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Dynamic decision making: what do people do?

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Abstract Potentially dynamically-inconsistent individuals create particular problems for economics, as their behaviour depends upon whether and how they attempt to resolve their potential inconsistency. This paper reports on the results of a new experiment designed to help us distinguish between the different types that may exist. We classify people into four types: myopic, naïve, resolute and sophisticated. We implement a new and simple experimental design in which subjects are asked to take two sequential decisions (interspersed by a random move by Nature) concerning the allocation of a given sum of money. The resulting data enables us to classify the subjects. We find that the majority are resolute, a significant few are sophisticated, rather few are naïve and similarly few are myopic.

Keywords Dynamic inconsistency · Sequential choice · Myopic · Naïve · Resolute · Sophisticated

JEL Classification D90 · D81

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Dynamic consistency is a central assumption in economics and is essential to many of the key results and policy prescriptions in important fields, such as those of investment, saving and pensions. However, there is ample evidence that people are not dynamically consistent. In the context of discounting, it seems that many people do not discount exponentially; so that their relative evaluation of consumption at two given points of time depend upon the time at which the evaluation is being made. In the context of decision-making under risk, it seems that many people do not have Expected Utility (EU) preferences; so that their risk aversion about choices in some period vary depending upon the time at which their risk evaluation is being made. In both cases, as time passes and uncertainty resolves, people may wish to change previously planned decisions, thus displaying inconsistencies between planned and actual decisions.

While non-exponential discounting and non-Expected Utility may lead to inconsistent behaviour, whether this possibility actually arises or not depends on how people react to potential dynamic inconsistency. Are they aware of it? Do they ignore it? Do they take it into account in planning their future behaviour? Do they somehow constrain themselves to act consistently? In this paper, we develop a new experimental design to shed light on these issues by first investigating the choices individuals should make depending on their reactions to potential inconsistency and then by verifying how they actually behave in the experiment.

Reactions to potential dynamic inconsistencies will vary depending on which type of behaviour subjects follow. Therefore, the first step in our analysis consists in identifying a set of types that can arise in the context of dynamic decision making under risk. In describing the various types, we use a terminology that it is by now rather standard.[1] The first type of behaviour we consider is the *naïve* one. This behaviour pertains to individuals who are supposed to work through time always choosing the best decision as viewed from the present perspective, even though this may lead to actual choices which differ from planned ones. The second type of behaviour we consider is the *resolute* one. It pertains to individuals who are supposed not to change their first period decisions. This behaviour can be interpreted in two quite different ways. One refers to the idea of commitment. Metaphorically, resolute individuals follow the example of Ulysses (about to be confronted by the sirens) by binding themselves to the mast, that is by imposing their first period preferences. The other one refers to the criticism of the consequentialist approach that is at the heart of the previous type of behaviour put forward by Machina (1989) and McClennen (1990). In this case, individuals are supposed

[1] This terminology goes back to the first studies of dynamic consistency in choice problems without risk. See Hammond (1976), who in turn refers to Blackorby et al. (1973), Pollak (1968) and Strotz (1955–1956).

to evaluate the decision at each stage by taking into account not only the possibilities still available to them, but also those that, while excluded by the passage of time and the resolution of uncertainty, represent risks that have been borne. The third type of behaviour we consider is the *sophisticated* one. It pertains to people who *backwardly induct* and it can be interpreted by assuming that potentially dynamically inconsistent people realise that they will want to change their minds in the future, and anticipate this behaviour *ex ante*. It is as if Ulysses decided to travel home by a different route. Indeed this is the prevailing model used in economics. We remark that the differences in choices arising from the above types of behaviour are only relevant for potentially dynamically inconsistent subjects. In particular, they do not exist for an individual with Expected Utility preferences, since in this case dynamic inconsistency would not be an issue.

The previous types of behaviour have the common property that, at the beginning of the decision problem, individuals realise they are facing a multiple-stage decision problem and take this into account. However, it is not uncommon for individuals to fail to realize the complete sequence of stages that are involved in a dynamic choice problem. In this case, they may neglect subsequent stages of the problem. As will be clearer in the following section, our design allows us to consider also this type of behaviour, which we call *myopic*. For future reference, it is useful to note that both naïve and myopic subjects are short-sighted, although for quite different reasons.

The experimental design we propose allows us to discriminate between the different types and therefore to answer the questions posed above. This is particularly relevant not only for economic theory but also for public policy. For example, if people are myopic, then the state might feel obliged to take action to ensure adequate pensions for the population. Therefore, the issues this paper addresses go well beyond the immediate analysis of individual behaviour.

We consider the simplest type of dynamic decision problem under risk – one with just two stages and two alternatives in each stage. We adopt an experimental method pioneered by Loomes (1991) and extend it to a dynamic decision problem. In the first stage, each subject allocates money between two alternatives and then Nature moves and selects at random one of them. In the second stage, each subject has to allocate the money that Nature's move implies between two further risky alternatives. After a final move by Nature, the subject earns the money that Nature's move implies. In our experiment, 71 subjects were asked to repeat this decision task on 27 problems with different probabilities for Nature's moves.

The data gathered in the lab enable us to see which type subjects are. For each subject, we assume Rank Dependent Expected Utility preferences, with a Constant Relative Risk Aversion utility function and a Quiggin (1982) weighting function. Two parameters are involved to describe the preferences: the risk aversion parameter of the utility function and the weighting parameter of the weighting function.[2] In addition, we need to estimate the precision of the probability distribution describing the noisiness of the implementation of the subjects' optimal strategy. We estimate each type separately and see which type fits best—that is, which describes best the decisions of the subject.

To summarize our results, we see that the resolute type is the best for 55% of our 71 subjects, with 23% sophisticated, 13% myopic and 10% naïve. If we restrict attention to those subjects for whom the best-fitting model is *significantly* better than the others at the 5% (1%) level, using the Clarke (2007) test, we find that of the 37 (21) for which this is true, 62% (76%) are resolute, 16% (14%) are sophisticated, 16% (10%) are myopic and 5% (none) are naïve. This first set of results shows that the majority of our subjects are resolute, a significant minority are sophisticated, rather few are naïve and similarly few are myopic.

In addition to the previous analysis, we consider those subjects who are significantly not EU as a result of a direct test on the magnitude of the estimated parameter of weighting functions. There are 28 such subjects and in this case the resolute type is the best for 64% of them, the sophisticated type for 14%, the naïve type for 14% and the myopic type for 7%. Finally, if we restrict attention to those subjects for whom the best-fitting model is *significantly* better than the others at the 5% (1%) level, we find that of the 18 (11) for which this is true, 72% (91%) are resolute, 17% (9%) are sophisticated, 11% (none) are myopic and none (none) are naïve. These results confirm the prevalence of resolute subjects, which is therefore robust to the exclusion of EU subjects, as well as the relative paucity of naïve and myopic subjects. Moreover, the above results show that the naïve and myopic types are rather rare, when taken separately. However, if we consider them together as short-sighted subjects, they are as common as the sophisticated ones over the whole sample and they are slightly more frequent than the latter if we take into account only those subjects who are significantly non-EU.[3]

We describe the related literature in the next section and the new design in the following one. We then define the various types of economic agent and discuss how they should behave in this experiment. We then describe our econometric analyses and present our results. A final section concludes.

1 Related literature

In this section, we briefly relate our approach and results to those available in the existing literature. As mentioned in the previous paragraph, we are

[2] We recall that if the weighting parameter is equal to 1, so that the weighting function is linear, then the individual has EU preferences and thus is not dynamically inconsistent.

[3] For simplicity, this last observation refers to the figures obtained before performing the Clarke tests.

interested in identifying different types of behaviour in dynamic problems under risk. In this sense, our approach is different from the idea of testing theories of choice by investigating which of the principles they are based on survive the experimental evidence. This approach is chosen in particular by Cubitt et al. (1998), who test which principles of dynamic choice are involved in common-ratio type violations of the Independence Axiom. Indeed, their strategy is based on the observation that, since the Independence Axiom can be shown to follow from specific principles of dynamic choice,[4] when it is violated, at least one of those principles must be violated as well. Following this approach, they set-up a between-subject experimental design and find evidence of failure of the *time independence* principle, and therefore those theories of choice based which are based on it.[5,6] A similar approach is followed by Cubitt and Sugden (2001), who are however particularly interested in controlling for the role affective experiences have in dynamic choice under risk. They propose evidence that the time independence and the separability principles are jointly rejected by the data.[7] Finally, a related paper by Busemeyer et al. (2000) uses a within-subject design to investigate violations of a set of consistency principles in a dynamic choice problem. They find robust evidence of violation of dynamic and strategic consistency but not of consequential consistency.[8]

In contrast with the previous analyses, Hey and Paradiso (2006) focus on how preferences, and not behaviour, differ for decision trees that are strategically identical. By appropriately adapting some of the choice problems proposed by Cubitt et al. (1998), they use evaluations of different trees to test whether individuals use the strategy method or the backward induction method when tackling dynamic decision problems. The authors find evidence not only of dynamic inconsistency, as Cubitt et al. (1998) do, but more importantly they find evidence that subjects value more those choice problems where

[4]See Hammond (1988), McClennen (1990) and Cubitt (1996), as well as Karni and Schmeidler (1991) and Volij (1994).

[5]As explained by Cubitt et al., this is the principle "according to which an agent, if required to precommit to an action to be taken conditional on a prior act of nature, precommits to the action which would be chosen if the moment of choice was delayed until after that act of nature". See Cubitt et al. (1998, p. 1366).

[6]Recent unpublished work by Dubois and Nebout (2009) extends the previous analysis by using a within-subject experimental design and essentially confirms the main findings.

[7]According to Cubitt and Sugden, the principle of separability "asserts that, at any point of choice in a dynamic problem, the history of how that point was reached is irrelevant to the choice made there: on this view, choice is forward-looking and unaffected by contingencies that can no longer occur". See Cubitt and Sugden (2001, p. 111).

[8]As explained by Busemeyer et al., "intuitively, dynamic consistency requires the decision maker to follow through on plans to the end. This is required for the working-backward planning strategy. Consequential consistency requires the decision maker to focus solely on the future events and final consequences given the current state. [...] Strategic consistency results when both dynamic and consequential consistency are satisfied." See Busemeyer et al. (2000, p. 531).

pre-commitment is available.[9] Expanding on these findings, Hey and Lotito (2009) propose an experiment where both behaviour and preferences are investigated. They use data on tree evaluations together with data on choices and find evidence that the strategy method, as opposed to that of backward induction, is followed by the majority of subjects. Our work improves substantially upon this strand of literature, since it uses an homogeneous and more informative type of data, namely choices which are expressed as a continuous variable. Moreover, it adds a new type of behaviour, that of myopia, which has been until now neglected in the literature.

Issues of dynamic inconsistency have been intensively investigated also in a related branch of literature interested in studying the consequences of abandoning the hypothesis of exponential discounting because of its failure to match empirical evidence.[10] Indeed, in those theories that explain observed behavioural anomalies in dynamic choice by assuming a rate of time preference that declines over time, dynamic inconsistency naturally emerges.[11] Therefore, it is possible to find in that literature analysis of behaviour that refer to some of the categories we have described in the previous paragraph, in particular the sophisticated and the naïve behaviour.[12] Within this branch of literature, the empirical studies are mainly focussed either on the estimation of the discount function or on the identification of behavioural strategies that can reveal either sophistication or naïvety.[13] Therefore, the approach we follow is starkly different. Not only do we focus on the case of non-Expected Utility, but more importantly we identify a set of types of behaviour that are observationally different and we test which one fits the best the experimental data we have gathered.

2 The experimental design

Our experimental design was inspired by one pioneered by Loomes (1991) and subsequently extended by Choi et al. (2007). This design was developed

[9] Interestingly this result can be interpreted as suggesting that decision-makers would like to be resolute.

[10] See e.g. Angeletos et al. (2001), Frederick et al. (2002) and Chabris et al. (2008) and the references therein for an ample discussion of these issues. See also in Loewenstein and Prelec (1991) a discussion of similarities among behavioral anomalies in models of choice over time and under uncertainty.

[11] See Akerlof (1991), Loewenstein and Prelec (1992), Laibson (1997) and O'Donoghue and Rabin (1999) to name just some of the seminal contributions. Of course, a declining rate of time preference is not the only possible explanation for the available evidence, see, for example, Rubinstein (2003).

[12] See, for example, O'Donoghue and Rabin (2000) for a general discussion.

[13] For an ample discussion of the former, see, for example, Frederick et al. (2002) and, for a very recent analysis, Benhabib et al. (2009). As for the latter, see, for example, Ariely and Wertenbroch (2002), Ashraf et al. (2006), Della Vigna and Malmendier (2006) and Thaler and Benartzi (2004).

for a different decision problem in a different context. In Loomes's design subjects were simply asked to allocate a sum of money between two risky alternatives. Choi et al. extended the design by endowing subjects with tokens and having different exchange rates between tokens and money for the different alternatives. We do not use that feature but extend Loomes's design in a different direction by having a dynamic allocation problem. In our design, subjects are presented with a set of N decision problems, all with the same two-stage structure. In each problem, subjects are given a sum of money to allocate between two probabilistic options with known and stated probabilities. In the first stage, subjects are asked to allocate the initial amount of money between these first two options. Then Nature moves at random—thereby 'selecting' one of the two options. In the second stage, starting after Nature's random move, subjects are asked to allocate the amount available, which depends on their choices in the first stage and Nature's move, between two further probabilistic options with known and stated probabilities. In the final stage, Nature moves again at random (thereby 'selecting' one of these further options) and the subject earns whatever he or she allocated to that option at the beginning of the second stage. At the end of the experiment one of the decision problems is chosen at random and the subject is paid his or her earnings on that particular problem.

We call options 1 and 2 the two probabilistic options in the first stage and we denote their probability respectively by p and $(1-p)$. Moreover, we call option 1A and option 1B the two second-stage probabilistic options if Nature chooses option 1 at end of the first stage and we denote their probability by p_1 and $(1-p_1)$. Similarly, we call option 2A and 2B the two second-stage probabilistic options if Nature chooses option 2 at the end of the first stage and we denote their probability by p_2 and $(1-p_2)$.

The design is simple and informative. To illustrate how the design discriminates between different types of potentially dynamically inconsistent people, let us assume that the decision-maker has Rank Dependent Expected Utility (RDEU) preferences, with utility function $u(x)$ and weighting function $w(p)$. If $w(p) = p$, then the model reduces to that of Expected Utility (EU) theory and the decision-maker is not dynamically inconsistent. However suppose that $w(.)$ is not linear. Then potential dynamic inconsistencies arise.

Consider the problem as viewed from when the decision-maker must make the first decision. The decision maker has to allocate the initial amount of money, denoted by m, between options 1 and 2 at the first stage and then allocate the amount available after Nature's choice between either options 1A and 1B or between options 2A and 2B. Thus, as viewed from the first stage, the decision-maker has to choose x_{1A}, x_{1B}, x_{2A} and x_{2B}—which denote the amounts allocated respectively to options 1A, 1B, 2A and 2B − to maximise his or her Rank Dependent Expected Utility under the constraint that $x_{1A} + x_{1B} + x_{2A} + x_{2B} = m$.

As it is known, with Rank Dependent Expected Utility, the weight attached to each possible outcome depends on the ranking of all available outcomes. As an illustration, suppose the outcomes satisfy the ranking $x_{2B} \leq x_{2A} \leq x_{1B} \leq x_{1A}$.[14] In this case, the weights attached to the outcomes are given by

$$\begin{aligned} v_{1A} &= w(q_{1A}) \\ v_{1B} &= w(q_{1A} + q_{1B}) - w(q_{1A}) \\ v_{2A} &= w(q_{1A} + q_{1B} + q_{2A}) - w(q_{1A} + q_{1B}) \\ v_{2B} &= 1 - w(q_{1A} + q_{1B} + q_{2A}), \end{aligned}$$

where $q_{1A} = pp_1$, $q_{1B} = p(1 - p_1)$, $q_{2A} = (1 - p)p_2$. Therefore, given the utility function $u(.)$ and the weighting function $w(.)$, the problem individuals face at the beginning of the first stage is to choose x_{1A}, x_{1B}, x_{2A} and x_{2B} to maximize the objective function

$$v_{1A}u(x_{1A}) + v_{1B}u(x_{1B}) + v_{2A}u(x_{2A}) + v_{2B}u(x_{2B}) \tag{2}$$

under the constraint $x_{1A} + x_{1B} + x_{2A} + x_{2B} = m$ and the given ranking constraint. Denote by $x^*_{1A}, x^*_{1B}, x^*_{2A}$ and x^*_{2B} the optimal values. Given these, in the first stage the individual allocates $m_1 = x^*_{1A} + x^*_{1B}$ to option 1 and $m_2 = x^*_{2A} + x^*_{2B}$ to option 2.

The potential dynamic inconsistency arises when the individual gets to the second stage and reconsiders his or her choices. Suppose for concreteness that Nature 'chooses' option 1, so that the individual has the amount m_1 to allocate between the second-stage probabilistic options. Suppose, in addition, that he or she reconsiders the first-stage decision about x_{1A} and x_{1B} and chooses to allocate m_1 to maximize his or her Rank Dependent Expected Utility *as viewed from the present point*. In this case, the weights he or she will use to evaluate the risky prospect are denoted by $\widehat{v}_{1A}$ and $\widehat{v}_{1B}$. Given that there are no further stages, the subject's problem is to choose x_{1A} and x_{1B} to maximise the objective function

$$\widehat{v}_{1A}u(x_{1A}) + \widehat{v}_{1B}u(x_{1B}) \tag{3}$$

under the constraint $x_{1A} + x_{1B} = m_1$ and the appropriate ranking constraint. Denote by $\widehat{x}_{1A}(m_1)$ and $\widehat{x}_{1B}(m_1)$ the subject's optimal choices, which depend on m_1.

If $w(p) = p$, then it must be that $\widehat{x}_{1A} = x^*_{1A}$ and $\widehat{x}_{1B} = x^*_{1B}$. Otherwise, by a standard revealed preferences argument, one should conclude that the allocation $(x^*_{1A}, x^*_{1B}, x^*_{2A}, x^*_{2B})$ was *not* optimal.[15] However, if $w(p) \neq p$, this

[14] Here we illustrate only the case of a single ranking. Of course, in the main analysis we consider all possible rankings. We also recall that the ranking is endogenous, for it relates to the allocation chosen by the subjects.

[15] If $(\widehat{x}_{1A}, \widehat{x}_{1B}) \neq (x^*_{1A}, x^*_{1B})$, then it must be that $p_1u(\widehat{x}_{1A}) + (1 - p_1)u(\widehat{x}_{1B}) > p_1u(x^*_{1A}) + (1 - p_1)u(x^*_{1B})$ and therefore that $pp_1u(\widehat{x}_{1A}) + p(1 - p_1)u(\widehat{x}_{1B}) > pp_1u(x^*_{1A}) + p(1 - p_1)u(x^*_{1B})$. This implies that the allocation $(\widehat{x}_{1A}, \widehat{x}_{1B}, x^*_{2A}, x^*_{2B})$ is preferable to the allocation $(x^*_{1A}, x^*_{1B}, x^*_{2A}, x^*_{2B})$. Since $\widehat{x}_{1A} + \widehat{x}_{1B} + x^*_{2A} + x^*_{2B} = m$, the former allocation is feasible and therefore the latter could have not been optimal at the beginning of the problem. Note that in case of linear weighting function, the ranking constraint does not affect the solution and therefore can be neglected in the above reasoning.

argument breaks down and there is no reason why $\widehat{x}_{1A}$ and $\widehat{x}_{1B}$ need to be equal to x^*_{1A} and x^*_{1B}, and in general they will be different.[16]

As mentioned in the introduction, we are interested in understanding how individuals deal with this inconsistency. To describe their behaviour in the context of our model, we say that an individual is *resolute* if he or she allocates x^*_{1A} to option 1A and x^*_{1B} to option 1B, thus implementing the choices made at the beginning of the first stage. If instead the individual allocates $\widehat{x}_{1A}$ to option 1A and $\widehat{x}_{1B}$ to option 1B, we say that he or she is *naïve*, because at the first stage he or she did not take into account the fact that he or she would choose differently at the second stage.

This observation suggests a natural way to represent *sophisticated* behaviour in the context of our model. Since a sophisticated individual takes into account that he or she will reconsider the optimal choices at the beginning of the second stage, we assume that he or she actually solves the allocation problem in two steps. First, taking as given an allocation of money between options 1 and 2, say m_1 and m_2, he or she chooses x_{1A} and x_{1B} to maximize his or her second-stage RDEU function (Eq. 3) under the constraint that $x_{1A} + x_{1B} = m_1$ and the appropriate ranking constraint, and similarly he or she chooses x_{2A} and x_{2B}. Denote by $x_{1A}(m_1), x_{1B}(m_1)$ and $x_{2A}(m_2), x_{2B}(m_2)$ the optimal values, where the notation makes clear the dependence on the given allocation (m_1, m_2). In the second step the individual chooses m_1 and m_2 to maximize

$$v_{1A}u(x_{1A}(m_1)) + v_{1B}u(x_{1B}(m_1)) + v_{2A}u(x_{2A}(m_2)) + v_{2B}u(x_{2B}(m_2))$$

subject to the constraint that $x_{1A}(m_1) + x_{1B}(m_1) + x_{2A}(m_2) + x_{2B}(m_2) = m$ and the appropriate ranking constraint. The optimal allocation, which we denote m_1^{**} and m_2^{**}, will then determine the final allocations $x^{**}_{jA} = x_{jA}(m_1^{**})$ and $x^{**}_{jB} = x_{jB}(m_2^{**})$ for $j = 1, 2$. Note that the sophisticated behaviour differs from the resolute, and *a fortiori*, from the naïve one because it takes the second stage optimal choices as a constraint. However, when $w(p) = p$, sophisticated and resolute behaviour will result in the same optimal choices.

As mentioned in the introduction, the previous types of behaviour have the common property that, at the beginning of the decision problem, the individuals realise that there are multiple stages and take this fact into account. Therefore, to model *myopic* behaviour, we assume that the individual ignores the second stage and takes his or her decisions as if the outcome at the first stage will be the actual payment. Therefore, at the beginning of the problem he or she chooses $\widetilde{x}_1$ and $\widetilde{x}_2$ to maximise

$$v_1 u\left(\widetilde{x}_1\right) + v_2 u\left(\widetilde{x}_2\right)$$

under that constraint that $\widetilde{x}_1 + \widetilde{x}_2 = m$ and the appropriate ranking constraint. When asked to actually make a decision at the second stage, the myopic individual will choose, x_{jA} and x_{jB} by maximising his or her Rank Dependent

[16] A similar reasoning holds in case Nature randomly selects move 2.

Table 1 An example of choices of the different types

pn	$\overline{m}$	Probabilities			Myopic			Naïve			Resolute			Sophisticated		
		p	p_1	p_2	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}
1	40	.60	.60	.60	20.00	10.00	10.00	25.55	12.78	7.22	25.55	18.33	7.22	20.00	10.00	10.00
2	40	.70	.60	.60	22.09	11.04	8.96	27.25	13.63	6.37	27.25	20.88	6.37	22.09	11.04	8.96
3	40	.80	.60	.60	27.60	13.80	6.20	28.88	14.44	5.56	28.88	23.32	5.56	27.60	13.80	6.20
4	40	.60	.70	.60	20.00	11.04	10.00	27.25	15.05	6.37	27.25	20.88	6.37	21.10	11.65	9.45
5	40	.70	.70	.60	22.09	12.19	8.96	29.14	16.09	5.43	29.14	23.72	5.43	22.63	12.49	8.69
6	40	.80	.70	.60	27.60	15.24	6.20	30.95	17.09	4.52	30.95	26.43	4.52	28.01	15.46	6.00
7	40	.60	.80	.60	20.00	13.80	10.00	28.88	19.93	5.56	28.88	23.32	5.56	24.69	17.04	7.65
8	40	.70	.80	.60	22.09	15.24	8.96	30.95	21.36	4.52	30.95	26.43	4.52	24.69	17.04	7.65
9	40	.80	.80	.60	27.60	19.04	6.20	32.92	22.71	3.54	32.92	29.38	3.54	29.23	20.17	5.38
10	40	.60	.60	.70	20.00	10.00	11.04	25.55	12.78	7.98	25.55	18.33	7.22	18.90	9.45	11.65
11	40	.70	.60	.70	22.09	11.04	9.89	27.25	13.63	7.04	27.25	20.88	6.37	18.90	9.45	11.65
12	40	.80	.60	.70	27.60	13.80	6.85	28.88	14.44	6.14	28.88	23.32	5.56	27.78	13.89	6.75
13	40	.60	.70	.70	20.00	11.04	11.04	27.25	15.05	7.04	27.25	20.88	6.37	22.09	12.19	9.89
14	40	.70	.70	.70	22.09	12.19	9.89	29.14	16.09	5.99	29.14	23.72	5.43	22.86	12.62	9.46
15	40	.80	.70	.70	27.60	15.24	6.85	30.95	17.09	5.00	30.95	26.43	4.52	28.18	15.56	6.53
16	40	.60	.80	.70	20.00	13.80	11.04	28.88	19.93	6.14	28.88	23.32	5.56	23.64	16.31	9.03
17	40	.70	.80	.70	22.09	15.24	9.89	30.95	21.36	5.00	30.95	26.43	4.52	25.62	17.68	7.94
18	40	.80	.80	.70	27.60	19.04	6.85	32.92	22.71	3.91	32.92	29.38	3.54	29.39	20.28	5.86
19	40	.60	.60	.80	20.00	10.00	13.80	25.55	12.78	9.97	25.55	18.33	7.22	15.31	7.65	17.04
20	40	.70	.60	.80	22.09	11.04	12.36	27.25	13.63	8.80	27.25	20.88	6.37	18.49	9.25	14.84
21	40	.80	.60	.80	27.60	13.80	8.56	29.02	14.51	7.58	29.02	23.32	5.70	27.80	13.90	8.42
22	40	.60	.70	.80	20.00	11.04	13.80	27.25	15.05	8.80	27.25	20.88	6.37	16.36	9.03	16.31
23	40	.70	.70	.80	22.09	12.19	12.36	29.14	16.09	7.49	29.14	23.72	5.43	24.26	13.39	10.86
24	40	.80	.70	.80	27.60	15.24	8.56	30.95	17.09	6.24	30.95	26.43	4.52	28.20	15.57	8.14
25	40	.60	.80	.80	20.00	13.80	13.80	28.88	19.93	7.67	28.88	23.32	5.56	25.49	17.59	10.01
26	40	.70	.80	.80	22.09	15.24	12.36	30.95	21.36	6.24	30.95	26.43	4.52	27.60	19.04	8.56
27	40	.80	.80	.80	27.60	19.04	8.56	32.92	22.71	4.89	32.92	29.38	3.54	29.41	20.29	7.31

Parameters assumed: $r = 2.0$ and $g = 0.6$
m_1: amount allocated to option 1 in the first stage
x_{1A}: amount allocated to option 1 in the second stage if Nature selects at random option 1
x_{2A}: amount allocated to option 1 in the second stage if Nature selects at random option 2

Table 2 An example with linear utility

$\overline{m}$	Probabilities			Myopic			Naïve			Resolute			Sophisticated		
	p	p_1	p_2	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}
40	.60	.60	.70	20.00	10.00	20.00	40.00	20.00	0.00	40.00	40.00	0.00	0.00	0.00	40.00

Parameters assumed: $u(x) = x$ and $g = 0.6$
m_1: amount allocated to option 1 in the first stage
x_{1A}: amount allocated to option 1 in the second stage if Nature selects at random option 1
x_{2A}: amount allocated to option 1 in the second stage if Nature selects at random option 2
In this example, since $u(x) = x$ the effect of risk aversion on optimal choices is absent and therefore the different decisions are driven by only the weighting function, given the different types of behaviour. In particular, it is easy to see that a resolute subject chooses $x^*_{1B} = 40$ and $x^*_{1A} = x^*_{2A} = x^*_{1B} = 0$. Therefore, in the first stage, he or she will choose $m^*_1 = m$ and $m^*_2 = 0$. A naïve subject chooses, in the first stage, the same allocation as the resolute and therefore he or she sets $\widehat{m}_1 = 40$ and $\widehat{m}_2 = 0$. In the second stage, if Nature selects at random option 2, then the subject has no option but to choose $\widehat{x}_{2A} = \widehat{x}_{2B} = 0$. However, if Nature selects at random option 1, then he or she will reconsider his or her choice and opt for $\widehat{x}_{1A} = \widehat{x}_{1B} = 20$. A sophisticated subject anticipates that, given m_1 and m_2, he or she will choose $x_{1A}(m_1) = x_{1B}(m_1) = m_1/2$ and $x_{1A}(m_2) = m_2$ and $x_{2B}(m_2) = 0$. Therefore, in the first stage, he or she will choose $m^{**}_1 = 0$ and $m^{**}_2 = 40$, so that $x^{**}_{1A} = x^{**}_{1B} = x^{**}_{2B} = 0$ and $x^{**}_{2A} = 40$. Finally, a myopic subject will choose $\widetilde{m}_1 = \widetilde{m}_2 = 20$ in the first stage and, depending on Nature's choice, $\widetilde{x}_{1A} = \widetilde{x}_{1B} = 10$ and $\widetilde{x}_{2A} = 20$ and $\widetilde{x}_{2B} = 0$ in the second stage

Expected Utility as viewed from that point and under the constraint that $x_{jA} + x_{jB} = \widetilde{x}_j$, where $j = 1$ or $j = 2$ depending on Nature's choice.

It follows from the above analysis, therefore, that a crucial feature of our experiment is that different types—resolute, naïve, sophisticated and myopic—do different things, if they have non-EU preferences.[17] This fact enables us to discriminate between the types and hence identify the type of each individual. This is the main purpose of the experiment. We note that much of economic theory usually assumes sophisticated behaviour. This is not an innocuous assumption, as it requires quite elaborate planning. We test whether subjects do this, and if not, what they actually do.

3 The experimental implementation

Subjects were given written instructions (in the Non-Mathematical Appendix) and then they were presented with 27 problems, all with the same structure, and all with the same amount of money (€40 in the experiment though £40 in the screen shots) to be allocated, but with different probabilities in the various problems. An example of the opening screen-shot of a problem is shown in Fig. 1. We used the words 'Left' and 'Right', rather than Options 1 and 2, because of the physical layout of the problem on the screen. In the problem

[17] We refer the reader to Tables 1 and 2 for examples of how different types of behaviour lead to different choices.

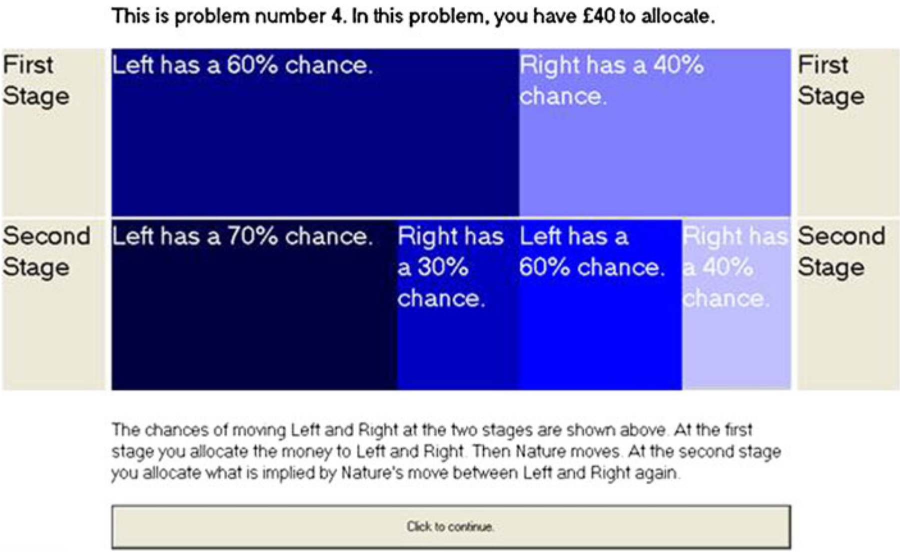


Fig. 1 The opening screen of a problem

pictured in Fig. 1, the probability of Nature moving Left at the first stage is 60% and that of moving Right 40%. In this particular problem, if Nature chooses Left after the first decision, then the probability of Nature moving Left (Right)

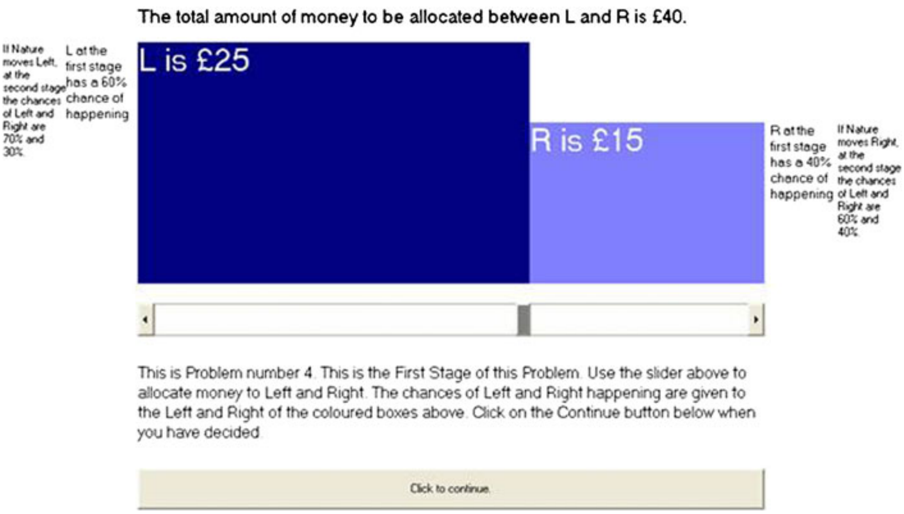


Fig. 2 The first-stage decision

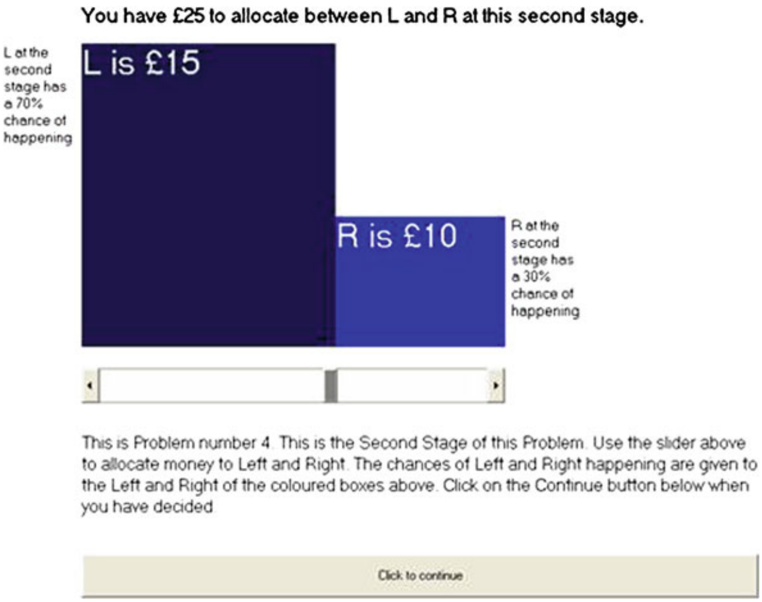


Fig. 3 The second-stage decision

after the second is 70% (30%); whereas if Nature chooses Right after the first decision, then the probability of Nature moving Left (Right) after the second is 60% (40%). We note that this, and the other screen-shots, is in English, though the experiment itself was conducted in Italian, at CESARE, the Centro di Economia Sperimentale A Roma Est, at LUISS in Rome. This first screen gives information about probabilities and the sum to allocate.

Then the subject is asked to allocate the £40 (€40 in the actual experiment) between Left and Right at this first stage. Figure 2 illustrates. The initial allocation (that is, the allocation when the screen is first displayed) is decided at random by the computer. As will be seen, there is a slider on the screen and the subject can use this to show his or her preferred allocation. The subject

Table 3 The problem set

pn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
p	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8	.6	.7	.8
p_1	.6	.6	.6	.7	.7	.7	.8	.8	.8	.6	.6	.6	.7	.7	.7	.8	.8	.8	.6	.6	.6	.7	.7	.7	.8	.8	.8
p_2	.6	.6	.6	.6	.6	.6	.6	.6	.6	.7	.7	.7	.7	.7	.7	.7	.7	.7	.8	.8	.8	.8	.8	.8	.8	.8	.8

Probability of Left at first stage: p
Probability of Left at second stage if Nature moves Left at first stage: p_1
Probability of Left at second stage if Nature moves Right at first stage: p_2

Table 4 Fitted and actual decisions for subject 65

pn	Myopic			Naïve			Resolute			Sophisticated			Actual		
	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}	m_1	x_{1A}	x_{2A}
1	26.27	17.25	9.02	26.49	17.29	8.81	27.01	20.51	6.50	26.16	17.01	9.00	27	*	7
2	31.80	20.88	5.38	31.74	20.71	5.39	30.97	24.18	4.52	31.55	20.52	5.49	32	27	*
3	36.07	23.69	2.58	35.98	23.48	2.62	35.37	26.70	2.32	35.82	23.30	2.72	36	*	2
4	26.27	20.88	9.02	27.25	21.54	8.32	29.52	24.27	5.24	27.00	21.24	8.45	25	21	*
5	31.80	25.28	5.38	32.17	25.42	5.11	32.08	28.12	3.96	31.95	25.13	5.23	36	31	*
6	36.07	28.68	2.58	36.21	28.62	2.47	35.77	31.09	2.11	36.04	28.34	2.58	35	32	*
7	26.27	23.69	9.02	28.72	25.79	7.36	31.40	27.59	4.78	28.29	25.30	7.62	27	24	*
8	31.80	28.68	5.38	33.06	29.69	4.53	34.23	31.35	2.88	32.76	29.30	4.71	34	32	*
9	36.07	32.53	2.58	36.54	32.81	2.26	36.27	34.29	1.87	36.35	32.51	2.37	38	35	*
10	26.27	17.25	10.92	26.42	17.24	10.73	27.61	20.46	7.14	21.89	14.24	14.24	28	23	*
11	31.80	20.88	6.52	31.38	20.48	6.81	30.82	24.07	5.92	31.21	20.30	6.91	32	*	5
12	36.07	23.69	3.12	35.78	23.35	3.33	35.27	26.62	3.08	35.63	23.17	3.44	36	31	*
13	26.27	20.88	10.92	26.95	21.30	10.31	28.70	24.18	6.79	26.69	20.99	10.46	28	23	*
14	31.80	25.28	6.52	32.11	25.37	6.24	32.53	28.06	4.47	31.62	24.87	6.59	33	28	*
15	36.07	28.68	3.12	36.03	28.47	3.14	35.68	31.01	2.82	35.86	28.20	3.26	35	30	*
16	26.27	23.69	10.92	27.96	25.10	9.52	30.36	27.31	6.59	27.39	24.50	9.91	32	30	*
17	31.80	28.68	6.52	32.86	29.50	5.65	33.90	31.31	3.52	32.55	29.11	5.86	33	*	5
18	36.07	32.53	3.12	36.37	32.66	2.87	36.45	34.23	2.22	36.18	32.36	3.00	35	31	*
19	26.27	17.25	12.38	25.82	16.85	12.73	28.04	20.00	9.07	23.16	15.06	15.06	28	*	10
20	31.80	20.88	7.39	30.72	20.05	8.33	30.87	23.52	7.35	23.16	15.06	15.06	32	*	6
21	36.07	23.69	3.54	35.41	23.10	4.12	34.82	26.29	4.28	35.27	22.94	4.23	35	25	*
22	26.27	20.88	12.38	26.32	20.79	12.29	28.42	23.74	8.86	21.28	16.74	16.74	28	24	*
23	31.80	25.28	7.39	31.95	25.25	7.23	32.99	27.64	5.35	31.65	24.89	7.46	33	*	5
24	36.07	28.68	3.54	35.67	28.19	3.89	35.27	30.65	3.91	35.52	27.93	4.01	37	33	*
25	26.27	23.69	12.38	27.02	24.26	11.66	29.01	26.70	8.67	26.74	23.92	11.86	30	28	*
26	31.80	28.68	7.39	32.41	29.10	6.82	33.83	31.01	4.67	32.10	28.71	7.06	30	*	7
27	36.07	32.53	3.54	36.25	32.56	3.36	36.61	33.96	2.66	35.87	32.08	3.70	35	32	*

m_1: amount allocated to option 1 in the first stage
x_{1A}: amount allocated to option 1 in the second stage if Nature selects at random option 1
x_{2A}: amount allocated to option 1 in the second stage if Nature selects at random option 2
$*$: unavailable data pertaining to the option *not* selected at random by Nature

then clicks on "Click to Continue" to see how Nature moves at this first stage and to proceed to the second stage.[18]

The random move by Nature was played out in a visually appealing and convincing way, which was explained in detail in the instructions provided to subjects.[19] Suppose in this problem Nature moved Left and the preferred allocation of the subject was that in Fig. 2. Then the subject would have £25 (€25 in the experiment) to allocate at the second stage. The second stage screen would then open as in Fig. 3. Once again, the initial allocation displayed on the screen of the £25 (€25 in the experiment) is decided at random by the computer. Again the subject can use the slider to indicate his or her preferred allocation, and click on the "Click to Continue" button (which appeared after 15 seconds) to confirm his or her allocation. Once again the random move by Nature and the subject's payoff for that problem were displayed on the screen. This procedure was repeated for all 27 problems, which appeared in a random order, with Left and Right at both stages randomly ordered. At the end of all 27 problems, one of the problems was chosen at random and the subject paid the outcome on that particular problem. Before we ran the experiment we carried out intensive simulations to ensure that we had a number of problems that would enable us to discriminate between the various types. Fewer problems would imply less discriminatory power. The actual set of problems is listed in Table 3. As we have already noted, the order of the problems and the left/right juxtaposition were determined at random.

4 Estimation and identification of types

Our analysis is by subject, as subjects are clearly different. For each subject, we assume that their preferences over risky prospects can be represented by a Rank Dependent Expected Utility function as in Eq. 2.[20] To characterize explicitly the allocations corresponding to the different types of behaviour, we assume an explicit form both for the function $u(x)$ and the weighting function $w(p)$. As for the former, we assume that it belongs to the Constant Relative Risk Aversion (CRRA) class and therefore can be written as follows[21]

$$u(x) = \begin{cases} \frac{x^{1-\frac{1}{r}}-1}{1-\frac{1}{r}} & \text{for } r \neq 1 \\ \ln\ x & \text{for } r = 1 \end{cases} \tag{4}$$

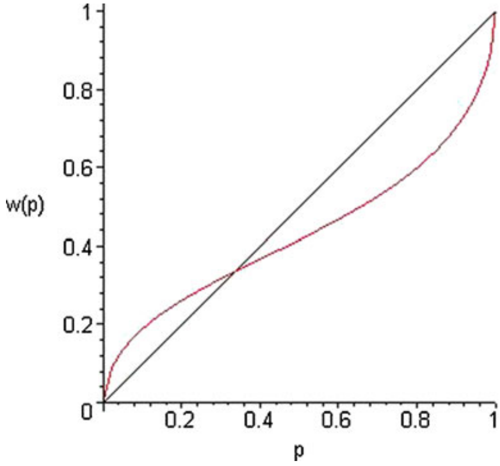
[18] We should note that we forced the subjects to wait 45 seconds before the "Click to Continue" button appeared on the screen.

[19] We refer the reader to the Non-Mathematical Appendix for a complete version of the instructions.

[20] This is the most widely-accepted non-EU preference functional in the literature. It contains EU as a special case.

[21] See Wakker (2008) for an excellent discussion of the properties of this utility function.

Fig. 4 The weighting function behind Table 1 ($g = 0.6$)



We note the special case when the parameter r takes the value 1. As for the weighting function, we assume that it takes the Quiggin (1982) form[22]

$$w(p) = \frac{p^g}{(p^g + (1-p)^g)^{\frac{1}{g}}} \tag{5}$$

If the individual has EU preferences, then $g = 1$ and $w(p) = p$. In this case, the individual will not be dynamically inconsistent.[23] Suppose instead that the individual is RDEU. In this case, the optimal allocations depend on how the individual behaves with respect to the issue of dynamic inconsistency. While we refer to the Mathematical Appendix 1 for a full characterization of the solutions to the different choice problems described in Section 2, we illustrate the differences in behaviour among the different types in Table 1, where a numerical example is provided for the case $r = 2.0$ and $g = 0.6$. With these parameters, an individual is moderately risk-averse and over-weights small probabilities and under-weights large ones (see Fig. 4 for a graph of the weighting function corresponding to this case).

In order to estimate the best-fitting values of r and g for each type of subject, and hence identify the best-fitting type for each subject, we need to make some assumption about the stochastic structure of the data. This is necessary because subjects' behaviour is noisy—there is a stochastic component to the data. Because the optimal decisions (see Mathematical Appendix 1) imply

[22]To be strictly correct, we should attribute this to Tversky and Kahneman (1992), who proposed this variation on the original specification proposed by Quiggin, namely: $w(p) = p^g/[p^g + (1-p)^g]$.

[23]To help clarify this point, in the Mathematical Appendix 1 we present an explicit example using the above functional forms.

given values for the *ratio* between the optimal allocation and the amount to allocate, it is natural to make some assumption about the empirical counterpart: the *ratio* between the actual amount allocated and the actual amount to allocate. Obviously this is a *proportion* and therefore lies between 0 and 1. The natural statistical distribution to assume is thus the *Beta* distribution. A random variable R with such a distribution satisfies $0 \leq R \leq 1$, and has two parameters—which we denote generically by α and β. The mean is given by $\frac{\alpha}{\alpha+\beta}$ and the variance by $\frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}$. An interesting property is that, if we want

$$E(R) = R^* \quad \text{and} \quad var(R) = \frac{R^*(1-R^*)}{s} \tag{6}$$

to be satisfied for some value of s, then we should put $\alpha = R^*(s-1)$ and $\beta = (1-R^*)(1-s)$. Here the parameter s is an indicator of the precision of the distribution of R. From Eq. 6 it can be seen that the variance of R tends to zero as s tends to infinity or as R^* tends to either 0 or 1.

We assume that each empirical proportion (the proportion of m allocated to 1; the proportion of $x_{1A} + x_{1B}$ allocated to 1A if Nature chooses 1; and the proportion of $x_{2A} + x_{2B}$ allocated to 2A if Nature chooses 2) has such a Beta distribution. Moreover we assume that the parameters of each of these three distributions are such that the means and variances are given by Eq. 6 where R^* is the corresponding optimal proportion. Note in passing the usefulness of the property that the variance of R tends to zero as R^* tends to either 0 or 1: this implies that if it is optimal to allocate all or nothing, then subjects do not make a mistake. Details are given in Mathematical Appendix 2.

There is one final point about the estimation that we should mention before we proceed to the results. The above discussion has assumed that the decision variables are continuous. In the experiment, subjects were forced to choose integer values at all stages. This implies that if the optimal decision is, for example, x^*, and the decision variable including error is x, then the value indicated by the subject is the nearest integer to x. This has important consequences, in that when the subject chooses, for example to allocate nothing then the decision variable (including the error) is not necessarily zero but some number less than or equal to 0.5. The estimation program takes this into account.

We proceed subject by subject as we believe that subjects are different and because we want to see how many of each type there are. We fit the above model (RDEU with CRRA utility function and Quiggin weighting function combined with a Beta distribution stochastic specification) to each subject individually, for each of the four types of individuals. We used GAUSS's maximum likelihood procedure to estimate the parameters.[24] We thus get, for each subject and for each type estimates of the parameters r, g and s. We also obtain a maximised log-likelihood for each type. This enables us to identify, for each subject, the best-fitting type.

[24]The program is at www-users.york.ac.uk/~jdh1/hey and panaccione/rnsm estimation final.est.

5 Results

Clearly the results reported in this section rely, as in all empirical analyses, on the various assumptions that we make concerning functional forms and stochastics. These comprise: the specific non-EU functional preference function—RDEU; the specific utility function—CARA; the specific weighting function—'Quiggin'; and the stochastic specification—Beta. Some of these assumptions have some *a priori* justification: the use of RDEU through many studies suggesting that this is a particularly good non-EU functional (and that it does not violate dominance which some other 'simpler' non-EU functionals—such as Prospect Theory—do); the Beta stochastics through consideration of the error process of the subjects; but the others less so. What we have rather informally done, and which we now intend to do more formally, is to see how robust our results are to different assumptions about these functionals and stochastics. Very preliminary findings suggest that other specifications lead to considerably poorer fits.[25]

We should also note that we assume that all subjects remain the same type, with the same preferences, throughout the duration of the experiment. This is in keeping with usual practice. We could, of course, explore the possibility that the type and the preference changed at some point in the experiment. We are reluctant at this stage to follow this line of enquiry, as the number of alternative hypotheses is so large.

A final preliminary is a comment to the effect that the attainment of our objective—to find the best type for each subject—may well be confounded by the closeness of the behaviour of different types of subjects (on the set of problems that the subjects tackled). Suppose, for example, that Types 1, 2 and 3 behave 'similarly' over this set, while Type 4 behaves differently from all other types. Now suppose that subject i is truly Type 1 while subject j is truly Type 4. Then our data analysis might conclude that we cannot identify the type of subject i but that subject j is of Type 4. While this is true, we cannot really use this information since the 'closeness' of any two types depends upon the parameters of the utility function and of the weighting function.

There were 71 subjects in our experiment. Full details of the estimates are available on request, but it may be helpful to give an example here. This is subject number 65 (full estimates for this subject are given in Table 4).

Type	Estimate of r	Estimate of g	Estimate of s	Log-likelihood
Myopic	1.600	1.000	26.364	−119.180
Naïve	1.681	0.940	26.960	−118.426
Resolute	1.966	0.797	72.770	−91.675
Sophisticated	1.630	0.950	21.424	−123.947

[25] Which is presumably some evidence in favour of our conclusions: we are looking for the highest log-likelihood over all types and all sets of assumptions. If Type 1, under a particular set of assumptions, has a lower log-likelihood than Type 2, under the same or a different set of assumptions, then it is unlikely that Type 1 is the true type.

The log-likelihood is largest for the Resolute type. For this type the estimate of the parameter *s*, which can be interpreted as the precision of the Beta distribution generating the stochastic component of behaviour, is large—suggesting that errors had a low magnitude. The estimate of the *g* parameter for the Resolute is 0.797—which can be shown to be significantly different from 1.[26]

We now concentrate on the overall results, which are summarised in Table 5. We begin our discussion of this table with the three left-hand columns, headed "All subjects". In the first of these three columns we simply allocate the subjects to the four types on the basis of the highest maximised log-likelihood. On this basis, we classify 39 (55%) as resolute, 16 (23%) as sophisticated, 9 (13%) as naïve and 7 (10%) as myopic. Hence it would appear that more than one-half are resolute, around a quarter sophisticated, and one-fifth naïve and one-fifth myopic. However, one might legitimately want to ask whether the fit is *significantly* better for one of the types; that is, whether the maximised log-likelihood is *significantly* higher for the best-fitting type. To this end, we carried out *Clarke* tests.[27] The results are given in the second and third columns of Table 5. It will be seen that if we require significance at 5% then 34 (48%) of the subjects do not have any type with a log-likelihood significantly better than all the others; and if we require significance at 1% then 50 (70%) cannot be classified. However, of those classified at the 5% level, 62% are resolute, 16% are sophisticated, 16% myopic and 5% are naïve; at the 1% level the corresponding figures are 76%, 14%, 10% and 0%. There is increasing evidence of resolute behaviour amongst those classified.

At this stage we should remember that people with EU preferences can not be dynamically inconsistent. In a sense we have already carried out an indirect test of whether subjects are EU or not in the above analysis: if an individual is EU then the four types should fit the data approximately equally well, and thus one type should not fit significantly better than the others. However, there is an obvious direct test: whether the estimated parameter *g*

[26]In order to constrain the parameters to be within the appropriate bounds, the GAUSS program transformed the parameters before estimation. The raw estimated parameters and their standard errors are given in the following table:

Parameter	*r*	*g*	s
Myopic raw estimates	−1.129	−0.916	−2.710
Standard errors of myopic raw estimates	0.226	0.369	0.222
Naïve raw estimates	−1.033	−1.138	−2.685
Standard errors of naïve raw estimates	0.136	0.212	0.222
Resolute raw estimates	−0.727	−1.811	−1.524
Standard errors of resolute raw estimates	0.064	0.089	0.255
Sophisticated raw estimates	−1.093	−1.099	−2.944
Standard errors of sophisticated raw estimates	0.177	0.290	0.216

[27]See Clarke (2007). An alternative test is the Vuong test, though Clarke shows that his test is more powerful.

Table 5 A summary of the main results

Type	All subjects			Subjects for whom g parameter significantly different from 1 at 5% for the best-fitting type		
	Number of subjects with highest log-likelihood	Number of subjects with significance on Clarke Test at 5%	Number of subjects with significance on Clarke Test at 1%	Overall	Number of subjects with significance on Clarke Test at 5%	Number of subjects with significance on Clarke Test at 1%
Myopic	7	6	2	2	2	0
Naïve	9	2	0	4	0	0
Resolute	39	23	16	18	13	10
Sophisticated	16	6	3	4	3	1
None of these	0	34	50	43	53	60
Total	71	71	71	71	71	71

of the weighting function is significantly different from 1.[28] Obviously this is a different test, and we cannot expect the direct and indirect tests necessarily to agree. Nevertheless, we report in the final three columns of Table 5, an analysis of the data restricted to those subjects for whom the g parameter was significantly different from 1 at the 5% level for the best-fitting type. There were 28 (39%) of such subjects. Of these, the highest maximised log-likelihood was for the resolute type for 18 (64%) of the subjects, for the sophisticated type for 4 (14%), for the naïve type for 4 (14%) and for the myopic type for 2 (7%). Finally, if we carry out Clarke tests for these 28 subjects with g parameters significantly different from 1, we find that: at 5%, ten are unclassifiable and of the 18 that are classifiable, 13 (72%) are resolute, 3 (17%) sophisticated, 2 (11%) myopic and 0 naïve; and that at 1%, 17 are unclassifiable and of the 11 that are classifiable, 10 (91%) are resolute, 1 (9%) sophisticated, and none is myopic or naïve. Once again there is increasing evidence of resolute behaviour amongst those classified.

6 Conclusions

This paper has been concerned with dynamic inconsistency. The issue is important in any economic analysis of behaviour through time. In a sense, this includes all types of economic behaviour, and includes particularly important examples such as saving, investment and pension decisions. Typically economists employ backward induction as their modelling of dynamic behaviour. Alternative methods include the strategy method, wherein the decision-maker is conceived of as considering all possible strategies and choosing the best one. Backward induction can be considered computationally simpler, as the dimensionality of the strategy method can be formidable, and perhaps beyond most decision-makers' capability. Nevertheless the backward induction

[28] Recall that if $g = 1$ then the RDEU model reduces to EU.

method does implicitly assume that the decision-maker can project him or herself forward to the final decision node and then backwardly induct from there. Again this is computationally intense.

If the decision-maker is dynamically consistent, then these two methods lead to the same solution. However, for dynamically inconsistent people, the two methods may lead to different solutions. The root cause of this result is that dynamically inconsistent people have different preferences at different points of time, and hence what appears to be optimal depends upon the point from which one is viewing the problem. There seems to be no right or wrong way to decide which is the best way to solve the problem—simply because the preferences of a dynamically inconsistent person change through time. It is exactly as if the individual is schizophrenic. Who is to say which are the true preferences of the individual?

In the context of discounting, dynamic consistency is equivalent to exponential discounting, while (potential) dynamic inconsistency is equivalent to non-exponential (for example, hyperbolic) discounting. In the context of decision-making under risk, dynamic consistency is equivalent to having Expected Utility preferences, while (potential) dynamic inconsistency is equivalent to having non-EU preferences, for example Rank Dependent Expected Utility (RDEU) preferences.

For potentially dynamically inconsistent individuals, since normative analysis seems impossible, all we can do is carry out a descriptive analysis and see what such people actually do. This is the objective of this paper. We classify subjects in our experiment into different types. These different types have different ways of reacting to their dynamic inconsistency. Following the literature, we considered three types: *naïve* (who simply ignore their inconsistency); *resolute* (who somehow impose their first period preferences on their future selves); and *sophisticated* (who plan in the present taking into account what they know they will do in the future).[29] We also add a fourth type, *myopic*, who act as if each period is the last.

As explained in the previous section, the majority of our dynamically inconsistent subjects are resolute, a significant minority are sophisticated; and rather few are naïve or myopic when taken separately, while they are at least as common as the sophisticated when interpreted together as short-sighted subjects.[30] The fact that we have few naïve or myopic is good news for economic theory and policy. We are, however, rather surprised by the preponderance of resolute types. It could be argued that our experimental software is such that it encourages resolute behaviour, but we see no reason why that is so. We did not ask subjects to state, at the first stage, what amounts

[29] We note that sophisticated types backwardly induct.

[30] In Hey and Lotito (2009) it was found that "...the majority of subjects are either naive or resolute...but very few are sophisticated." There were somewhat more naive subjects than resolute, which is at odds with the results of the experiment reported in this paper, but both papers agree on the relative paucity of sophisticated subjects. This suggests that economic theorists should be rather more cautious before assuming that economic agents can backwardly induct.

they wished to allocate to each of the four possible outcomes (1A, 1B, 2A and 2B), and indeed it was rather the opposite. Perhaps the statement of the probabilities in the form of Fig. 1 encouraged them to think about these final outcomes, but note that the software did not tell them the probabilities of 1A, 1B, 2A and 2B. On the contrary, the problem was very much stated in a sequential way. Indeed, it might be argued that the software actually discouraged resolute play.

We would like to extend the experiment in two ways. The first is straightforward: to run the experiment with a random sample from some population, since it might be argued that student subjects are not representative of the population as a whole.[31] Second, we would like to run the same experiment with more than two stages, and perhaps with endowments of money every period. In this way, we would get closer to a savings problem. The problem then is in calculating the optimal strategies for each type of subject. Even with just two periods it is computationally difficult. However one needs to calculate the optimal strategies in order to distinguish between the types. With just two stages, we think that we have been successful—and have a conclusion that is rather surprising. If the majority of people are resolute, then state intervention may be less necessary. That is, of course, if the first period preferences are the true ones. But who knows?

Non-mathematical appendix: the experimental instructions

(We omit the screen shots as they are the same as in the paper. References to figures below are to the figures in the main text of this article.)

Preamble

Welcome to this experiment. It is an experiment on the economics of dynamic decision making under risk. The Ministry for Education, University and Research of Italy (MIUR) has provided the funds to finance this research. Thank you for taking part. Please read these instructions carefully. It is important that you do so, as your payment for taking part in this experiment will depend upon the decisions that you take. The payment will be made, in cash, at the end of the experiment. The payment will consist of whatever money you earn as a result of the decisions you make during the experiment. You will be asked to sign a receipt for the payment, and to acknowledge that you participated voluntarily in the experiment. The results of the experiment will be used for the purpose of academic research and will be published in such a way that your anonymity will be preserved.

[31] Though it might be argued that non-students are likely to be even more resolute.

The experiment

In the experiment you will be presented with 27 dynamic decision problems, all of the same form. Each problem has two stages. At the beginning of each of these problems you will be given an allocation of £40. At the first stage you will be asked to allocate the money between two options, which, because of the way that they are presented on the computer screen, will be called Left and Right. When you have made the allocation, a random device, which we call Nature, will determine whether you move Left or Right. The chances of each will be told to you before you make your allocation. This is the first stage of the problem. At the second stage, you will have a similar decision: to allocate the money that is implied by your first stage decision again between Left and Right. The chances of each will be told to you before you make your first allocation. Once again, when you have made the allocation, the random device which we call Nature will determine whether you move Left or Right. The amount of money that you allocated to the realised outcome will be your payoff for that particular decision problem. At the end of all the decision problems, one will be chosen at random, and your payoff for that particular problem will be your payment for the experiment.

An example

Look at Fig. 1. This a screen shot of a particular decision problem. Like in every problem the amount of money with which you are initially allocated is £40. You will see that in this problem there is a 60% chance that Nature will choose Left and a 40% chance that she will choose Right after your first decision. What happens at the second decision node depends upon the move that Nature made at the first decision node. In this example, if Nature moves Left after your first decision, then there is a 70% chance that Nature will move Left and a 30% chance that she will move Right after your second decision. If instead Nature moves Right after your first decision, then there is a 60% chance that Nature will move Left and a 40% chance that she will move Right after your second decision. Note that these probabilities change from problem to problem.

Figure 1 here.

As in every problem you initially have £40 to allocate. When you click on "Click to Continue" the computer shows a random allocation. In the picture the allocation of the £40 is £25 to Left and £15 to Right. You must decide your preferred allocation by moving the slider under the boxes.

Figure 2 here.

When you have decided on and shown your preferred allocation, you should click on 'Click to Continue'. At this point the random move by Nature will be played out: the computer generates a series of random numbers, the last of which determines the move by Nature, and the stated chances are respected. Suppose that the outcome of this random process is that Nature chooses Left. Then, because £25 in this example has been allocated to Left, this is the amount of money which you are asked to allocate at the second stage. Figure 3 illustrates. It is restated here that there is a 70% chance that Nature will move Left and a 30% chance that she will move Right after this second decision (these are obviously the same chances that you were told when you started this problem). Once again, when the screen opens the allocation is random and you must decide and show your preferred allocation by moving the slider under the boxes. In the figure the allocation of the £25 is £15 to Left and £10 to Right.

Figure 3 here.

When you have shown your preferred allocation, you should click on 'Click to continue'. Once again, the random move by Nature will be played out: the computer generates a series of random numbers, the last of which determines the move by Nature, and the stated chances are respected. Suppose that the outcome of this random process is that Nature chooses Left. Then your payoff for that decision problem would be £15. Your payment for the experiment will be a randomly chosen one of the payoffs on all the decision problems.

Nature

Here we give some more detail about Nature and the random process that the computer uses. Nature is our word for a random process. Nature operates completely independently of your decisions. When, for example, there is a 60% chance of Nature moving Left and a 40% chance of Nature moving Right, then what Nature does depends only on these chances and not on your decision.

Nature is implemented by the computer in the following way: the computer has a routine for generating a sequence of random numbers that are equally likely to be anywhere between 0 and 1. The program generates a sequence of 10 of these and the last of these determines Nature's move: in this 60%/40% example, if the last number is less than 0.6 then Nature's move is Left and if the last number is greater than 0.6, then Nature's move is Right. In general if there is a p% chance of Left and a $(100 - p)$% chance of Right, then Nature moves Left if the random number is less than $p/100$ and moves Right if the random number is greater than $(100 - p)/100$.

Implementation

If anything is unclear after reading the Instructions, you should ask for clarification from one of the experimenters. Then you should turn to the computer. When you click on 'Click to Start' a PowerPoint presentation, which goes at a pre-determined speed, will be shown. After that, or indeed at any stage of the experiment, you can ask clarification from the experimenters. When you are ready you can start the experiment. You will see that the software forces you to wait a certain amount of time before you can confirm any decision. This is to ensure that you always state your preferred allocation. But of course you should do anyhow, as your payment depends upon your decisions. At the end of the experiment, you should call over one of the experimenters, and in front of him or her, you will randomly determine the decision problem which will determine your payoff. The experimenter will pay you in cash after you have completed a brief questionnaire and signed a receipt for the payment. You will then be free to go. We estimate that the whole experiment will last some 90 minutes.

Thank you for your participation.

Mathematical appendix 1: the optimal strategies for the various types

The reader should note that the notation in the main text differs slightly from that used in this mathematical appendix. The symbols q_{1A}, q_{1B}, q_{2A} and q_{2B} in the main text correspond to q_1, q_2, q_3 and q_4 in this mathematical appendix, and the symbols x_{1A}, x_{1B}, x_{2A} and x_{2B} in the main text correspond to x_1, x_2, x_3 and x_4 in this mathematical appendix.

Preamble

Let us consider the following utility function

$$u(x) = \begin{cases} \frac{x^{1-\frac{1}{r}}-1}{1-\frac{1}{r}} & \text{for } r \neq 1 \\ \ln(x) & \text{for } r = 1 \end{cases},$$

which implies that marginal utility is given by

$$u'(x) = x^{-1/r}.$$

Strict concavity requires $0 < r < \infty$. The RDEU function is

$$U = v_1 u(x_1) + v_2 u(x_2) + v_3 u(x_3) + v_4 u(x_4),$$

where x_1, x_2, x_3 and x_4 are respectively the outcomes for options 1A, 1B, 2A, and 2B. The weights attached to the different outcomes depend on actual probabilities adjusted using the weighting function w with parameter g.

As it is known, these weights depend on the ranking of the outcomes. Therefore, it is necessary to consider all possible 24 different rankings of the final outcomes x_k for $k = 1, ..., 4$. Each of these will identify a different objective function and a different admissible range for the solution to the maximization problem.

To make this point clear, we consider a simple example. Assume that $x_1 > x_2 > x_3 > x_4$. In this case, the weights are as follows

$$\begin{aligned} v_1 &= w(q_1) \\ v_2 &= w(q_1 + q_2) - w(q_1) \\ v_3 &= w(q_1 + q_2 + q_3) - w(q_1 + q_2) \\ v_4 &= 1 - w(q_1 + q_2 + q_3), \end{aligned}$$

where q_i is the compound probability of outcome i. If the ranking contains an equality, e.g. $x_1 = x_2 > x_3 > x_4$, then—letting $x_1 = x_2 = x_{21}$—the weights are as follows

$$\begin{aligned} v_{21} &= w(q_1 + q_2) \\ v_3 &= w(q_1 + q_2 + q_3) - w(q_1 + q_2) \\ v_4 &= 1 - w(q_1 + q_2 + q_3). \end{aligned}$$

Therefore for each possible ranking of the final outcomes, there is a different set of weights, hence a different RDEU function.

As mentioned in Section 5, we propose an explicit example to verify that when $w(p) = p$, that is when the individual is EU, there is no issue of dynamic inconsistency. Therefore, we first consider the choice problem as viewed from the beginning of the first stage. In this case, the individual optimally allocates the available amount of money m by choosing x_k^* for $k = 1, ..., 4$ to maximise his or her preferences. In this case, he or she will solve the following problem

$$\max_{x_1, x_2, x_3, x_4,} \quad q_1 u(x_1) + q_2 u(x_2) + q_3 u(x_3) + q_4 u(x_4) \quad s.t. \quad x_1 + x_2 + x_3 + x_4 = m,$$

whose solution is

$$x_k^* = \frac{m q_k^r}{\sum_{j=1}^4 q_j^r} \quad \text{for} \quad k = 1, 2, 3, 4 \tag{7}$$

Suppose for concreteness that the individual arrives at the second stage after Nature has chosen option 1 (a similar reasoning holds if Nature has chosen option 2). If the individual reconsiders his or her choices at this stage, he or she will solve the following problem

$$\max_{x_1, x_2} \; p_1 u(x_1) + (1 - p_1) u(x_2) \quad s.t. \quad x_1 + x_2 = x_1^* + x_2^*,$$

whose solution is

$$\widehat{x}_1 = \frac{(x_1^* + x_2^*) p_1^r}{p_1^r + (1 - p_1)^r} = \frac{m q_1^r}{\sum_{j=1}^4 q_j^r} \quad \text{and} \quad \widehat{x}_2 = \frac{(x_1^* + x_2^*)(1 - p_1)^r}{p_1^r + (1 - p_1)^r} = \frac{m q_2^r}{\sum_{j=1}^4 q_j^r}$$

Therefore $\widehat{x}_k = x_k^*$ for $k = 1, 2$ (and similarly for $k = 3, 4$) and dynamic consistency is guaranteed.[32]

Resolute

In this section we consider the choice problem for a resolute decision maker. Recall that there are $i = 1, ..., 24$ possible rankings of the final outcomes that

[32] Of course, this is the case for all EU decision-makers, not just those with a CRRA utility function.

must be taken into account. The maximization problem in this case should be written as follows

$$\max_{x_k} \quad U \quad \text{s.t.} \quad \sum_{k=1}^{4} x_k = \overline{m} \quad \text{and} \quad \{x_k\}_{k=1}^{4} \text{ respect ranking } i.$$

Since each ranking defines an admissible range for the choice variables that can be described by a system of inequalities, we should consider a different constrained maximization problem for each ranking as follows. Consider for example the following ranking

$$x_1 \geq x_2 \geq x_3 \geq x_4. \tag{8}$$

In this case, the maximization problem can be written as[33]

$$\max_{x_k} \; v_1 u(x_1) + v_2 u(x_2) + v_3 u(x_3) + v_4 u(x_4) \tag{9a}$$

$$\text{s.t.} \qquad x_2 - x_1 \leqslant 0 \tag{9b}$$

$$x_3 - x_2 \leqslant 0 \tag{9c}$$

$$x_4 - x_3 \leqslant 0 \tag{9d}$$

$$x_1 + x_2 + x_3 + x_4 = \overline{m} \tag{9e}$$

Let γ be the multiplier for constraint (9e) and $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$ be the multipliers respectively for constraints (9b), (9c) and (9d). The first order conditions for this problem are given by

$$v_1 u'(x_1) + \lambda_1 - \gamma = 0 \tag{10a}$$

$$v_2 u'(x_2) - \lambda_1 + \lambda_2 - \gamma = 0 \tag{10b}$$

$$v_3 u'(x_3) - \lambda_2 + \lambda_3 - \gamma = 0 \tag{10c}$$

$$v_4 u'(x_4) - \lambda_3 - \gamma = 0 \tag{10d}$$

$$\lambda_1 (x_2 - x_1) = 0 \tag{10e}$$

$$\lambda_2 (x_3 - x_2) = 0 \tag{10f}$$

$$\lambda_3 (x_4 - x_3) = 0 \tag{10g}$$

$$x_1 + x_2 + x_3 + x_4 - \overline{m} = 0 \tag{10h}$$

Depending on which inequality constraint is binding at the solution, we have different admissible cases which vary with the parameters. While this procedure allows us to completely characterize the solution set for each possible ranking and therefore to identify the global optimum by finding the highest value for the indirect expected utility, it cannot be easily coded in a program to run the necessary simulations.

[33] We neglect the non-negativity constraints, as they are not binding for the utility function we consider.

Therefore, we choose to tackle the problem in a different way, to be explained in what follows. For concreteness, assume the ranking is given by Eq. 8, as all the other rankings can be treated in the same way. The first step is to realize that for each possible ranking of the final outcomes there are four possible configurations that can arise: no equality among outcomes, equality among two outcomes (with two sub-cases to be described in what follows), equality among three outcomes and equality among all outcomes.

Let us refer to these cases as case 1, 2, 3, 4 and 5 respectively. In case 1, none of the inequality constraints (9b)–(9d) is binding and therefore $\lambda_j = 0$ for $j = 1, 2, 3$. In case 2, only one inequality constraint is binding and therefore $\lambda_j > 0$ for some j and $\lambda_i = 0$ for $i \neq j$. In case 3 and 4, two inequality constraints are binding and therefore $\lambda_i = 0$ for some $i = 1, 2, 3$ and $\lambda_j > 0$ for $j \neq i$, the difference between the two sub-cases being whether the binding constraints alternate or not. Finally in case 5, all constraints are binding and therefore the solution is $x_k = \overline{m}/4$ for $k = 1, ..., 4$.

In each of the remaining cases, the solution can be explicitly computed using Eqs. 10a–10h. If case 1 holds, the solution is

$$x_k = \left(\frac{v_k^r}{\sum_{l=1}^4 v_k^r} \right) \overline{m}. \tag{11}$$

If case 2 holds (suppose again for concreteness that $x_1 = x_2$, the other cases being symmetric), the solution is

$$x_1 = x_2 = \left(\frac{\left(\frac{v_1+v_2}{2}\right)^r}{2\left(\frac{v_1+v_2}{2}\right)^r + v_3^r + v_4^r} \right) \overline{m}, \tag{12a}$$

$$x_k = \left(\frac{v_k^r}{2\left(\frac{v_1+v_2}{2}\right)^r + v_3^r + v_4^r} \right) \overline{m} \quad \text{for} \quad k = 3, 4. \tag{12b}$$

If case 3 holds (suppose again for concreteness that $x_1 = x_2 = x_3$, the other case being symmetric), the solution is

$$x_1 = x_2 = x_3 = \left(\frac{\left(\frac{v_1+v_2+v_3}{3}\right)^r}{3\left(\frac{v_1+v_2+v_3}{3}\right)^r + v_4^r} \right) \overline{m}, \tag{13a}$$

$$x_4 = \left(\frac{v_4^r}{3\left(\frac{v_1+v_2+v_3}{3}\right)^r + v_4^r} \right) \overline{m}. \tag{13b}$$

Finally if case 4 holds, the solution is

$$x_1 = x_2 = \left(\frac{\left(\frac{v_1+v_2}{2}\right)^r}{2\left(\frac{v_1+v_2}{2}\right)^r + 2\left(\frac{v_3+v_4}{2}\right)^r} \right) \overline{m}, \tag{14a}$$

$$x_3 = x_4 = \left(\frac{\left(\frac{v_3+v_4}{2}\right)^r}{2\left(\frac{v_1+v_2}{2}\right)^r + 2\left(\frac{v_3+v_4}{2}\right)^r} \right) \overline{m}. \tag{14b}$$

The second step in the procedure requires keeping track of all possible configurations of cases 2, 3 and 4 that can arise. Depending on the various rankings considered, case 2 can arise in six possible configurations

$$\begin{array}{ccc} x_1 = x_2 & x_1 = x_3 & x_1 = x_4 \\ x_2 = x_3 & x_2 = x_4 & x_3 = x_4 \end{array}$$

On the other hand, case 3 can arise in four possible configurations

$$\begin{array}{cc} x_1 = x_2 = x_3 & x_1 = x_2 = x_4 \\ x_1 = x_3 = x_4 & x_2 = x_3 = x_4 \end{array}$$

Finally, case 4 can arise in three possible configurations

$$\begin{array}{ccc} x_1 = x_2 & \text{and} & x_3 = x_4 \\ x_1 = x_3 & \text{and} & x_2 = x_4 \\ x_1 = x_4 & \text{and} & x_2 = x_3 \end{array}$$

Therefore, for each ranking i we can compute the solution set $\{x_k\}_{k=1}^4$ for each possible case and verify if it indeed satisfied the given ranking. If it does, it is considered an admissible solution set, otherwise it is discarded. Finally, the third step of the procedure consists in computing the utility level corresponding to each admissible solution set and then picking as the optimal choice the one that gives the highest value.

Sophisticated

In this section we consider the choice problem for a sophisticated decision maker. The individual who acts in a sophisticated way solves the problem in two steps. First, given an allocation m_1 and m_2 such that $m_1 + m_2 = \overline{m}$, he or she solves the maximization problem at the final decision nodes, that is those at the second stage.[34] Denote the solution by

$$(x_1^*, x_2^*) = (x_1(m_1), x_2(m_1)) \quad \text{and} \quad (x_3^*, x_4^*) = (x_3(m_2), x_4(m_2)),$$

as it depends on m_1 and m_2. In the second stage, he or she solves for the optimal value of m_1 and m_2, taking into account the optimal choices in the final node.

Let w_j denote the weight for outcome j. Given (m_1, m_2), the sophisticated individual solves

$$\max_{x_1, x_2} \; w_1 u(x_1) + w_2 u(x_2) \quad \text{s.t.} \quad x_2 - x_1 \leqslant 0 \quad \text{and} \quad x_1 + x_2 = m_1 \tag{15}$$

$$\max_{x_3, x_4} \; w_3 u(x_3) + w_4 u(x_4) \quad \text{s.t.} \quad x_4 - x_3 \leqslant 0 \quad \text{and} \quad x_3 + x_4 = m_2 \tag{16}$$

[34] In this appendix, we replace the notation $\overline{m}_1$ and $\overline{m}_2$ with m_1 and m_2 as no confusion should arise.

Let (γ_1, μ_1) and (γ_2, μ_2) be the multipliers for the first and second constraint respectively in problems (15) and (16), and consider the first order conditions for problem (15), the others being analogous:

$$
\begin{aligned}
w_1 u'(x_1) + \gamma_1 - \mu_1 &= 0 \\
w_2 u'(x_2) - \gamma_1 - \mu_1 &= 0 \\
\gamma_1 (x_2 - x_1) &= 0 \\
x_1 + x_2 - m_1 &= 0
\end{aligned}
$$

To compute the solution of the first stage problem, which in turn will be used in the second stage problem, we have to consider all possible combinations of equality and inequalities. Using the same classification as in the previous section, we consider the single configuration for both cases 1 and 5, six configurations for case 2, four configurations for case 3 and finally three configurations for case 4.

Consider first case 1, so that $x_1 > x_2$ and $x_3 > x_4$. As in this case $\gamma_1 = 0$, the solution to problem (15) is

$$x_i^* = x_i(m_1) = \frac{w_i^r m_1}{w_1^r + w_2^r} \quad \text{for} \quad i = 1, 2. \tag{17}$$

By symmetry, the solution to problem (16) is

$$x_i^* = x_i(m_2) = \frac{w_i^r m_2}{w_3^r + w_4^r} \quad \text{for} \quad i = 3, 4. \tag{18}$$

In the second stage, given x_i^* for $i = 1, 2, 3, 4$, the sophisticated individual solves

$$\max_{m_1, m_2} \quad v_1 u(x_1^*) + v_2 u(x_2^*) + v_3 u(x_3^*) + v_4 u(x_4^*) \tag{19}$$

$$\text{s.t.} \quad m_1 + m_2 = \overline{m}, \tag{20}$$

where the weights depend on the actual ranking. The first order conditions for this problem are

$$v_1 u'(x_1^*) \frac{\mathrm{d}x_1}{\mathrm{d}m_1} + v_2 u'(x_2^*) \frac{\mathrm{d}x_2}{\mathrm{d}m_1} - \gamma = 0 \tag{21}$$

$$v_3 u'(x_3^*) \frac{\mathrm{d}x_3}{\mathrm{d}m_2} + v_4 u'(x_4^*) \frac{\mathrm{d}x_4}{\mathrm{d}m_2} - \gamma = 0 \tag{22}$$

$$m_1 + m_2 - \overline{m} = 0, \tag{23}$$

where γ is the multiplier for Eq. 20. From the previous stage, we know that for $i = 1, 2$

$$u'(x_i^*) = \left(\frac{w_i^r m_1}{w_1^r + w_2^r} \right)^{-1/r}$$

and

$$\frac{\mathrm{d}x_i}{\mathrm{d}m_1} = \frac{w_i^r}{w_1^r + w_2^r},$$

and similarly for $i = 3, 4$. Therefore, from Eq. 21 we get

$$v_1 \left(\frac{w_1^r m_1}{w_1^r + w_2^r} \right)^{-1/r} \left(\frac{w_1^r}{w_1^r + w_2^r} \right) + v_2 \left(\frac{w_2^r m_1}{w_1^r + w_2^r} \right)^{-1/r} \left(\frac{w_2^r}{w_1^r + w_2^r} \right) = \gamma,$$

hence

$$m_1 = \left[v_1 \left(\frac{w_1^r}{w_1^r + w_2^r} \right)^{1-1/r} + v_2 \left(\frac{w_2^r}{w_1^r + w_2^r} \right)^{1-1/r} \right]^r \gamma^{-r} = A\gamma^{-r}$$

and

$$m_2 = \left[v_3 \left(\frac{w_3^r}{w_3^r + w_3^r} \right)^{1-1/r} + v_4 \left(\frac{w_4^r}{w_3^r + w_4^r} \right)^{1-1/r} \right]^{\frac{1}{r}} \gamma^{-r} = B\gamma^{-r},$$

where

$$A = \left[v_1 \left(\frac{w_1^r}{w_1^r + w_2^r} \right)^{1-1/r} + v_2 \left(\frac{w_2^r}{w_1^r + w_2^r} \right)^{1-1/r} \right]^r \tag{24}$$

and

$$B = \left[v_3 \left(\frac{w_3^r}{w_3^r + w_3^r} \right)^{1-1/r} + v_4 \left(\frac{w_4^r}{w_3^r + w_4^r} \right)^{1-1/r} \right]^r. \tag{25}$$

Using Eq. 23, we get

$$\gamma^{-r} = \frac{\overline{m}}{A + B},$$

and therefore we conclude that

$$m_1^* = \left(\frac{A}{A + B} \right) \overline{m} \quad \text{and} \quad m_2^* = \left(\frac{B}{A + B} \right) \overline{m}.$$

By plugging these values into Eqs. 17 and 18 we obtain the desired optimal choices of x_k for $k = 1, 2, 3, 4$.

Consider now case 2, that is the case of a single equality. Three relevant sub-cases will be considered in what follows, namely $x_1 = x_2$ and $x_3 > x_4$ (case 21), $x_1 > x_2 = x_3 > x_4$ (case 22) and finally $x_1 > x_2$ and $x_3 = x_4$ (case 23). The other sub-cases can be treated in a similar fashion.

In case 21, we have $x_1^* = x_2^* = \frac{m_1}{2}$, while x_3^* and x_4^* are still given by Eq. 18. This implies that $m_2 = B\gamma^{-r}$, where B is given by Eq. 25, while using Eq. 21 we get

$$m_1 = 2 \left(\frac{v_1 + v_2}{2} \right)^r \gamma^{-r}$$

Finally, using Eq. 23 we get

$$m_1^* = \left(\frac{C}{C+B}\right)\overline{m} \quad \text{and} \quad m_2^* = \left(\frac{B}{C+B}\right)\overline{m},$$

where

$$C = 2\left(\frac{v_1+v_2}{2}\right)^r$$

By symmetry, in case 23 we get $x_3^* = x_4^* = \frac{m_2}{2}$ and

$$m_1^* = \left(\frac{A}{A+D}\right)\overline{m} \quad \text{and} \quad m_2^* = \left(\frac{D}{A+D}\right)\overline{m},$$

where A is given by Eq. 24 and

$$D = 2\left(\frac{v_3+v_4}{2}\right)^r.$$

Finally, in case 22 we have

$$x_i^* = x_i(m_1) = \frac{w_i^r m_1}{w_1^r + w_2^r} \quad \text{for} \quad i = 1, 2$$

$$x_i^* = x_i(m_2) = \frac{w_i^r m_2}{w_3^r + w_4^r} \quad \text{for} \quad i = 3, 4.$$

Given that $m_1 + m_2 = m$, we get

$$m_1^* = \frac{w_3^r(w_1^r + w_2^r)\overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)}$$

$$m_2^* = \frac{w_2^r(w_3^r + w_4^r)\overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)}$$

and hence that

$$x_1^* = \frac{w_1^r w_3^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)},$$

$$x_2^* = \frac{w_2^r w_3^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)},$$

$$x_3^* = \frac{w_2^r w_3^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)},$$

$$x_4^* = \frac{w_2^r w_4^r \overline{m}}{w_3^r(w_1^r + w_2^r) + w_2^r(w_3^r + w_4^r)}.$$

Consider now case 3, that is the case of a two equalities. Two relevant sub-cases will be considered in what follows, namely $x_1 = x_2 = x_3 > x_4$ (case 31) and $x_1 > x_2 = x_3 = x_4$ (case 32), as the other sub-cases can be treated in a similar fashion.

In case 31, it is easy to see that

$$x_1^* = x_2^* = x_3^* = \frac{m_1^*}{2} = \frac{w_3^r \overline{m}}{3w_3^r + w_4^r} \quad \text{and} \quad x_4^* = \frac{w_4^r \overline{m}}{3w_3^r + w_4^r}.$$

By symmetry, in case 32 we have that

$$x_1^* = \frac{w_1^r \overline{m}}{w_1^r + 3w_2^r} \quad \text{and} \quad x_2^* = x_3^* = x_4^* = \frac{m_2^*}{2} = \frac{w_2^r \overline{m}}{w_1^r + 3w_2^r}$$

Consider finally case 4, and in particular the sub-case: $x_1 = x_2 > x_3 = x_4$. In this case, we have

$$x_1^* = x_2^* = \frac{m_1}{2} \quad \text{and} \quad x_3^* = x_4^* = \frac{m_2}{2},$$

and, using Eqs. 21–23, we easily get

$$m_1^* = \left(\frac{C}{C+D}\right)\overline{m} \quad \text{and} \quad m_2^* = \left(\frac{D}{C+D}\right)\overline{m},$$

where C and D are defined above.

Naïve

In this section we consider the choice problem for a naïve decision maker. From the discussion in the main text, it should be clear that a naïve individual will behave like a resolute decision maker in the first stage of the choice problem. However, at the second stage, he or she will solve the same first-step problem that a sophisticated individual solves. Therefore, while in the first stage the optimal choices are the same as those derived for the case of a resolute individual, in the second stage the optimal choices are the solution to problems (15) and (16), where m_1 and m_2 are those implied by the first stage decision.

It follows that for this case no new computations are needed, as all the relevant optimal choices have been derived above.

Myopic

In this section we consider the choice problem for a myopic decision maker. While this type of decision maker will solve in the second stage a problem analogous to Eqs. 15 and 16, in the first stage he or she will solve the decision problem actually ignoring the second stage.

Let $\hat{x} = x_1 + x_2$ and $\tilde{x} = x_1 + x_2$. Assuming $\hat{x} \geq \tilde{x}$, the myopic decision maker will solve

$$\max_{\hat{x},\tilde{x}} v_1 u(\hat{x}) + v_2 u(\tilde{x}) \quad s.t. \quad \tilde{x} + \hat{x} = \overline{m} \quad and \quad \tilde{x} - \hat{x} \leqslant 0,$$

where the weight now only involves the probabilities of the first node, namely p and $(1 - p)$ depending on the ranking. As the above problem has the same structure as problems (15) and (16), the optimal choices can be computed following an analogous procedure.

Mathematical appendix 2: the stochastic specification

Preamble

This document discusses the stochastic specifications employed in the estimation. Throughout we assume Beta distributions. We begin with a discussion of the general properties of that distribution.

We use x^*, y^* and z^* to denote the optimal values of the decision variables (under some decision rule) and x, y and z the actual values. We also use m to denote the initial amount of money to be allocated.

Properties of beta distribution

Suppose that x has a Beta distribution with parameters α and β. The parameters must be positive. The distribution has the following properties:

$$0 \le x \le 1 \tag{26}$$

The *pdf* (probability density function) $f(.)$ is given by

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \tag{27}$$

The function $\Gamma(.)$ is what is known as the GAMMA function. It is defined by

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt \tag{28}$$

The *cdf* (cumulative distribution function) we denote by $F(.)$– is the integral of $f(.)$ from minus infinity to x.

The mean of x is given by

$$Ex = \frac{\alpha}{\alpha+\beta} \tag{29}$$

The variance of x is given by

$$var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{30}$$

We note that if we want the mean and variance to be such that they are equal to y and $y(1-y)/s$ for some y, then the following must hold

$$\alpha = y(s-1) \tag{31}$$

$$\beta = (1-y)(s-1) \tag{32}$$

Note that here the parameter s is the inverse of the variance (divided by $y(1-y)$). We note that the variance depends upon y—approaching 0 when y approaches 0 and 1. Note that s must be greater than 1. These are very nice properties.

Stochastic specification

(1) We make the following assumption about the distribution of $\frac{x}{m}$:

A1 $\frac{x}{m}$ has a Beta distribution with parameters α and β given by

$$\alpha = \frac{x^*}{m}(s-1) \tag{33}$$

$$\beta = \left(1 - \frac{x^*}{m}\right)(s-1) \tag{34}$$

These restrictions on the parameters imply that the mean of $\frac{x}{m}$ is equal to $\frac{x^*}{m}$ and that the variance of $\frac{x}{m}$ is $\frac{x^*}{m}(1-\frac{x^*}{m})/s$. So the variance goes to zero at the extremes and the parameter s characterises the *precision* of the distribution. Also x is bounded to lie between 0 and m .

(2) We now make the following assumptions about the distributions of $\frac{y}{x}$ and $\frac{z}{m-x}$:

A2.1 $\frac{y}{x}$ has a Beta distribution with parameters α and β given by

$$\alpha = \frac{y^*}{x^*}(s-1) \tag{35}$$

$$\beta = \left(1 - \frac{y^*}{x^*}\right)(s-1) \tag{36}$$

These restrictions on the parameters imply that the mean of $\frac{y}{x}$ is equal to $\frac{y^*}{x^*}$and that the variance of $\frac{y}{x}$ is $\frac{y^*}{x^*}(1-\frac{y^*}{x^*})/s$. And that $\frac{y}{x}$ is bounded between 0 and 1; and hence that y is bounded between 0 and x.

A2.2 $\frac{z}{m-x}$ has a Beta distribution with parameters α and β given by

$$\alpha = \frac{z^*}{m-x^*}(s-1) \tag{37}$$

$$\beta = \left(1 - \frac{z^*}{m-x^*}\right)(s-1) \tag{38}$$

These restrictions on the parameters imply that the mean of $\frac{z}{m-x}$ is equal to $\frac{z^*}{m-x^*}$ and that the variance of $\frac{z}{m-x}$ is $\frac{z^*}{m-x^*}(1-\frac{z^*}{m-x^*})/s$. And that $\frac{z}{m-x}$ is bounded between 0 and 1—and hence that z is bounded between 0 and $m-x$.

We note that the means of x/m, y/x and $z/(m-x)$ always are between 0 and 1.

A discretised specification

We now note that the experimental software forces the subjects to choose integer values for the decision variables. So we should estimate a discretised version. Take x for example. This is between 0 and m. The value reported in the experiment is rounded to the nearest integer. So we get:

- [0, 0.5] becomes 0
- [0.5, 1.5] becomes 1
- [1.5, 2.5] becomes 2

...

- $[m-1.5, m-0.5]$ becomes $m-1$
- $[m-0.5, m]$ becomes m.

In terms of proportions, this means that

- $[0, 0.5/m]$ becomes 0
- $[0.5/m, 1.5/m]$ becomes $1/m$
- $[1.5/m, 2.5/m]$ becomes $2/m$

...

- $[(m-1.5)/m, (m-0.5)/m]$ becomes $(m-1)/m$
- $[(m-0.5)/m, 1]$ becomes 1.

We can write this in general as

- If $x \leq 0.5/m$ then $x = 0$
- If $(i-0.5)/m < x \leq (i+0.5)/m$ then $x = i$ (for $i = 1, 2, ..., (m-1)$)
- If $(m-0.5)/m < x$ then $x = m$

So the probabilities are as follows (where cdf denotes the cumulative distribution function):

- $P(x=0) = cdf[0.5/m]$
- $P(x=i) = cdf[(i+0.5)/m] - cdf[(i-0.5)/m]$ for $i = 1, 2, ..., (m-1)$
- $P(x=m) = 1 - cdf[(m-0.5)/m]$

By analogy, we have the following for y:

- $P(y=0) = cdf[0.5/x]$
- $P(y=i) = cdf[(i+0.5)/x] - cdf[(i-0.5)/x]$ for $i = 1, 2, ..., (x-1)$
- $P(y=x) = 1 - cdf[(x-0.5)/x]$,

and for z:

- $P(z=0) = cdf[0.5/(m-x)]$
- $P(z=i) = cdf[(i+0.5)/(m-x)] - cdf[(i-0.5)/(m-x)]$ for $i = 1, 2, ..., (m-x-1)$
- $P(z=m-x) = 1 - cdf[((m-x)-0.5)/(m-x)]$

So the contributions to the likelihood are (where $cdf\,Beta(x, \alpha, \beta)$ denotes the cumulative density up to x under a Beta distribution with parameters α and β):

– for x:

$$\begin{aligned} &\ln\left(cdf\,Beta\left(\frac{0.5}{m}, \alpha, \beta\right)\right) \text{ if } x = 0 \\ &\ln\left(cdf\,Beta\left(\frac{x+0.5}{m}, \alpha, \beta\right)\right) \\ &-\ln\left(cdf\,Beta\left(\frac{x-0.5}{m}, \alpha, \beta\right)\right) \text{ if } 0 < x < m \\ &\ln\left(1 - cdf\,Beta\left(\frac{m-0.5}{m}, \alpha, \beta\right)\right) \text{ if } x = m \end{aligned} \tag{39}$$

– for y (if observed):

$$\begin{aligned} &\ln\left(cdf\,Beta\left(\frac{0.5}{x}\alpha, \beta\right)\right) \text{ if } y = 0 \\ &\ln\left(cdf\,Beta\left(\frac{y+0.5}{x}, \alpha, \beta\right)\right) \\ &-\ln\left(cdf\,Beta\left(\frac{x-0.5}{m}, \alpha, \beta\right)\right) \text{ if } 0 < y < x \\ &\ln\left(1 - cdf\,Beta\left(\frac{x-0.5}{x}, \alpha, \beta\right)\right) \text{ if } y = x \end{aligned} \tag{40}$$

Rather trivially if $x = 0$ then y has to be zero and there is no contribution to the likelihood from y.

– for z (if observed):

$$\begin{aligned} &\ln\left(cdf\,Beta\left(\frac{0.5}{m-x}, \alpha, \beta\right)\right) \text{ if } z = 0 \\ &\ln\left(cdf\,Beta\left(\frac{z+0.5}{m-x}, \alpha, \beta\right)\right) \\ &-\ln\left(cdf\,Beta\left(\frac{z-0.5}{m-x}, \alpha, \beta\right)\right) \text{ if } 0 < z < m-x \\ &\ln\left(1 - cdf\,Beta\left(\frac{z-0.5}{m-z}, \alpha, \beta\right)\right) \text{ if } z = m-x \end{aligned} \tag{41}$$

Rather trivially if $x = m$ then z has to be zero and there is no contribution to the likelihood from z.

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