Department of Mathematics
Formal Languages and Automata 2003
Solutions for Exercises 8

1. Let $\mathscr{A} = (A, Q, \delta, q_0, F)$ be a DFA, and consider the reduced DFA $\bar{\mathscr{A}} = (A, \overline{Q}, \overline{\delta}, [q_0], \overline{F})$ obtained from $\mathscr{A}$. Prove that

$$\overline{\delta}([q], w) = [\delta(q, w)] \text{ for all } q \in Q \text{ and } w \in A^*.$$

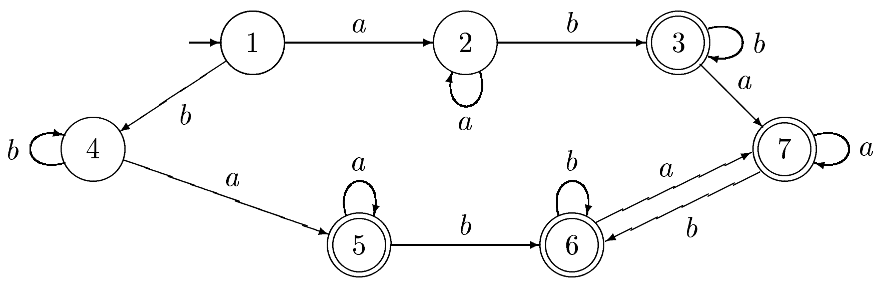
First, for any $q \in Q$, we have $\overline{\delta}([q], \varepsilon) = [q] = [\delta(q, \varepsilon)]$, so the equation holds when $|w| = 0$. Now suppose it holds for all states and words of length k. Let $|w| = k + 1$. Then $w = va$ for some word v of length k and some letter a. Hence

$$\begin{aligned}
\overline{\delta}([q], w) &= \overline{\delta}([q], va) \\
&= \overline{\delta}(\overline{\delta}([q], v), a) \\
&= \overline{\delta}([\delta(q, v)], a) \text{ (by the induction hypothesis)} \\
&= [\delta(\delta(q, v), a)] \text{ (definition of } \overline{\delta}) \\
&= [\delta(q, va)] \\
&= [\delta(q, w)].
\end{aligned}$$

Hence result by induction.

2. For each of the following DFAs, find a reduced DFA which is equivalent to it.

(a)



The following table gives the transition function:

	a	b
1	2	4
2	2	3
3	7	3
4	5	4
5	5	6
6	7	6
7	7	6

The set of nonfinal states and the set of final states are the $\sim_0$-classes, i.e., the $\sim_0$-classes are

$$\{1, 2, 4\} \text{ and } \{3, 5, 6, 7\}.$$

From the table, we see that the $\sim_1$-classes are

$$\{1\},\ \{2\},\ \{4\},\ \{3, 5, 6, 7\},$$

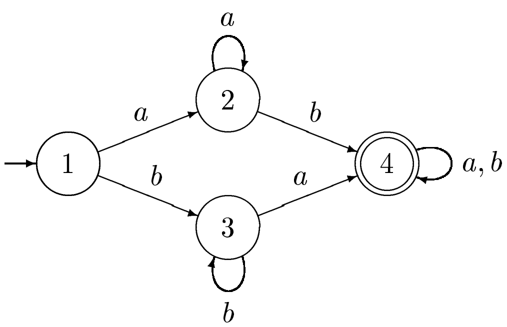
and the the $\sim_2$-classes are

$$\{1\},\ \{2\},\ \{4\},\ \{3, 5, 6, 7\}.$$

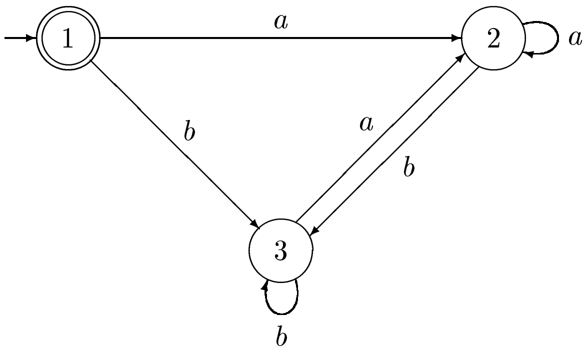
Since $\sim_1 = \sim_2$, we have $\sim = \sim_1$, and we can construct a reduced DFA whose states are the $\sim$-classes, and which recognises the same language as the given DFA. We relabel the states:

$$1 = \{1\},\ 2 = \{2\},\ 3 = \{4\},\ 4 = \{3, 5, 6, 7\},$$

and the required reduced DFA is:



(b)



The following table gives the transition function:

	a	b
1	2	3
2	2	3
3	2	3

The set of nonfinal states and the set of final states are the $\sim_0$-classes, i.e., the $\sim_0$-classes are

$$\{1\} \text{ and } \{2, 3\}.$$

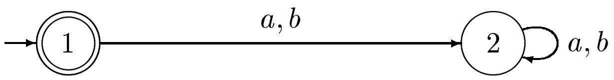
From the table, we see that the $\sim_1$-classes are

$$\{1\} \text{ and } \{2, 3\}.$$

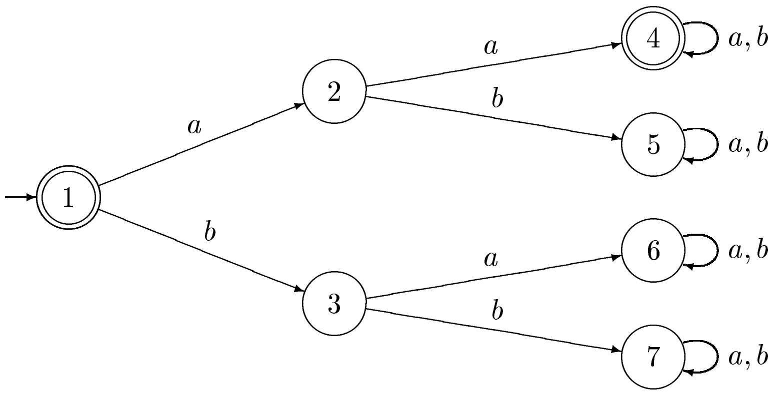
Since $\sim_0 = \sim_1$, we have $\sim = \sim_0$, and we can construct a reduced DFA whose states are the $\sim$-classes, and which recognises the same language as the given DFA. We relabel the states:

$$1 = \{1\} \text{ and } 2 = \{2, 3\},$$

and the required reduced DFA is:



(c)



The following table gives the transition function:

	a	b
1	2	3
2	4	5
3	6	7
4	4	4
5	5	5
6	6	6
7	7	7

The set of nonfinal states and the set of final states are the $\sim_0$-classes, i.e., the $\sim_0$-classes are

$$\{1,4\} \text{ and } \{2,3,5,6,7\}.$$

From the table, we see that the $\sim_1$-classes are

$$\{1\},\ \{2\},\ \{4\},\ \{3,5,6,7\},$$

and the the $\sim_2$-classes are

$$\{1\},\ \{2\},\ \{4\},\ \{3,5,6,7\}.$$

Since $\sim_1 = \sim_2$, we have $\sim\, = \,\sim_1$, and we can construct a reduced DFA whose states are the $\sim$-classes, and which recognises the same language as the given DFA. We relabel the states:

$$1 = \{1\},\ 2 = \{2\},\ 4 = \{4\},\ 3 = \{3,5,6,7\},$$

and the required reduced DFA is: