Detecting a single event

The problem is this. If we have a series of cases where no event has taken place, what is the estimated event rate?

Our best estimate of the proportion of cases which have an event is zero.

We need a small sample confidence interval for the estimate, based on the Binomial distribution.

The Binomial distribution has two parameters, \( p \), the proportion of observations which are an event, and \( n \), the number of observations.

We find \( p \) which would give a probability 0.025 of having zero events. This is the upper limit of the 95% confidence interval.

Detecting a single event

For example, suppose we observe \( n = 40 \) cases with no events. What is the estimated proportion in the population who would experience the event?

95% confidence interval = 0 to 0.088.
Upper limit = 8.8%.

Suppose we observe \( n = 100 \) cases with no events.
95% confidence interval = 0 to 0.036,
upper limit = 3.6%.

Suppose we observe \( n = 1000 \) cases with no events.
95% confidence interval = 0 to 0.0037,
upper limit = 0.37%.
Detecting a single event

It is easy to tabulate this for selected sample sizes.
Follow link: http://martinbland.co.uk/ and follow menu to “Detecting a single event”.

<table>
<thead>
<tr>
<th>Number</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000</td>
</tr>
<tr>
<td>1</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>0.0002</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Detecting a single event

There is a program, biconf.exe, but it is a bit difficult to use as Microsoft have dropped MS-DOS from Windows 7.
Go to http://martinbland.co.uk/ and follow the link to Simple statistical software on the menu.
Can be used when there are events, too.

Detecting a single event

How does it work?

Binomial distribution formula, probability of \( r \) events out of \( n \):

\[
Pr(r) = \frac{n!}{r!(n-r)!}p^r(1-p)^{n-r}
\]

(but we only do that to make ourselves look clever).
Then to find the probability of being less or equal to than some number of events, we sum all the probabilities for \( r \) up to that number.
We find \( p \) so that this is 0.025.
Detecting a single event

How does it work?
We find $p$ so that this is 0.025.
It is all built into the program.

A power calculation

How big a sample do we need to have a 90% chance of finding an event?
We postulate an event probability for the population, $p$.
What is the probability (proportion of possible samples) of no events in a sample of size $n$?
Probability that a given observation has no adverse event $= 1 - p$.
Probability of no observations in $n$ observations $= (1 - p)^n$.
We set this equal to $1 - power = 1 - 0.90 = 0.1$.

A power calculation

Probability of no observations in $n$ observations $= (1 - p)^n$.
We set this equal to $1 - power = 1 - 0.90 = 0.1$.
For example, suppose adverse events happen once in 100 trials, $p = 0.01$. What sample do we need to have 90% chance of seeing an event?

$(1 - 0.01)^n = 0.1$

$n \log(0.99) = \log(0.1)$
$n = \log(0.1)/\log(0.99) = 229.1$
We need 229 observations to have a 90% chance of an adverse event.
A power calculation
We can do this for any value of $p$.

A power calculation
For more practical sample sizes:

A power calculation
For more practical sample sizes, with grid lines:

Go to http://martinbland.co.uk/ and follow menu to "Detecting a single event".