normalised Beta**
- Based on simulations, the limit $\beta_N < 2.8$ is called the **Troyon limit**
- Further optimisation of current profile $(l_i)$

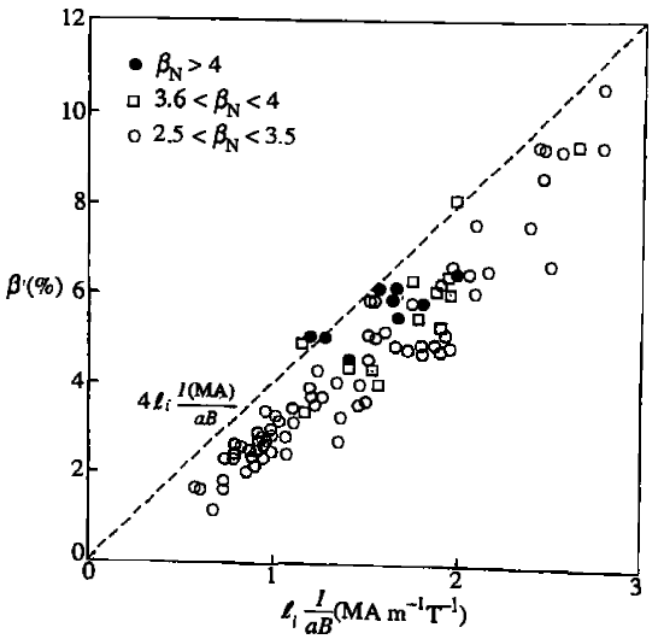
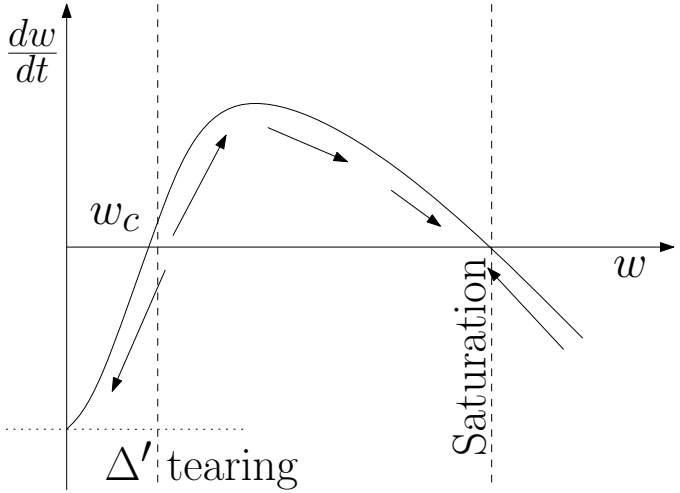



Figure : Wesson 16.6.4

# Beta limits: Neoclassical Tearing Modes

Last lecture we looked at tearing modes (magnetic islands). Modification of the pressure profile leads to changes in the bootstrap current. This gives a **modified Rutherford equation** for the width $w \sim B_r^2$ of the island:

$$\frac{dw}{dt} \simeq \frac{\eta}{2\mu_0}\left[\Delta' - \frac{\alpha}{r_s}\frac{\sqrt{\epsilon}\,(w/r_s)}{(w_c/r_s)^2 + (w/r_s)^2}\frac{d\beta_p}{dr}\right]$$




- At small $w$ stability is determined by $\Delta'$ (usually stable)
- Above $w_c$ profiles are flattened, mode grows
- At some amplitute the mode saturates

Called **Neoclassical Tearing Modes** because they are driven unstable by the bootstrap current

# Beta limits: Neoclassical Tearing Modes

As the pressure (beta) is increased, the critical size for an NTM to grow gets smaller. At some point a sufficient "kick" will occur to start the mode

- First observed on TFTR ("supershot" scenarios), now observed in all high-performance tokamaks
- Common triggers are sawteeth, fishbones, and ELMs
- Tend to be what limits $\beta$

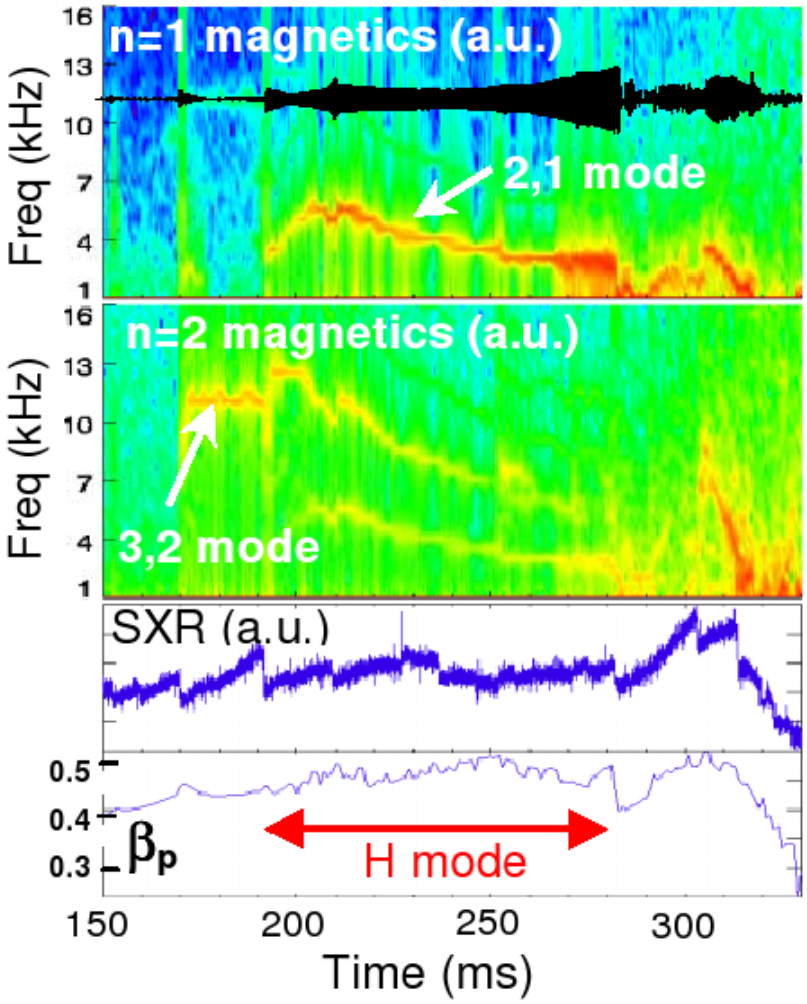



Figure : MAST shot 2952

# Wall stabilisation

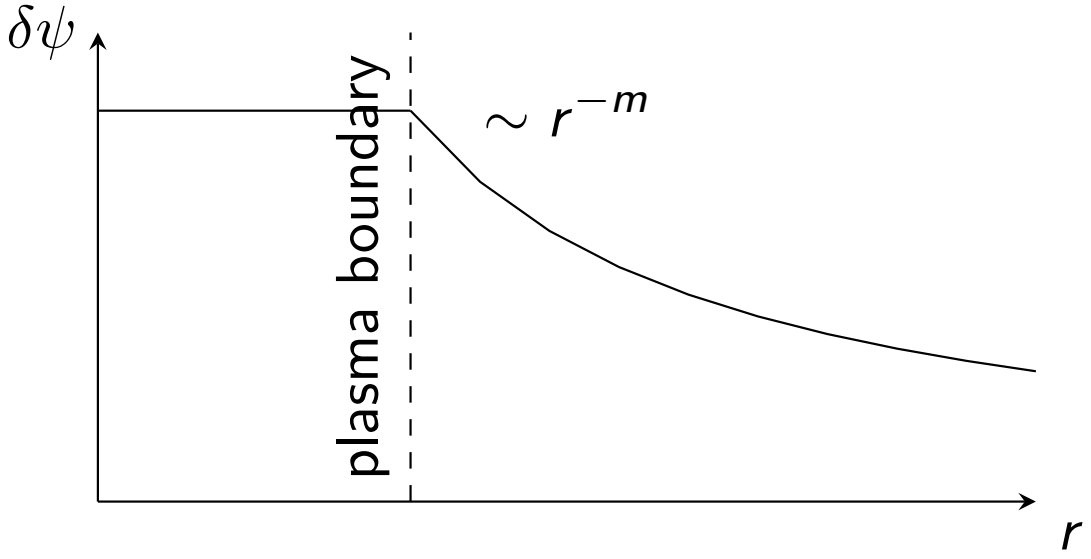
In the last lecture we saw that tearing and kink modes are governed by the Cylindrical Tearing Mode equation:

$$\nabla^2 \delta\psi - \frac{\mu_0 \frac{dJ_\phi}{dr}}{B_\theta \left[1 - qn/m\right]} \delta\psi = 0$$

The solutions to this equation (and so $\Delta'$) depends on the boundary conditions.

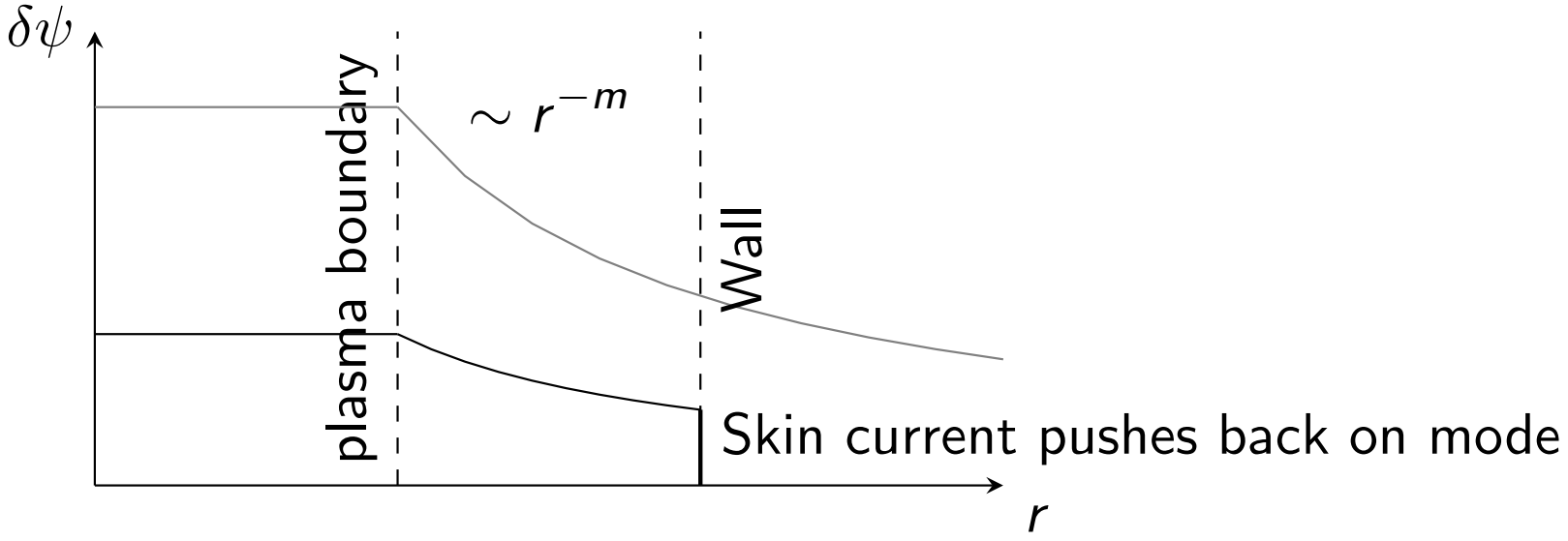
- **No wall** is a situation where the wall is far away and the perturbation goes like $\delta\psi \sim r^{-m}$. This is the most unstable situation
- An **ideal wall** is a perfectly conducting boundary where the perturbation is forced to zero
- If an ideal wall could be put at the plasma boundary, all external modes would be fully stabilised
- In practice, vessel walls are not ideal

# Resistive walls



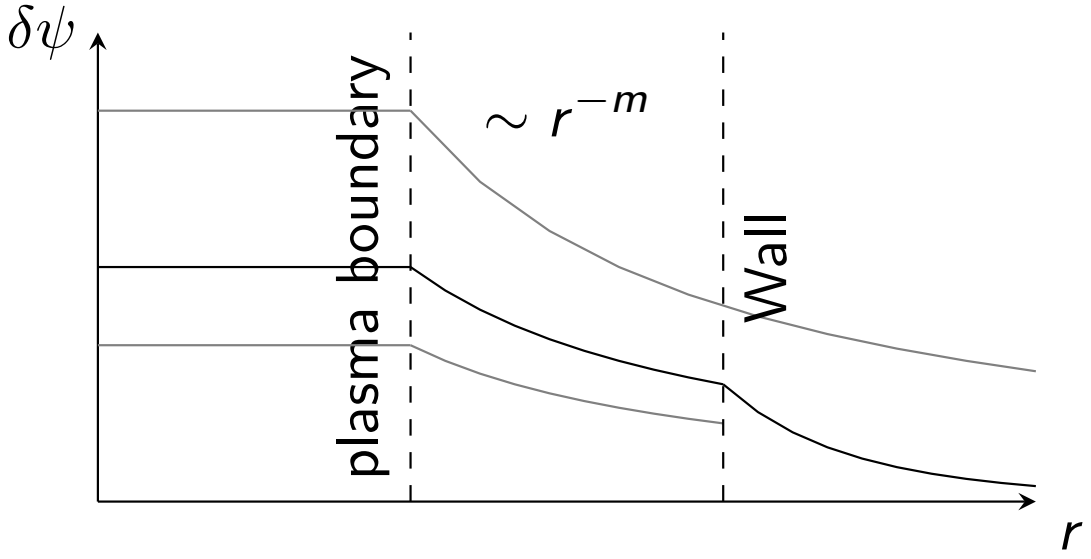

- If there is no wall (wall is far away) then the

# Resistive walls




- If there is no wall (wall is far away) then the
- An ideal wall is a superconductor which forces the perturbation to zero at the boundary. This corresponds to a current sheet which pushes back on the mode.

# Resistive walls




- If there is no wall (wall is far away) then the
- An ideal wall is a superconductor which forces the perturbation to zero at the boundary. This corresponds to a current sheet which pushes back on the mode.
- In real machines, the vessel walls always have some finite resistivity. The current and hence radial magnetic field can diffuse into the wall

# Resistive walls

The resistive diffusion is the same as we've seen before

$$\frac{\partial \underline{B}}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 \underline{B}$$

and since we're interested in diffusion into the wall:

$$\frac{\partial \underline{B}}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 \underline{B}$$

If the thickness of the wall is $L$ and the time for diffusion into the wall is $\tau_W$ then

$$\frac{\underline{B}}{\tau_w} \simeq \frac{\eta}{\mu_0} \frac{1}{L_w^2} \underline{B}$$

and so the **wall time** is $\tau_w \simeq \dfrac{L_w^2 \mu_0}{\eta}$. This is typically $\sim$ 10ms.

# Rotation stabilisation

- If an instability is rotating at a frequency $\Omega$ then the $\underline{B}$ field at the wall will reverse direction on a timescale $\tau = 2\pi/\Omega$
- If this is much faster than $\tau_w$ then the field doesn't have time to diffuse into the wall
- Hence if $\Omega \gg 1/\tau_w$ then the wall appears ideal

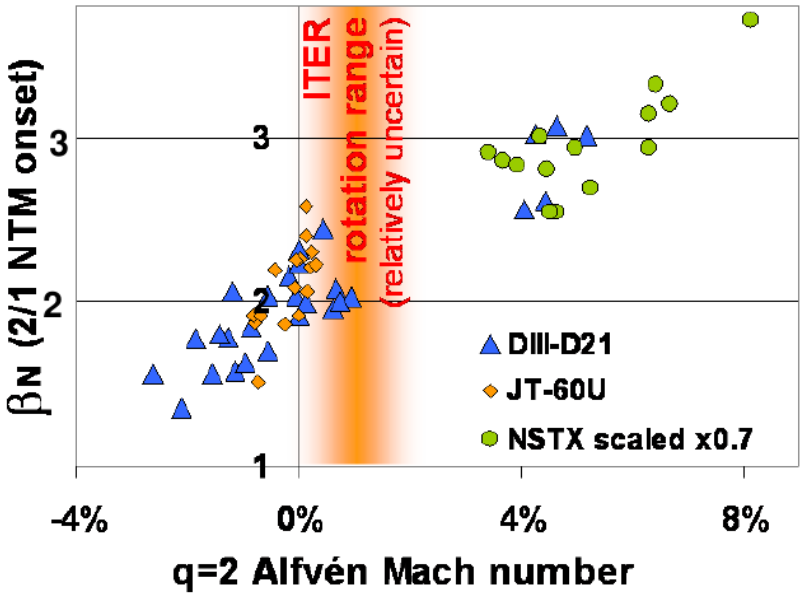



Figure : R.Buttery, IAEA 2008

# Mode locking

- If a current is being driven in a resistive wall then there must be some heating $W_\eta = \int \eta j^2 d^3x$
- This energy must come from plasma rotation. Currents in the wall produce a torque on the plasma which brakes the rotation
- Rotational energy $\propto \Omega^2$ so

$$\Omega \frac{d\Omega}{dt} \propto \int \eta j^2 d^3x \propto -\tau_w \left(\Omega B_{r,wall}\right)^2$$

- The magnetic field at the island and wall are approximately: $B^2_{r,island} \propto \left(1 + \Omega^2 \tau_w^2\right) B^2_{r,wall}$

Therefore,

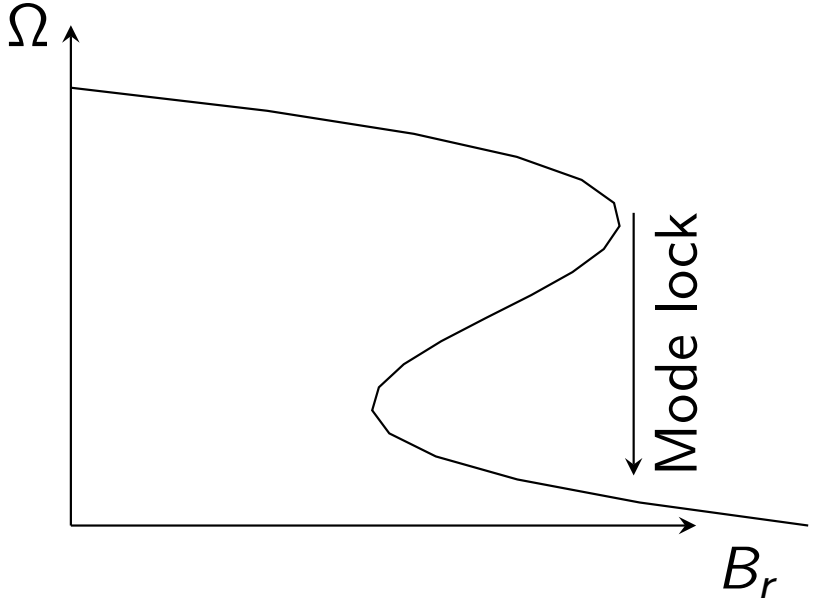
$$\frac{d\Omega}{dt} = -\frac{a\Omega\tau_w b_r^2}{1 + \Omega^2\tau_w^2}$$

For large tokamaks $\Omega\tau_w << 1$ and $\Omega \propto e^{t\sqrt{\tau_w B_r}}$

# Mode locking

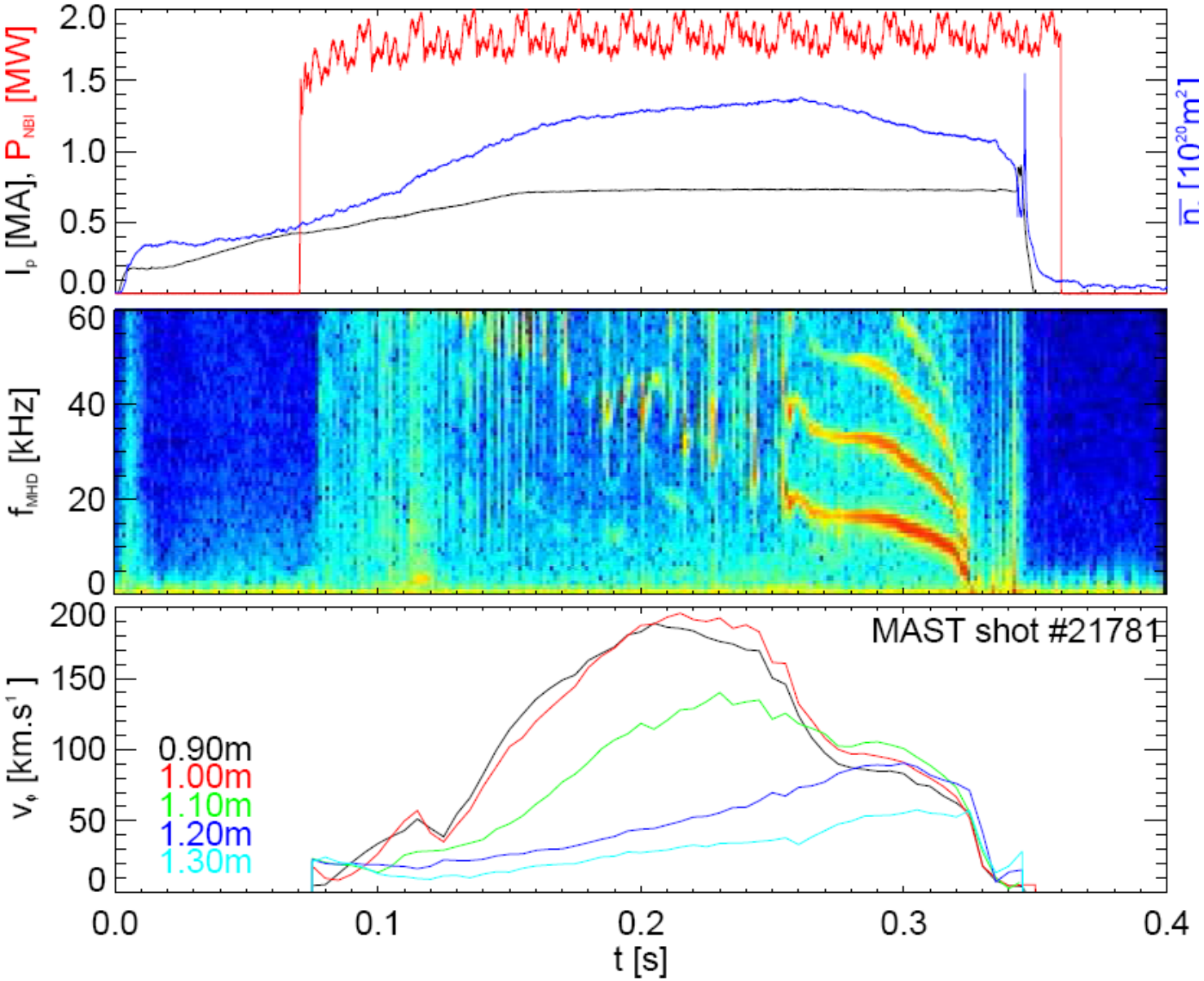
- Interaction between instability and the wall slows the mode
- Rotation slows and wall stabilisation becomes less effective
- A simple model is that an island rotation is driven by drag with the rest of the plasma, and braked by the wall

$$\frac{d\Omega}{dt} \propto \nu\left(\Omega_0 - \Omega\right) - \frac{\Omega \tau_w B_r^2}{1 + \Omega^2 \tau_w^2}$$




- Final braking caused by error fields: non-axisymmetric fields caused by finite number of coils or manufacturing imperfections
- Sudden braking of the plasma, often leading to a disruption

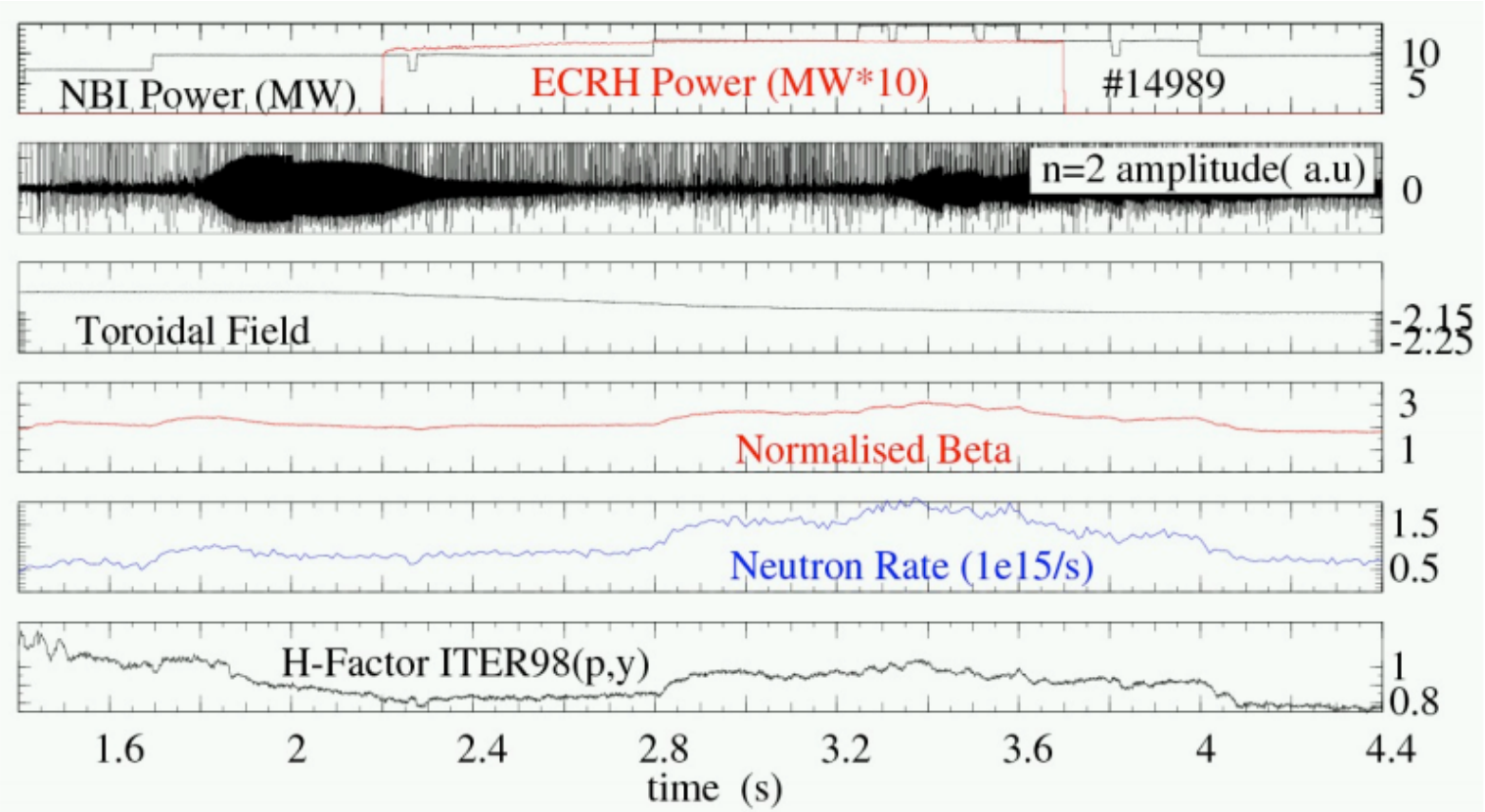
# Mode locking



# Stabilising NTMs

There are several schemes being investigated to control NTMs.

- The most successful one so far has been to use Electron Cyclotron Current Drive (ECCD).
- Idea is to restore the missing bootstrap current in the island
  - Localised current drive: ECCD, LHCD, ...
  - Localised heating: changes resistivity and so current profile



# Stabilising NTMs

- The absorption location of RF waves can be varied by changing the launcher angle, plasma major radius or toroidal field
- Suppression methods typically use magnetics signals to find start of an NTM
- "Search and suppress" methods scan the alignment, stopping when the island shows a response
- Active tracking uses experimental measurements to reconstruct the location of the $q = 3/2$ and $q = 2$ surfaces, then targets these with the launcher. Very complicated and needs fast calculations
- If the $q$ surface location is known, preemptive suppression can be used to prevent NTMs from starting
- Active suppression of 3,2 and 2,1 modes on ITER using upper ECRH launcher under development

# Stabilising NTMs: alternatives

In addition to active suppression, other things can be done to reduce NTMs

- The source of seed islands should be reduced. Suppressing sawteeth or making them smaller
- Driving plasma rotation or rotation shear helps suppress NTMs, but not so applicable to ITER
- Kinetic effects (fast particles) have been found to have an effect on mode stability, and being investigated

# Disruptions

So what happens when a tokamak hits one of these limits?

- In a tokamak the position of the plasma is controlled using a feedback system (see this week's problem sheet)
- If an event is violent enough it can move the plasma too quickly for the system to respond. Distortions to the plasma can also confuse the system so that it makes things worse
- At this point the control system may fail and give up ("FA cutout")
- The plasma then typically hits either the top or bottom of the vessel. This is called a **disruption**.
- These must be avoided in large tokamaks, so ways to arrange a "soft landing" are being developed e.g. Massive Gas Injection.

# Summary

- Ballooning modes provide a limit $\beta \sim r/\left(q^2 R\right) \sim \left(\frac{I_\phi}{rB_\phi}\right)^2 \frac{R}{r}$
- Experimentally the beta limit is found to vary linearly with $I_\phi/\left(rB_\phi\right)$ and so the **normalised beta** is defined as

$$\beta_N \equiv \frac{\beta\left(\%\right)}{I_\phi/\left(rB_\phi\right)}$$

- Because **Neoclassical Tearing Modes** are destabilised by the bootstrap current, they appear at high $\beta$, and tend to limit tokamak performance
- The vessel wall influences the growth-rate of NTMs and external kinks, and rotation makes a resistive wall appear ideal
- External kink modes which would be unstable without a wall, but stable with an ideal wall are called Resistive Wall Modes (RWMs)
- Mode locking brakes the plasma rotation and allows the mode to grow rapidly. This can then lead to violent disruptions