Small-scale instabilities

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The course so far...

- Plasma equilibria
  - Toroidal confinement devices: Tokamaks and Stellarators
  - MHD equilibrium and the Grad-Shafranov equation
  - Neoclassical currents: Pfirsch-Schlüter, banana and bootstrap currents
- Collisional transport
  - Collision processes in plasmas: small-angle deflection
  - Classical and Neoclassical transport
- Heating and current drive
  - Neutral Beam Injection
  - Waves in plasmas, RF heating and current drive
- Plasma instabilities and performance limits
  - Pressure-driven interchange and ballooning modes
  - Current-driven kink and tearing modes
  - NTMs and beta limits

This lecture

- We've looked at MHD instabilities, which tend to be the fastest and most dangerous for confinement
- In the next few lectures we'll look at other instabilities which degrade confinement but don't lead to catastrophic results.
- These result from treating electrons and ions separately and so are two-fluid or kinetic effects
- They are essentially electrostatic, and are driven by gradients in temperature and density
- These are thought to be the origin of turbulence in confinement devices, and so anomalous transport

References:

- J. Wesson "Tokamaks", sections 8.2 - 8.5
- J.W.Connor, H.R.Wilson "Survey of theories of anomalous transport" *Plasma Phys. Control. Fusion* **36** 719-795 (1994)
- B.D.Scott "Computation of turbulence in magnetically confined plasmas" *Plasma Phys. Control. Fusion* **48** B277 (2006)

Electron response

- Because electrons move quickly along magnetic fields, they are often assumed to quickly reach equilibrium on the timescale of instabilities.
- This simplifies the analysis as we can concentrate on the ions, and assume that the electrons follow.

The momentum equation for electrons is

$$n_e m_e \left(\frac{\partial \underline{v}_e}{\partial t} + \underline{v}_e \cdot \nabla \underline{v}_e\right) = -\nabla p_e - n_e e \left(\underline{E} + \underline{v}_e \times \underline{B}\right)$$

Parallel to the magnetic field, and setting the left side to zero

$$n_e e E_{||} + \nabla_{||} p_e = 0$$

Electron response

Linearising this equation, neglecting temperature variations because parallel thermal conduction is fast (so $\nabla_{||}T \simeq 0$), and assuming electrostatic perturbations so $E_{||} = -\nabla_{||}\phi$

$$-\left(n_0 + \delta n\right) e\nabla_{||}\left(\phi_0 + \delta\phi\right) + \nabla_{||}\left[T_0\left(n_0 + \delta n\right)\right] = 0$$

$$-n_0 e\nabla_{||}\delta\phi - \delta n e\cancelto{0}{\nabla_{||}\phi_0} + \nabla_{||}\left(\delta n T_0\right) = 0$$

Since there are no parallel gradients of equilibrium ϕ_0 and n_0, this becomes

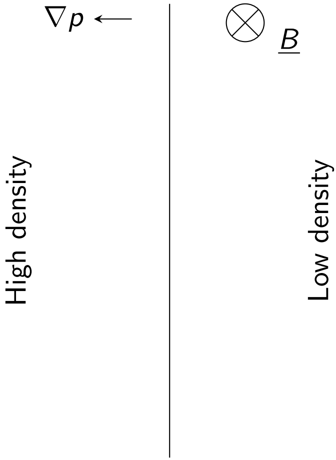
$$-\nabla_{||}\left(n_0 e\delta\phi\right) + \nabla_{||}\left(\delta n T_0\right) = 0$$

and so:

$$n_0 e\delta\phi = \delta n T_0 \qquad \Rightarrow \frac{\delta n}{n_0} = \frac{e\delta\phi}{T_0}$$

This is called the **adiabatic** or **Boltzmann response**

Consider a slab of plasma with the $\underline{B}$ into the page, and density increasing from right to left



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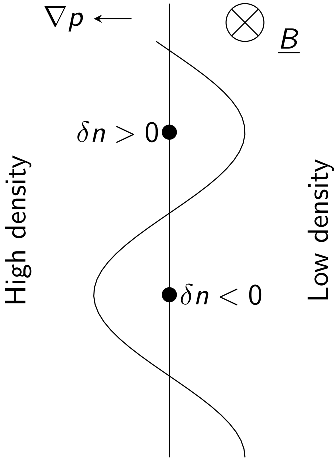

- A small density perturbation

Electron drift waves

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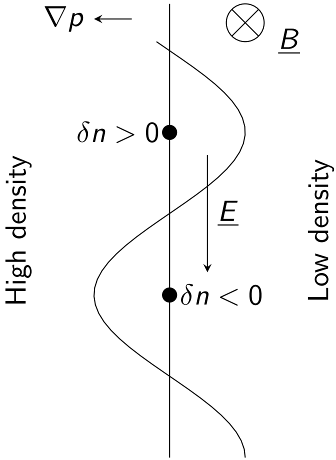

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- The electrons move along the field and establish force balance and so $\delta\phi \simeq \frac{T_0}{e}\frac{\delta n}{n_0}$

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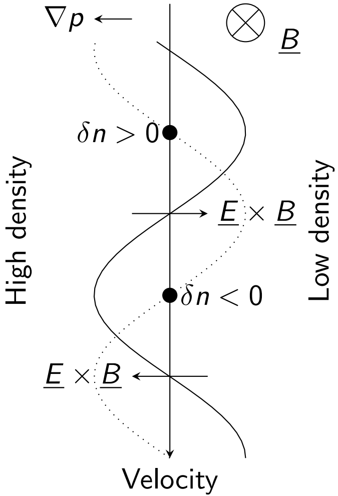

- A small density perturbation
- The electrons move along the field and establish force balance and so $\delta\phi \simeq \frac{T_0}{e}\frac{\delta n}{n_0}$
- This gives an $\underline{E} \times \underline{B}$ velocity which is 90^o out of phase with the density
- This is a wave which propagates perpendicular to ∇n and $\underline{B}$
- Because $\delta \underline{v}$ and δn are out of phase, there is no net radial transport.

Electron drift waves

We can derive the dispersion relation of this wave using the ion density (continuity) equation

$$\frac{\partial \delta n_i}{\partial t} = -\nabla \cdot \left[\left(n_0 + \delta n_i\right) \delta \underline{v}\right]$$

As before, put in a solution $\delta n_i \propto \exp\left(-i\omega t\right)$. so that $\frac{\partial \delta n_i}{\partial t} \rightarrow -i\omega$. For the ions, assume that radial $\underline{E} \times \underline{B}$ is the dominant motion (not true for electrons!)

$$\Rightarrow -i\omega\delta n_i = v_{ExB,r}\frac{dn_0}{dr}$$

Electron drift waves

The radial $\underline{E} \times \underline{B}$ velocity is given by the poloidal gradient of the electrostatic potential (since B field is mainly toroidal).
Taking a single wave of the form $\exp\left(ik_\theta r\theta\right)$

$$v_{ExB,r} = -\frac{1}{B}\frac{\partial\delta\phi}{\partial r\theta} = -\frac{1}{B}ik_\theta\delta\phi \Rightarrow n_i = \frac{k_\theta\delta\phi}{\omega B}\frac{dn_0}{dr}$$

Using quasi-neutrality, $n_i \simeq n_e$ and so

$$\frac{k_\theta\cancel{\delta\phi}}{\omega B}\frac{dn_0}{dr} = \frac{n_0 e\cancel{\delta\phi}}{T_0}$$

The wave frequency is therefore

$$\omega = \frac{k_\theta T_0}{eBn_0}\frac{dn_0}{dr} = \frac{k_\theta}{eBn_0}\frac{dp_0}{dr} = k_\theta v_* = \omega_*$$

This velocity v_* is the **diamagnetic drift** velocity, and is the reason why this category of waves are known as **drift waves**.

Electron drift wave growth

- If velocity and density perturbations are out of phase then there is no net radial transport and the wave doesn't grow.
- If the electrons can't keep up with the wave then this leads to a phase shift and growth of the mode
- Finite electron mass, or just about any form of dissipation e.g. resistivity or Landau damping will have this effect.
- This is often called the **Universal instability** because all useful plasmas have density gradients and some dissipation.
- In fact magnetic shear (change in q with radius) can stabilise these modes
- In toroidal geometry these waves are destabilised by trapped particles and become unstable above a threshold given by the magnetic shear

Ion Temperature Gradient (ITG or η_i) mode

- Start with a small fluctuation in temperature
- The sum of curvature and grad-B drifts is

$$\underline{v}_{\nabla B} + \underline{v}_R \simeq \frac{\left(v_\perp^2/2 + v_{||}^2\right)}{R\Omega}\underline{e}_z$$

so hotter particles drift faster

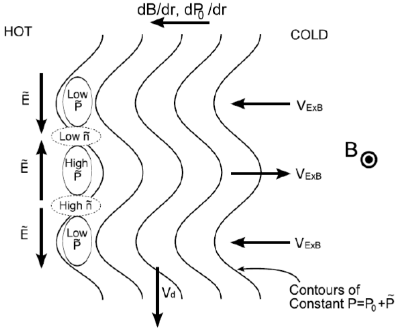

Figure : D.Applegate's thesis

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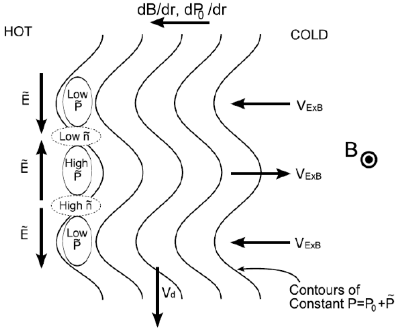
  so hotter particles drift faster
- This leads to regions of higher and lower density
- Using the electron adiabatic response, this gives an $\underline{E} \times \underline{B}$ drift which amplifies the original perturbation



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Ion Temperature Gradient (ITG or η_i) mode

- The ITG mode is stabilised by magnetic shear, but less so than the electron drift wave
- A proper analysis gives a threshold for stability (see figure)
- The mode gets its name because there is a threshold in ion temperature gradient at a given density gradient
- This is thought to be why tokamak profiles are "stiff" - the gradient tend to be fixed.

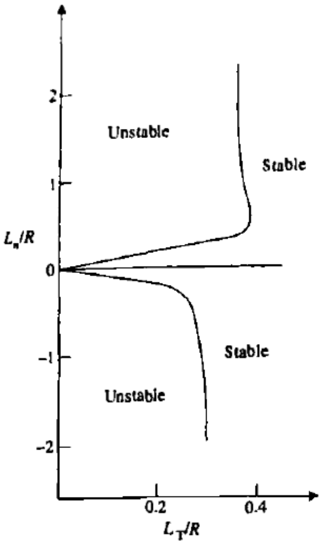


Figure : Wesson 8.3.3

Electron Temperature Gradient (ETG)

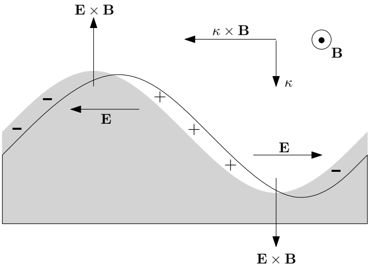
- Because the ITG instability depends on the ion ∇B and curvature drift, it has wavelengths of a few ion Larmor radii.
- There is another instability which has a scale between the electron and ion Larmor radii.
- Because it is smaller than the ion Larmor radius, the instability doesn't "know" that ions are on orbits. Instead, it sees a Boltzmann-like response for the ions, similar to the electron behavior in the ITG mode.
- The ETG mode is therefore very similar to the ITG mode, but with the roles of the electrons and ions reversed
- **Sidenote:** there has been a long and ongoing debate over whether ETG or ITG is more important for tokamak transport

Trapped particle modes

Particle trapping in toroidal plasmas means there are two populations of particles:

- passing particles which have a net parallel velocity and which explore all parts of the torus
- trapped particles which have little net parallel motion, and which only explore part of the torus

This leads to an instability similar to sausage / interchange modes



- For passing particles the good and bad curvature averages out and is stable
- Trapped particles only see the bad curvature side so have a net drift
- The passing particles act like a background with a Boltzmann response

The perpendicular scales of these instabilities are usually given in terms of the ion Larmor radius ρ_i

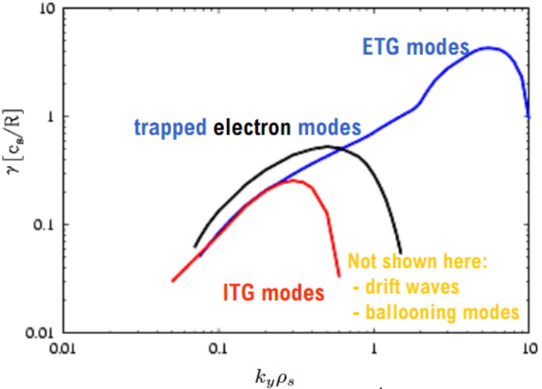


Figure : F.Jenko, GENE, 16^{th} Sep 2008

Can transform into each other by continuous parameter changes[1]

[1] Kammerer *et.al.* Phys. Plasmas 15, 052102 (2008)

Interacting modes

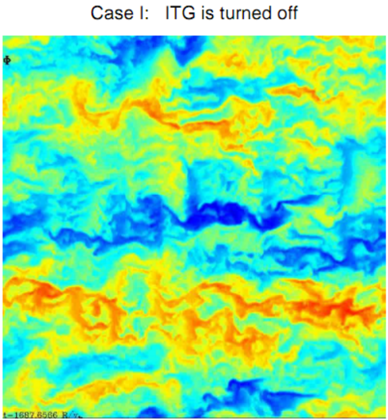


Figure : F.Jenko, GENE, 16^{th} Sep 2008

- GENE simulations, 100,000 CPU-hours
- Box size $64\rho_i$
- Resolution $\sim 2\rho_e$ with reduced mass ratio of 400
- Switching off ITG modes gives combination of ETG and TEM

Interacting modes

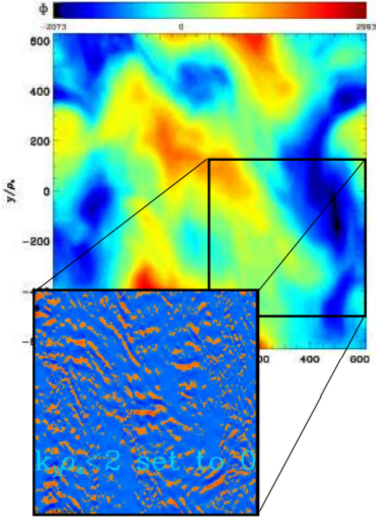


Figure : F.Jenko, GENE, 16^{th} Sep

- Adding ITG drive modifies the results
- Small-scale structures (ETG and TEM) broken up by large-scale structures (ITG)
- Complicated nonlinear dynamics with interacting modes
- Can't just superimpose different instabilities

Summary

- There are many instabilities in plasmas which arise because the plasma is composed of different populations of particles, rather than a single homogenous fluid
  - Different response of ions and electrons along the field gives rise to the electron drift wave and ITG mode
  - Different Larmor radius scales for electrons and ions leads to a different perpendicular response and the ETG mode
  - Trapped and passing particles respond differently to fluctuations, leading to trapped particle modes like the TEM
- These instabilities are mainly electrostatic: magnetic fluctuations are present, and can be an important effect at high β, but are not essential.
- At large amplitude the dynamics are highly nonlinear $\Rightarrow$ turbulence (Next lecture)
- Thought to be the origin of **anomalous transport** in tokamaks