**Bootstrapping the Battle of Britain**

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**ABSTRACT**

The Battle of Britain is the focus of much historical controversy. We show here how the statistical technique of *weighted bootstrapping* can be used to create a new quantitative basis to help address such controversies. Bootstrapping facilitates the exploration of alternative campaign possibilities with different tactics. This results in comparative probabilities of “victory” for the actual and various counterfactual campaigns, providing a quantified assessment of the likelihood of German achievement of air superiority, thereby facilitating invasion. We find this more likely had the Luftwaffe targeted airfields more heavily, and greatly more likely had Germany brought forward its air campaign.

1. **Introduction**

The Battle of Britain continues to generate fierce controversy amongst historians as its eightieth anniversary approaches. The conflict between critics and advocates of “Big Wings” now has a considerable pedigree and has reached popular culture by many avenues, particularly the reputations of the personalities involved.[[1]](#footnote-1) However, debates about the British conduct of the battle can distract attention from other fundamental questions. For example, some German veterans and historians deny that any such definitive conflict as a “Battle of Britain” occurred.[[2]](#footnote-2) Whilst this is a minority view, victory and defeat are difficult to define with clarity even if we accept the existence of a discrete campaign. The German decision to postpone invasion of England might be an obvious yardstick, but some deny that invasion was ever Hitler’s intention and this of course returns us to the question of whether or not there was ever a “Battle” in the classical sense, rather than simply an active front.[[3]](#footnote-3)

Nevertheless, there is no doubt that the Luftwaffe was instructed to realize a definite goal of peace with the British Empire on German terms. Whether this was to be achieved through an invasion or political pressure caused by public susceptibility to “terror” bombing is perhaps beside the point. A German victory was intended and this would follow the definite achievement of air supremacy over at least southern England. The point at which German aerial success would change the political situation is impossible to quantify objectively, however. A decision by the British government to sue for peace would be an obvious measure – and did not occur. It can also be discounted as a realistic policy during the timeframe of the Battle, as the protracted and intense “Blitz” which followed also failed to achieve a moral collapse in the UK. Nor was an invasion attempted, although this remains the only realistic outcome to end the war quickly. Historians need to conceptualize, therefore, the Luftwaffe’s proximity to a success enabling invasion if they are to possess a fully-rounded understanding of historical developments between May and October 1940.

At this point the consideration of unrealized possibilities becomes unavoidable, and this is a difficult issue to discuss using academic historical techniques. The field has largely been left to popular writers willing to discuss the counterfactual possibility of invasion. These accounts require some sort of trigger for German action and fall broadly into two camps. One posits an invasion in September 1940 after German success in the prolonged air campaign, perhaps described in terms of a retreat by Fighter Command north of the Thames into the 12 Group Area.[[4]](#footnote-4) This would enable an invasion attempt based on air supremacy over territory previously defended by 11 Group. Such a scenario possesses scholarly credentials as it most closely matches German planning and instructions to the Wehrmacht from Hitler. The second option, less frequently visited, favours an early German invasion, without a lengthy campaign to achieve air supremacy, to ‘bounce’ the British defences and exploit their temporary disarray in the aftermath of Dunkirk.[[5]](#footnote-5) Intermediate options are not popular topics for speculative writing or academic historians, but they do fall within the scope of our analysis.[[6]](#footnote-6) How then might real data help in addressing the probability of a decisive German success and avoid historical analysis based on simple guesswork?

In recent years historians have demonstrated that quantitative analysis has a useful role in addressing circular arguments and apparently intractable issues. The purpose of this paper is to explain and advocate a quantitative analytical technique new to historical analysis, and to demonstrate its usefulness in addressing central issues in the Battle of Britain. The technique in statistics is called *bootstrapping*.[[7]](#footnote-7) The essential idea is to regard a set of historical data points as a sample from a single unknown distribution and then to re-sample, with replacement, from this distribution.

In this approach, the record of daily events and attrition in a military campaign constitute all that we can know about the distribution of possible outcomes. From this daily record we can create alternative campaigns by re-sampling – choosing days at random, resulting in a set of days’ fighting, but in a different order, and perhaps with some days appearing more than once or not at all. Such re-samples, all taken together, form a “bell curve” of possibilities for how the campaign might have turned out. The validity of this approach depends crucially on all days’ being, statistically speaking, alike. These conditions are rarely realized – for example, in the actual “Hundred Days” of the Waterloo Campaign, each day radically affected the next. However, this obvious *caveat* aside, close analysis of real data can be valuable in, for example, a geographically fixed campaign with a recurrent daily pattern of action such as the Battle of Britain.

Nonetheless the historian’s focus must remain principally on the real factors that influenced the campaign: variations between the days; the units involved; the conditions of engagement, the weather, and so on. More fundamental questions affecting the outcome include the intentions and assumptions of the protagonists, the quality of their decision making, and the dilemmas they faced. The purpose of the present exercise is not to provide an alternative reality but rather to enable a more broad-based understanding of the historical outcome in all its complexity.

To address such questions we take a range of possible actions facing the protagonists, assign these decisions to particular days, and when we resample make such days more or less likely to be chosen depending on how a protagonist varies their decisions. This process is called *weighted bootstrapping*: we create alternative, counterfactual campaigns by varying the probabilities with which we choose individual days. As a historical technique, this wrings all that can be extracted from the known data with minimal assumptions and provides a standardized methodology for doing so. In any circumstance in which there is at least reasonable evidence that the underlying distribution is constant, the weighted bootstrap provides the best information that can be obtained from (nothing but) the data, and thus the best jumping-off point for the historian.

**2. Bootstrapping a military campaign**

The subtleties of mathematical statistics need not concern us here: in our historical context both the method and the result are simple and intuitive. Consider an extended military campaign, fought between Red and Blue, of days, say, on each of which the underlying conditions – of equipment, terrain, *etc*. – are much the same. Suppose that we have data for numbers and casualties on each day, and that Red eventually “won” the campaign according to some quantitative measure. One view might be that these days provide all that we can possibly know about the campaign, and that it is unproductive to consider alternatives. It can also truthfully be said, however, that these days are merely a sample from the distribution of all possible days’ fighting, which might profitably be explored, if a methodology could be created for doing so.

Bootstrapping allows the two views to be somewhat reconciled. We acknowledge that all we know – all we *can* know – about the campaign is contained in those days’ data. But we can create alternative campaigns by re-sampling, with replacement, from the original campaign. A single alternative campaign consists of days each chosen at random from among the actual days. In this alternative, some days, perhaps heavily favouring one side or the other, may be repeated, perhaps many times. Then with the computing power available to modern researchers we can create many such alternative campaigns – , say. These are our best estimate, using the actual campaign data, of the distribution of all the ways the campaign might have turned out, based on the assumption that all days’ outcomes were equally likely. Next we can ask: according to our quantitative victory criterion, under which Red won the actual battle, what proportion of the alternative battles did Red win? If, say, emerges, then this is our best estimate of the probability of Red’s victory.

The bootstrap as described above appears to take us little further forward. It offers control over what the data are telling us about the way the campaign actually fought might have played out differently, but at this level of implementation it offers no genuine counterfactuals. Rather it principally enables us to turn the answers (‘yes’ or ‘no’, ‘Red’ or ‘Blue’) to categorical questions about the campaign into probabilities.

Importantly, the bootstrap technique is *non-parametric:* no models are posited or underlying dynamics assumed; it relies on no assumptions about the nature of warfare in the campaign.[[8]](#footnote-8) In the bootstrap, there is nothing but the data, the record of what actually happened. The only artifice is our choice of a quantified threshold for victory: to the extent to which we differ about this, we will differ in the probabilities of victory we arrive at. So the Red victory probability will vary hugely according to our preconceptions about the narrowness of the margin and thereby the quantified victory criterion. The essence of our technique will be to *invert* this relationship: we begin with a victory probability, considering it as no more than an attempt to quantify, approximately, a historian’s belief about the narrowness of victory, and use this probability to reverse-engineer the threshold for victory. This threshold can then be used in various counterfactual scenarios to extract the *altered* victory probabilities associated with these scenarios.

The crucial step to perform such counterfactual history is non-uniform weighting, in which, when we re-sample, we choose different days with different probabilities. For example, suppose Blue used certain tactics for the first days, and changed them utterly (to its advantage) for the last – and, as we know, lost the actual campaign. Now suppose we construct our alternative campaigns on the assumption that Blue instead changed its tactics much earlier, – after about days, say. We do this by selecting the days of each alternative campaign as follows: each of the first days of the actual campaign is chosen with probability not but rather , and each of the last days is chosen with probability . The sum of all the probabilities is still one, but now about days will be fought with Blue’s earlier, worse tactics, and about with Blue’s later, better tactics.

Suppose that Blue’s proportion of victories in this set of alternative campaigns, using counterfactual tactics, is , in contrast to its victories using the actual (unweighted) campaign data. The difference – the turnaround, indeed – in these figures is a robust result. Our original Blue victory probability of only might be thought a result of our preconceptions about the size of Red’s victory margin – we have effectively calibrated it to them – but changes in those preconceptions will tend to move both of Blue’s probabilities of victory, in the real campaign and in the counterfactual, in the same direction. The difference in these two figures is what is informative – it is much less a product of the historian’s views than is either of the probabilities individually. This weighted bootstrap is no longer strictly non-parametric, for it introduces some ideas of our own in the form of the re-weightings, but it is still free of any assumptions about the process of combat attrition. So, with no further assumptions, the historian who believed that Red won the actual campaign decisively, with a chance of victory, should accept that if Blue had changed tactics earlier then it would have been the likely winner.

**3. The Battle of Britain**

The Battle of Britain, so evocative in British history, continues to generate academic debate. Historians, nevertheless, agree a core narrative: the German Luftwaffe fought an air campaign over the English Channel, south-eastern England and London in the summer and autumn of 1940 intended to achieve air supremacy through the neutralization of the Royal Air Force. Once they switched in October to mainly night-bombing it was clear that German strategy had failed, at least for 1940: Fighter Command remained in being, and was growing stronger.[[9]](#footnote-9)

German air supremacy was consistently regarded, by both sides, as a necessary (albeit not necessarily sufficient) pre-condition for the invasion of England. Hitler was initially – at the time of the Dunkirk evacuation, at the end of May – uncommitted to invasion. However, following meetings in late May and June, and resulting initial studies by the German High Command, Hitler issued on 16th July an order to prepare for invasion, Operation *Sea Lion*. For the British, simultaneous with the defensive campaign, RAF Bomber Command and light forces of the Royal Navy conducted offensive campaigns against German craft in Channel ports whose obvious role would have been to form an invasion force. The Royal Navy would clearly have posed an existential threat to an invasion fleet.[[10]](#footnote-10) The idea that the air battle was a “myth” because of this recurs in an almost cyclical way, and journalists find it particularly tempting. Before Cumming’s 2010 work in this vein appeared,[[11]](#footnote-11) three eminent military historians were (in their view) ambushed by the journalist Brian James in an article for *History Today* and portrayed as agreeing with this perspective.[[12]](#footnote-12) Their actual position, delivered in a riposte, was that invasion was thwarted by a holistic British defence involving all three services. One might say that this is an orthodox position and that no academic historian actively dismisses the importance of either the Army or the Navy in 1940. Such attempts constitute ‘a silly season story par excellence’.[[13]](#footnote-13) One would struggle to dismiss the Luftwaffe’s appearance over England in great strength as insignificant, or to argue that the obliteration of Fighter Command in the summer of 1940 would offer no implications for the course of the war. It is clear that Fighter Command won its defensive campaign, its “holding action”, and that this greatly reduced the probability of invasion.

A Fighter Command defeat would minimally have consisted of a forced withdrawal from forward-most airfields, then north of the Thames into 12 Group’s area, and finally effective abandonment of the skies south of the Thames. There was no obvious step-change for Fighter Command to make: rather its gradual failure would have appeared as first a steady, later an accelerating extension of RAF 11 Group commander Keith Park’s already-parsimonious defensive approach. Given existing relative rates of attrition and replacement, the best German outcome would probably have been a brief time-window of air superiority.[[14]](#footnote-14) If this triggered invasion, a fresh air battle would have been fought, in parallel with a massive naval action.[[15]](#footnote-15)

An invasion campaign is outside our scope: by the “Battle of Britain” we mean the first, purely-aerial action. An analysis of the scaling of combat attrition and its relationship with Lanchester’s laws and ideas of concentration has been conducted in a series of works by the present authors,[[16]](#footnote-16) and in the first of these we used bootstrap techniques to verify that the Luftwaffe benefited more from large sortie numbers than did the RAF. Further, the Battle of Britain was fought over a relatively short period using largely similar aircraft. Various tests of the data show no obvious trends over time. It has been shown previously that the only obvious change-point appears in a plot of (both sides’) losses as a proportion of total sorties, which shows that the battle was more intense before 15th September 1940 than thereafter.[[17]](#footnote-17) However, there is no asymmetric (between the two sides) change in the data, and nothing to invalidate our assumption that the data can be considered independent and identically distributed.

Our task is to apply bootstrap techniques to consider variations of Luftwaffe tactics, for it is clear that the Luftwaffe had no clear idea of how to achieve air supremacy.[[18]](#footnote-18) Early in the campaign bombers concentrated on Channel shipping. Later their targets were airfields and aircraft factories. Famously, on 6th September they altered their main target to London. This has been seen by many authors – among them contemporary actors[[19]](#footnote-19), airpower theorists[[20]](#footnote-20) and historians[[21]](#footnote-21) – as a crucial error, although from the contemporary Luftwaffe perspective it reflected a dilemma. Should the bombers destroy the RAF in production and on the ground, or were they merely bait to draw RAF fighters for the escorting Luftwaffe fighters to destroy? And what if the RAF declined action? Ultimately London was the target which the RAF must defend, a deceptive echo of German First World War naval attacks on east coast English towns designed to bring out the Royal Navy.[[22]](#footnote-22)

It has been argued that the most important component of the RAF’s victory was its information system, aided by a strategic defensive over home territory.[[23]](#footnote-23) The counterpoint of this is that the Luftwaffe, on the offensive over enemy territory, had very poor information, and thus little sense of which tactic was proving effective. A part of the British narrative, most famously argued in the eponymous work by Dempster and Wood, is that the victory was by a narrow margin.[[24]](#footnote-24) We have added our voice elsewhere to the chorus of claims that the margin was not so narrow, based on high British fighter production and the large number of fighters stationed elsewhere in the UK and not directly employed in the battle.[[25]](#footnote-25) The strongest constraint was probably pilot training, for although newly-trained pilot numbers were matching losses for most of the battle, Dowding was strongly of the view that neither a novice monoplane fighter pilot, nor an experienced pilot of other types, was worth a combat-experienced fast monoplane fighter pilot.[[26]](#footnote-26)

In our exercise, weighted bootstraps allow us to model alternative campaigns in which the Luftwaffe prolongs or contracts the different phases of the battle and varies its targets. To avoid the switch to London is an obvious counterfactual. A more subtle possibility is to alter Goering’s belief – analogous to Leigh-Mallory’s, on the RAF side – that a fighter force must be destroyed in the air.[[27]](#footnote-27) It is a common opinion that had the Luftwaffe prolonged its attacks on airfields, and indeed on any targets south of London, it would have had a much greater chance of victory.[[28]](#footnote-28) Here we can quantify this, with weighted resamples in which the Luftwaffe chooses its targets in different proportions. The third and perhaps the deepest counterfactual is to make Hitler strongly in favour of invasion from the outset. In fictionalized accounts this leads to an invasion within a month of Dunkirk. More realistically, we allow for planning and preparation by all three arms, including the achievement of air superiority prior to invasion – the Luftwaffe was only able to make the necessary bases in France available during June. So our counterfactual brings forward the air campaign by three weeks.

In all of our cases, a governing factor is the necessity for neap tides, which are optimal for both military and seafaring reasons: to give slow strings of barges freedom of movement, to reduce tidal races over Channel sandbanks, and so that an ebb tide can assist the barges down-Channel and allow beaching just before dawn with some moonlight.[[29]](#footnote-29) This results in a fortnightly cycle, with invasion possible for 3 days either side of the quarter-moons on 26th August, 8th September, 24th September and 8th October. Thus an initial decision for invasion needs to be made during one of the weeks beginning 13th August, 25th August, 11th September or 25th September. To allow for necessary preparations for invasion – in reality the first feasible date was mid-September – the window beginning 13th August is only included in those counterfactuals in which the drive for invasion is brought forward by three weeks.

**4. Methods**

**4.1 Data**

In order to better understand the Battle of Britain, we gathered a variety of data, presented in Tables 1 and 2. Table 1 consists of British and German total airframe losses (Hurricanes, Spitfires, or otherwise; and fighters, bombers or otherwise; respectively), British pilot casualties, and primary target type (docks, shipping and coastal, reconnaissance merely, aerodromes, London, Kent and Thames estuary). Table 2 additionally provides the number of British sorties and (British estimates of) German sorties as well as regional weather (rain, clear, overcast).[[30]](#footnote-30)

We also divide the campaign into four phases in Table 1, following the official history.[[31]](#footnote-31) These are (P1): 10th July - 7th August, principally of coastal attacks and armed reconnaissance; (P2): 8th August - 18th August, of heavy attacks on mostly coastal targets; (P3): 24th August - 6th September, of sustained attacks gradually concentrating on airfields; and finally (P4): 7th September - 31st October, following the Luftwaffe’s switch to London as its principal target. Note the five-day lull between (P2) and (P3) and denoted 0 in Table 1; we treat this lull as a part of (P3) in our scenarios. We do not include 25th September and 16th October when reweighting based on target; these days correspond respectively to an attack on Filton and to general German air sweeps.

**4.2 Victory criterion**

We now need a “victory” criterion, by which we mean a trigger for invasion. Within RAF and War Cabinet policy there is no one obvious change which might have occurred and would have constituted the defeat of Fighter Command; the need to retain a force capable of contesting invasion, combined with the ability to withdraw Fighter Command northwards, ensure this. Further, we find no evidence of a plan for such a single collective withdrawal, which would probably rather have been gradual, and not immediately obvious to the Germans, who sometimes erroneously assumed that unrelated airfields belonged to Fighter Command.[[32]](#footnote-32)

In terms of the constraints on Fighter Command, it was pilot supply that approached criticality. British monoplane fighter airframe production, running at about 400 per month,[[33]](#footnote-33) would always have been sufficient to provide a modern aircraft for every available pilot. In contrast the supply of newly-trained pilots was running at around 260 per month, supplemented by non-UK volunteers and refugees, and pilots re-allocated from other types. As noted above, Dowding considered a novice fighter pilot worth less than a combat-experienced one, and this is consistent with, for example, the 501 Squadron figures,[[34]](#footnote-34) with of novice pilots lost within a month compared with of experienced ones. If we therefore assign a novice pilot a value of of a pilot lost, a reasonable estimate of the strength of Fighter Command, BS(t), might therefore be

Here is the initial Fighter Command pilot strength, on 6th July 1940 or on 15th June, all assumed to be trained and experienced.[[35]](#footnote-35) The new pilot contribution is is the average daily complement of new pilots, and is the total number of British pilots lost, so that is the total Fighter Command pilot strength at time *t*. Pilot losses are computed using data from a number of sources, given in Table 1: essentially they include all pilots killed, seriously wounded or missing in action, and exclude the slightly wounded, who typically returned to action within a few days.

We now need to decide how the values of might be used to trigger invasion. Let (for “T-Tag”) be the planned date of invasion, which we recall must lie within days either side of the quarter-moon . Recall further that an initial decision for invasion needs to be taken on . For invasion to be triggered, the Luftwaffe must appear to have been gaining air superiority during (let us say) the days before this, and for invasion not to be cancelled or postponed the same must apply for days after the initial decision – beyond this, too much effort would have been put into preparations such as the sowing of minefields for poor air combat reports to cause cancellation. So we choose some critical value according to the procedure outlined in the next paragraph, and say that invasion is triggered if throughout the period from to for any between and . Thus the latest date for which the simulated air combat data are germane is 6th October.

What critical threshold value ofwould constitute the defeat of Fighter Command? As noted earlier, the crux of our method is not to attempt to answer this directly, but rather to calibrate it to prior beliefs using bootstrap methods. Imagine three historians of differing views. One of them believes that the British margin of victory was nil – that the battle was won on a coin toss – and thus that the Germans had a probability of victory. A second believes that the British had a modest margin of victory, that it would have taken a moderate amount of deviation from the expected (average) result for the Germans to win, and thus that the British probability of victory was , corresponding to one so-called “standard deviation” σ from the expected (average) value in a normal distribution (a “bell curve”). A third believes that a German victory was very unlikely, and would have taken double such a deviation from the average (a “2σ event”), and thus (on a bell curve) that the British probability of victory was . We then run a simple bootstrap on the Battle of Britain as actually fought, which results in a bell curve of outcomes centred on the actual outcome, and choose the three values of which generate the three historians’ British victory probabilities specified above.

We then use these three values of in our counterfactual scenarios, resulting for each scenario in three new probabilities. These are robust to small changes in the form of the victory criterion, since this merely mediates between the figures of interest, which are each historian’s belief (expressed as a victory probability estimate) about the actual battle, and the belief which it would then be rational for them to assign, on the basis only of the evidence from the actual fighting, to each counterfactual scenario.[[36]](#footnote-36)

**4.3 Counterfactual scenarios**

In the most radical counterfactual fiction the Luftwaffe’s initial hopes of swift achievement of (at least) air superiority are realized, followed by an early invasion.[[37]](#footnote-37) We cannot and do not pursue such ideas: rather our counterfactuals are air campaigns which depart from the actual campaign in their dates or targeting but which are built up using data from it. Instead we consider five counterfactuals that can be well-posed in terms of our bootstrapping method. These are summarised in Table 3.

**CF1: What if the switch to bombing London had not occurred?**

That the Luftwaffe switch to bombing London was an error is a standard argument, as noted earlier. To capture it here we simply extend P3 to 6th October, eliminating P4 entirely.

**CF2: What if Hitler had been fundamentally in favor of invasion from the outset**?

In this case we assume that planning would be brought forward: Raeder’s visit to Hitler on 21st May would, in its effects, have taken the place of that of 20th June; air campaign planning would have been initiated much earlier than the actual 30th June.[[38]](#footnote-38) We take the net result as bringing forward the air campaign by three weeks – as much as seems reasonable given the Luftwaffe’s need to make the Channel-littoral airbases operational. Thus we bring forward P1 to 16th June-17th July, and spread P2 and P3 proportionally over 18th July-6th September, with P4 thereafter. Since the battle begins early, this also gives time for the Germans to take advantage of the 26th August neap tides.

**CF3 combines CF1 (no fourth phase) and CF2 (early onset):**

We take CF2, but with no switch to London: bring forward P1 to 16th June-17th July, and spread proportionally P2 and P3 over 18th July-6th October.

For our next counterfactual we switch from contracting or prolonging phases to alterations of targeting. Recall that in the actual battle the numbers of days for the principal target types were

**CF4: What if Goering and his staff had believed that Fighter Command could be more easily destroyed on the ground than in the air?**

Townsend[[39]](#footnote-39) notes the belief of both Goering and staff officer Paul Deichmann that Fighter Command would be more easily destroyed in the air than on the ground (paralleling the beliefs of Big Wing advocate Trafford Leigh-Mallory in the RAF). Indeed, Townsend[[40]](#footnote-40) records Deichmann’s view that the Luftwaffe should not destroy radar stations, whose work would simply bring the RAF’s fighters to the Luftwaffe’s, facilitating their destruction. Thus for this counterfactual we take an 89-day battle terminating on 6th October, with unchanged, untargeted, and exceeding , with

**CF5 combines CF2 (an early start) with CF4 (targeting of Fighter Command on the ground)**:

We take to combine a commitment for invasion with a firm belief in the destruction of Fighter Command on the ground as a prerequisite.

In addition to these well-posed counterfactuals, we will also look at the problems caused by trying to determine the impact of the weather on the Battle of Britain. We use our attempt to create counterfactual weather for the Battle as a cautionary tale.

**5. Results**

**5.1 Unweighted bootstrapping: the Battle of Britain**

Before tackling the counterfactuals, we first need to apply the unweighted bootstrap, creating many new samples (henceforth we call these re-samples “trials”) of the Battle of Britain from the actual days’ fighting. We begin with the results of a bootstrap with trials. Compared to a standard trials this will allow us to better fit a normal distribution and thereby obtain suitable critical values . We then compare the effects of bootstrapping to the battle as actually fought with a standard trials. With these baselines in mind, we can then proceed to address the genuine counterfactuals.

As discussed earlier, to begin exploring the counterfactuals we must first obtain the threshold values which match various prior beliefs as to the probability of the Luftwaffe’s obtaining air superiority. To do so, we will impose a normal distribution on the results of a large number of trials without any reweighting.[[41]](#footnote-41) The results of such a calibration run are shown in Figure 1. We observe immediately that the normal distribution provides a good fit. Thus we can take the expected value (the average or mean) of the normal distribution as the threshold (median). The standard deviation and second standard deviation to the left of the average then agree well with our desired and probabilities of British victory. The corresponding thresholds are , , and pilots respectively. To summarize: the historian who believes that the German invasion decision was evenly-balanced would use a threshold pilot strength of in our victory criterion, while the historians who believe in moderate and large British margins of victory would use and respectively.[[42]](#footnote-42)

Figure : A calibration run in which 100,000 trials are run and then fitted with a normal distribution. We use this run to inform our choices of thresholds, here shown as the first three green vertical lines, representing two, one, and no standard deviations below the mean respectively.

0

1000

2000

3000

4000

1200

1300

1400

1500

1600

BS(t)

count

Scaled Normal and

Standard Deviations

Normal(1437.3, 54.32)

Victories and Thresholds

German Victories

British Victories > 1437.5

Next, we compare our now-calibrated bootstrap to the battle as actually fought. We use the same number of trials as for our counterfactuals: . Owing to the lower number of trials, we should expect more variation, meaning that the results will be less precise and may deviate from our ideal values. The results of this run can be seen in Figures 2 and 3. Figure 2 is equivalent to Figure 1, but now for the smaller number of trials. Figure 3 shows plots of the day-to-day number of British pilots, with the actual number of pilots (by day) superimposed.[[43]](#footnote-43)

Figure 2: Bootstrapping the Battle of Britain with sampling in proportion to the phases as actually fought. The thresholds correspond to 97.2%, 83.9%, and 49.5% probabilities of British victories.

0

250

500

750

1000

1200

1300

1400

1500

1600

1700

BS(t)

count

Victories and Thresholds

German Victories

British Victories > 1328.5

British Victories > 1383

British Victories > 1437.5

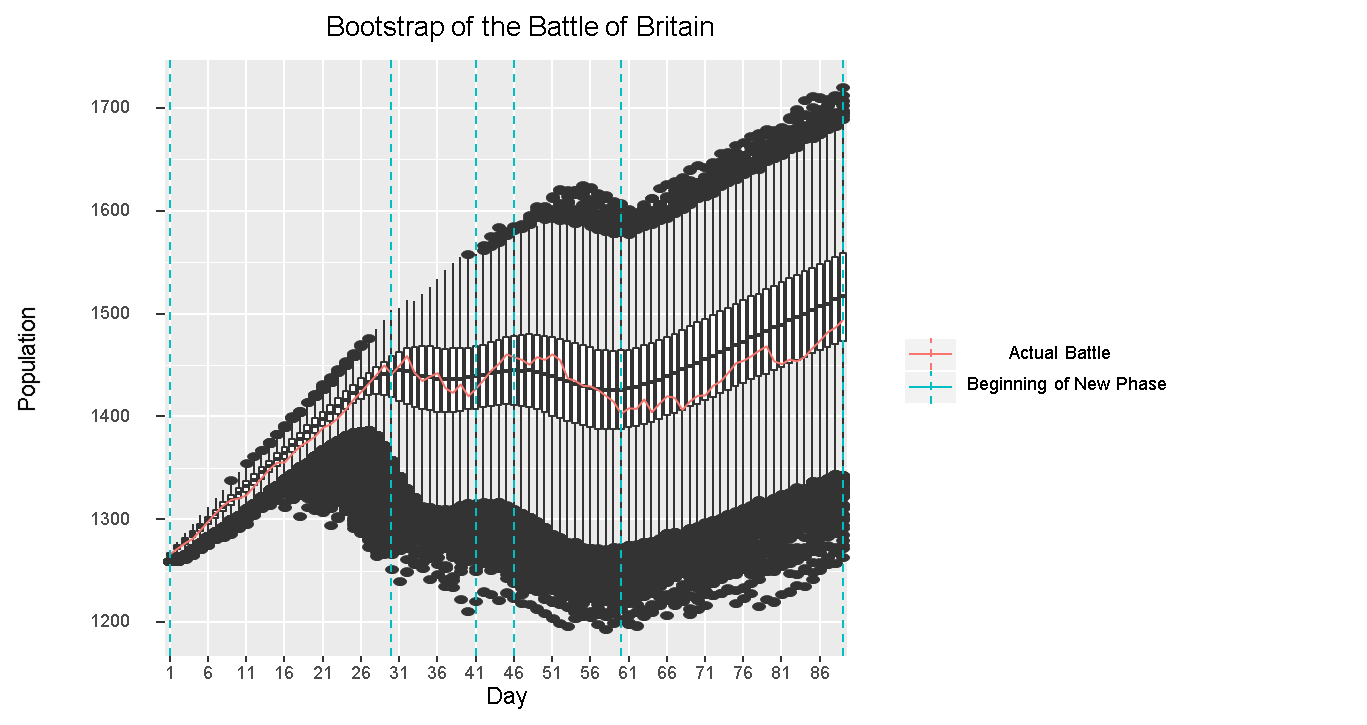


Figure : Bootstrapping the Battle of Britain with sampling in proportion to the phases as actually fought. The box-and-whisker plots show the day-to-day distributions of the number of pilots, the dashed vertical lines show boundaries between the phases of the battle as actually fought, and the solid curve is the number of pilots in the battle as actually fought.

**5.2 Weighted bootstrapping: operational counterfactuals**

**CF1: What if the switch to bombing London had not occurred?**

The results of our first counterfactual, where the Luftwaffe does not switch target to London (*i.e*. enter into phase 4), are shown in Figure 4. It is immediately clear that the probability of British victory has significantly decreased. If one believed that the British won the real battle with probability , the implied threshold now yields a British victory probability of just . Our lowest threshold, which had given the British a victory probability of in the real battle, brings this down to . This reinforces the common narrative that the switch to targeting London was a mistake.

0

200

400

600

800

1200

1400

1600

BS(t)

count

Victories and Thresholds

German Victories

British Victories > 1328.5

British Victories > 1383

British Victories > 1437.5

Figure : Bootstrapped results of the scenario where the Luftwaffe had not switched to targeting London by entering the fourth phase. Our thresholds now correspond to 63.7%, 31.2%, and 9.1% probabilities of British victory.

**CF2: What if Hitler had been fundamentally in favour of invasion from the outset**?

Our second counterfactual is grimmer still for the British: an eager Hitler pushes for an earlier beginning to the campaign, catching Fighter Command with approximately fewer pilots initially available. The constant threat allows the Luftwaffe to engage earlier RAF pilots who in the real battle would have had more training and combat experience. Additionally, it gives the German forces access to the earliest possible invasion date on 24th August. Figure 5 shows the damage this does to the probability of British victory: the victory possibility has now become , and the most optimistic threshold is now just .

Figure : Bootstrapped results of the scenario where the Luftwaffe began their assault earlier. Our thresholds now correspond to 18.0%, 3.8%, and 0.3% probabilities of British victory.

0

200

400

600

800

1000

1200

1400

1600

BS(t)

count

Victories and Thresholds

German Victories

British Victories > 1328.5

British Victories > 1383

British Victories > 1437.5

**CF3 combines CF1 (no fourth phase) and CF2 (early onset):**

As one might now expect, combining an early attack with no switch to London decreases further the viability of the British defence. This counterfactual also helps remind us of the prospect of diminishing returns: British chances are not utterly destroyed by the combined changes. The probability of British victory is simply reduced further: the victory probability is now (**CF2**: ), while the is now (**CF2**: ).

Figure : Bootstrapped results of the scenario where the Luftwaffe began their assault early and did not choose to switch to primarily targeting London by entering the fourth phase. Our thresholds now correspond to 8.3%, 1.0%, and 0.1% probabilities of British victory.

0

200

400

600

1000

1200

1400

1600

BS(t)

count

Victories and Thresholds

German Victories

British Victories > 1328.5

British Victories > 1383

British Victories > 1437.5

**CF4: What if Goering and his staff had believed that Fighter Command could be more easily destroyed on the ground than in the air?**

Instead of a change in the phasing of the battle, we now investigate the effects of not attacking London at all. We have already mentioned the German belief that it was easiest to destroy the RAF in the air, lured up by bombers, especially over London. The effects of this belief are brought out by our fourth counterfactual, in which the Luftwaffe focuses much more on the airfields. Results are shown in Figure 7. The results of **CF4** are a more drastic variation of **CF1** (no fourth phase), but are inevitably closely aligned with it. The victory probability is reduced to (**CF1**: ) and to (**CF1**: ). Not only should the Germans not have made their early-September switch; they paid dearly by choosing to attack London at all.

Figure : Bootstrapped results of the scenario where the Luftwaffe neglected to attack London entirely and had instead focused on the airfields. Our thresholds now correspond to 33.6%, 9.0%, and 1.1% probabilities of British victory.

0

250

500

750

1100

1200

1300

1400

1500

1600

BS(t)

count

Victories and Thresholds

German Victories

British Victories > 1328.5

British Victories > 1383

British Victories > 1437.5

**CF5 combines CF2 (an early start) with CF4 (targeting of Fighter Command on the ground):**

We now come to our last counterfactual: if Hitler had been eager for invasion, giving the Luftwaffe an early start, and if the Luftwaffe had been dedicated to targets associated with destroying the RAF on the ground. This is our most negative counterfactual for Britain, as shown in Figure 8: if one believes that the probability of British victory in the battle as actually fought was , or even 84%, then this alternative yields fewer than in victories for Britain. If one holds that the probability of British victory was , this situation yields just for the British. However much one might not believe in a “narrow margin” of British victory in the battle actually fought, the British aerial victory, it seems, depended very strongly on poor German choices. We summarize the results of all our counterfactuals in Table 4.

Figure : Bootstrapped results of the scenario where the Luftwaffe began their assault early as well as not attacked London and instead focused on the airfields. Our thresholds now correspond to 0.4%, 0.01%, and almost-nil probabilities of British victory.

0

200

400

600

800

900

1000

1100

1200

1300

1400

BS(t)

count

Victories and Thresholds

German Victories

British Victories > 1328.5

British Victories > 1383

**5.3 A cautionary tale: counterfactual weather**

Finally, we briefly discuss problems with bootstrapping. Natural objections to the use of bootstrap techniques include changes in the way the data are distributed or hidden underlying common factors. Weather exhibits precisely these problems, and is illustrative of some of the difficulties.

Suppose one wanted to investigate the impact of the weather on the Battle of Britain – to imagine that it might have been more or less rainy or cloudy in 1940 than is typical for July to October. There are many problems in doing so. First: most available quantitative data are very coarse, or qualitative in nature, or insufficiently geographically precise. The second and more fundamental problem is that, in order to use the bootstrap, we must assume that the data are drawn from some unchanging distribution. Weather is not so here. Above all, since weather is time-dependent, by altering it we may inadvertently alter the proportions of other variables. For example, the phases of the battle are also time-dependent, and simple statistical tests show that weather and phase are related. In general, if the weather were unusually sunny during one phase but cloudy and rainy during another, then clearly it would be impossible to alter phasing without also altering the weather, and *vice versa*. If we were not aware of this fact, we might falsely ascribe our results to the influence of weather.

For illustrative purposes only, we proceed with an ill-posed bootstrap in which we alter the weather from the actual 1940 weather to more typical (median) weather by changing the proportion of clear days.[[44]](#footnote-44) This leads to drops in the probabilities of British victory: our victory probability drops to and our to . However, this is probably due almost entirely to the correlation between weather and phase. More important for us is that there is no indication from the procedure that something has gone wrong. The bootstrap is an unthinking tool that does exactly what is asked of it, leaving the user to make sure that independent assumptions are justified.

**6. Conclusions**

We used the Battle of Britain data as a testbed for the bootstrapping technique in history. Our results match the standard historiographical view: the Germans were most successful when attacking airfields in August and early September, and they blundered when they changed target to London. A major problem with this retrospective assessment, however, is that it did not occur to the Germans. Indeed, Wood and Dempster argue in their classic account that the Luftwaffe assumed that the British retreat north of the Thames had already occurred, hindering their necessary objective of destroying Fighter Command.[[45]](#footnote-45) Thus, regardless of Hitler’s motives for bombing London, the Luftwaffe considered it strategically essential.

As we noted above, the bootstrap technique relies on the assumption that, had the earlier attacks been extended, combat attrition would have continued in the same way. Whether the Germans would really have obtained continuing good results is a question beyond our methods. We do not and cannot know British tactical and strategic responses to a counterfactually improved German performance, and we revert to our central observation above: that the bootstrap can only tell us the possibilities which result from the battle as actually fought.

However, we can certainly nuance the view that the Battle of Britain was a narrow victory. For the battle as actually fought it may have been so, but the sheer scale of the reversal of probabilities when we vary German targeting policy or prolong the earlier phases of the battle suggests that the British victory in the air campaign was sensitively dependent on German strategy and Luftwaffe tactics, and could easily have been reversed, even were RAF pilots and tactics improving rapidly. Among such possibilities the switch to London (**CF1**) may not have been the most significant of the German errors: it only results in a movement of just over one standard deviation σ, a switch from a balanced campaign to one moderately favouring the Germans, or from moderately favouring the British to balance. Bringing the campaign forward by three weeks (**CF2**) has a much greater, almost 3σ effect: from significantly favouring one side to moderately favouring the other (or vice versa). Yet this would have required a fundamentally more aggressive approach towards Britain, with an early strategic understanding of the necessity of militarily defeating Britain.[[46]](#footnote-46) Almost as significant – somewhere between 2σ and 3σ – would have been an understanding that Fighter Command should be defeated at least in part on the ground. The narrow margin, it seems, was not in the battle actually fought, but in German failure to pursue an optimal strategy.

That our analysis stresses the importance of an earlier beginning to the German campaign promotes a broader consideration of the topic. An Anglo-centric appreciation of the Battle of Britain tends implicitly to assume that the Battle of France ended effectively with the Dunkirk evacuation,[[47]](#footnote-47) liberating significant Luftwaffe assets to prepare for an early Sea Lion.[[48]](#footnote-48) In fact, the second phase of the German campaign in France, *Fall Rot*, was a hard-fought contest which occupied most of June and included a “second BEF” of 200,000 men. This German assault was initially repelled and, as with all “Blitzkrieg” campaigns, ultimate success depended on the whole-hearted participation of the Luftwaffe.[[49]](#footnote-49) The initiation of a costly air and land campaign against the UK with an unbeaten and increasingly competent French military in the field and finite German resources stretched to the limit would constitute a very bold initiative. Despite Hitler’s reputation as a gambler he was keenly aware of real possibilities of failure at this stage, and indeed his “halt” order before Dunkirk is often attributed to this well-founded anxiety. Heinz Guderian, the famously thrusting tank commander, was unpleasantly surprised by the disappearance of Hitler’s previous boldness as his XIX Corps broke through towards the Channel coast, noting that it had “never occurred” to him that Hitler “would now be the one to be frightened by his own temerity and would order our advance to be stopped at once”.[[50]](#footnote-50)

Had Hitler expressed greater enthusiasm slightly later than the capture of Dunkirk and after the breaking of French resistance, as assumed in our bootstrap, an early air campaign against Britain would need still more counterfactual support: an assumption that the Luftwaffe had suffered less badly in conquering France. Bungay notes that the Luftwaffe lost 1,428 aircraft in the campaign, “about half its operational strength” with another 488 damaged. Clearly it was not only the RAF which needed to recuperate, and moreover the Germans had to rebuild their full force of fighters and bombers for the coming battle, whereas the RAF could focus on fighters alone.[[51]](#footnote-51) Similar problems[[52]](#footnote-52) apply to alternative targeting policies. Had the Luftwaffe attacked the RAF and its infrastructure on the ground more effectively and with better concentration than it did, then our analysis suggests results would have been better. However, this would have required a vastly improved Luftwaffe intelligence organization with a greatly developed conception of the nature and ranked importance of the many targets available for attack and of the results that could be achieved by bombing them.

In our analysis, therefore, we have used real data to conceptualize possible alternative outcomes to the Battle of Britain and in effect reinforced the most conservative position based on the actual outcome: that invasion never had much prospect of success based on air supremacy without serious counterfactual changes. In terms of the air campaign the best prospects for invasion would have required an earlier start, though no such opportunity realistically presented itself in wider strategic terms. Less promising would have been an invasion initiated at the end of September without a previous switch to attacking London, reversing actual German consensus in favour of the London attack. The prospects for the invasion itself would not have been significantly improved in this case from those which caused Hitler and the High Command to decline it.

Beyond the Battle of Britain, a primary purpose of this paper is methodological innovation: to illustrate how weighted bootstrapping can provide a natural and intuitive tool for historians to investigate unrealized possibilities quantitatively and, by doing so, inform historical controversies and debates. The results can be presented, very simply, as the alteration in a historian’s probability estimate for the possible compared to the actual, promoting fuller analysis of actual historical outcomes.

However, this technique requires not only a sufficient number of data points – of actual events each drawn from an implied underlying distribution of possibilities – but that this distribution be unchanging and the events be independent. The technique is justified only to the extent to which one day’s events do not influence the next, and neither the sum of such influences nor other hidden trends causes the nature of the events to change over time. These conditions are almost unknown in the entirety of a land campaign, but at sea and in the air the position is less clear. Nevertheless, the bootstrap technique can provide a useful way of quantifying what can be said using nothing but the data: it has the virtues of being transparent – in the sense that the meaning of the reweighting of events is immediately clear – and free of any mathematical models. In a sense the bootstrap is the most transparent and conservative possible statistical technique to apply in a historical context: it accepts the data as all the events that are or can be known, and constructs alternative histories only as recombinations of these known events. But it thereby inevitably produces normal distributions (“bell curves”); it cannot access exceptional eventualities unseen in the actual data.

For future work, the main problem is the tension between the desire for a large number of data points, to better sample the underlying distribution, and the need for this distribution to be unvarying. A large number of data points will typically occur over either a long period of time or a large region of space, and either way there are likely to be important variations in the process which generates the data. Further, when the number of data points becomes too large, the bootstrap becomes less interesting precisely because all re-samples look much the same – most of them will be close to the mean. One natural possibility potentially balancing these tensions would be an investigation of the Battle of the Atlantic with each data point corresponding to a single convoy, with hierarchical modelling which takes account of changing equipment and tactics. Possibilities exist to use bootstrapping at higher resolution or in different scenarios, perhaps in political or economic aspects of strategy. Either way, if assumptions regarding independence are valid, then bootstrapping can be a valuable historical tool.

**Acknowledgements**

We should like to thank Professor Tom Lucas of the US Naval Postgraduate School for the original suggestion to bootstrap the Battle of Britain.

Table 1: Combat loss and target data. Phases are 1: 10 Jul - 7 Aug, principally of coastal attacks and armed reconnaissance, 2: 8 Aug - 18 Aug, of heavy attacks on mostly coastal targets, 0: 19th Aug - 23 Aug, of little interaction between the forces, 3: 24th Aug - 6th Sept, of attacks gradually concentrating on aerodromes and 4: 7th Sept - 31st Oct, following the switch to principally bombing London. Primary targets are A: aerodromes, C: docks, shipping and coastal or L: London, Kent, and Thames estuary. Note that pilots lost, wounded or slightly wounded are measured by incident. Thus some values represent pilots receiving a slight wound, flying again, and then receiving another wound.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Day** | **Date** | **British Airframe Losses** | **German Airframe Losses** | **British Pilots Lost** | **British Pilots Wounded** | **British Pilots Slightly Wounded** | **Phase** | **Primary Target** |
| 1 | 10/07/1940 | 2 | 11 | 2 | 0 | 0 | 1 | C |
| 2 | 11/07/1940 | 6 | 17 | 3 | 1 | 0 | 1 | C |
| 3 | 12/07/1940 | 5 | 9 | 4 | 0 | 1 | 1 | C |
| 4 | 13/07/1940 | 6 | 6 | 5 | 0 | 1 | 1 | C |
| 5 | 14/07/1940 | 1 | 3 | 1 | 0 | 0 | 1 | C |
| 6 | 15/07/1940 | 2 | 5 | 0 | 1 | 0 | 1 | C |
| 7 | 16/07/1940 | 1 | 4 | 1 | 0 | 0 | 1 | C |
| 8 | 17/07/1940 | 1 | 4 | 1 | 1 | 0 | 1 | C |
| 9 | 18/07/1940 | 5 | 6 | 4 | 0 | 0 | 1 | C |
| 10 | 19/07/1940 | 10 | 5 | 4 | 4 | 0 | 1 | C |
| 11 | 20/07/1940 | 9 | 12 | 6 | 0 | 2 | 1 | C |
| 12 | 21/07/1940 | 2 | 12 | 1 | 0 | 0 | 1 | C |
| 13 | 22/07/1940 | 2 | 4 | 1 | 0 | 0 | 1 | C |
| 14 | 23/07/1940 | 2 | 5 | 0 | 1 | 0 | 1 | C |
| 15 | 24/07/1940 | 5 | 15 | 3 | 0 | 1 | 1 | C |
| 16 | 25/07/1940 | 9 | 19 | 7 | 2 | 2 | 1 | C |
| 17 | 26/07/1940 | 1 | 5 | 1 | 0 | 0 | 1 | C |
| 18 | 27/07/1940 | 2 | 5 | 2 | 0 | 0 | 1 | C |
| 19 | 28/07/1940 | 6 | 11 | 1 | 4 | 1 | 1 | C |
| 20 | 29/07/1940 | 6 | 11 | 3 | 1 | 0 | 1 | C |
| 21 | 30/07/1940 | 1 | 9 | 0 | 1 | 0 | 1 | C |
| 22 | 31/07/1940 | 7 | 7 | 4 | 2 | 0 | 1 | A |
| 23 | 01/08/1940 | 4 | 13 | 3 | 0 | 0 | 1 | C |
| 24 | 02/08/1940 | 3 | 7 | 1 | 0 | 0 | 1 | C |
| 25 | 03/08/1940 | 0 | 6 | 0 | 0 | 0 | 1 | C |
| 26 | 04/08/1940 | 1 | 2 | 2 | 0 | 0 | 1 | C |
| 27 | 05/08/1940 | 2 | 8 | 1 | 0 | 1 | 1 | C |
| 28 | 06/08/1940 | 6 | 6 | 1 | 0 | 1 | 1 | R |
| 29 | 07/08/1940 | 4 | 3 | 0 | 0 | 1 | 1 | R |
| 30 | 08/08/1940 | 21 | 24 | 17 | 1 | 3 | 2 | C |
| 31 | 09/08/1940 | 3 | 6 | 1 | 0 | 0 | 2 | R |
| 32 | 10/08/1940 | 0 | 1 | 0 | 0 | 0 | 2 | C |
| 33 | 11/08/1940 | 28 | 38 | 25 | 1 | 3 | 2 | C |
| 34 | 12/08/1940 | 18 | 32 | 11 | 6 | 2 | 2 | C |
| 35 | 13/08/1940 | 15 | 39 | 4 | 1 | 5 | 2 | C |
| 36 | 14/08/1940 | 9 | 20 | 4 | 2 | 1 | 2 | C |
| 37 | 15/08/1940 | 35 | 76 | 16 | 8 | 5 | 2 | C |
| 38 | 16/08/1940 | 24 | 44 | 9 | 5 | 6 | 2 | A |
| 39 | 17/08/1940 | 2 | 5 | 0 | 1 | 0 | 2 | R |
| 40 | 18/08/1940 | 33 | 67 | 10 | 11 | 6 | 2 | A |
| 41 | 19/08/1940 | 5 | 11 | 2 | 0 | 1 | 0 | C |
| 42 | 20/08/1940 | 2 | 8 | 1 | 0 | 0 | 0 | C |
| 43 | 21/08/1940 | 4 | 14 | 0 | 0 | 1 | 0 | C |
| 44 | 22/08/1940 | 4 | 4 | 2 | 1 | 0 | 0 | C |
| 45 | 23/08/1940 | 1 | 8 | 0 | 0 | 0 | 0 | C |
| 46 | 24/08/1940 | 20 | 41 | 5 | 7 | 6 | 3 | R |
| 47 | 25/08/1940 | 18 | 23 | 11 | 1 | 3 | 3 | C |
| 48 | 26/08/1940 | 29 | 42 | 4 | 10 | 9 | 3 | R |
| 49 | 27/08/1940 | 7 | 11 | 2 | 0 | 0 | 3 | A |
| 50 | 28/08/1940 | 15 | 32 | 6 | 6 | 2 | 3 | R |
| 51 | 29/08/1940 | 10 | 24 | 2 | 2 | 4 | 3 | A |
| 52 | 30/08/1940 | 25 | 40 | 11 | 4 | 0 | 3 | A |
| 53 | 31/08/1940 | 41 | 39 | 9 | 18 | 7 | 3 | A |
| 54 | 01/09/1940 | 13 | 16 | 6 | 6 | 0 | 3 | A |
| 55 | 02/09/1940 | 14 | 37 | 4 | 10 | 2 | 3 | A |
| 56 | 03/09/1940 | 15 | 20 | 6 | 4 | 7 | 3 | A |
| 57 | 04/09/1940 | 17 | 28 | 11 | 2 | 4 | 3 | A |
| 58 | 05/09/1940 | 20 | 27 | 8 | 7 | 3 | 3 | A |
| 59 | 06/09/1940 | 20 | 33 | 8 | 7 | 8 | 3 | A |
| 60 | 07/09/1940 | 25 | 41 | 15 | 6 | 7 | 4 | A |
| 61 | 08/09/1940 | 5 | 16 | 2 | 2 | 0 | 4 | A |
| 62 | 09/09/1940 | 17 | 30 | 6 | 4 | 7 | 4 | A |
| 63 | 10/09/1940 | 3 | 13 | 0 | 0 | 0 | 4 | R |
| 64 | 11/09/1940 | 29 | 29 | 14 | 8 | 7 | 4 | L |
| 65 | 12/09/1940 | 1 | 7 | 1 | 0 | 0 | 4 | R |
| 66 | 13/09/1940 | 3 | 7 | 2 | 0 | 1 | 4 | L |
| 67 | 14/09/1940 | 13 | 13 | 4 | 7 | 0 | 4 | L |
| 68 | 15/09/1940 | 31 | 61 | 16 | 5 | 6 | 4 | L |
| 69 | 16/09/1940 | 1 | 10 | 0 | 1 | 0 | 4 | L |
| 70 | 17/09/1940 | 6 | 8 | 3 | 1 | 1 | 4 | L |
| 71 | 18/09/1940 | 12 | 20 | 4 | 5 | 3 | 4 | L |
| 72 | 19/09/1940 | 0 | 10 | 0 | 0 | 0 | 4 | L |
| 73 | 20/09/1940 | 8 | 8 | 5 | 0 | 2 | 4 | L |
| 74 | 21/09/1940 | 1 | 11 | 1 | 0 | 0 | 4 | C |
| 75 | 22/09/1940 | 1 | 6 | 0 | 0 | 0 | 4 | L |
| 76 | 23/09/1940 | 11 | 17 | 3 | 4 | 0 | 4 | C |
| 77 | 24/09/1940 | 6 | 11 | 2 | 3 | 3 | 4 | L |
| 78 | 25/09/1940 | 6 | 16 | 3 | 1 | 0 | 4 | (Filton) |
| 79 | 26/09/1940 | 8 | 9 | 3 | 2 | 1 | 4 | C |
| 80 | 27/09/1940 | 28 | 57 | 20 | 4 | 2 | 4 | L |
| 81 | 28/09/1940 | 17 | 12 | 10 | 2 | 2 | 4 | L |
| 82 | 29/09/1940 | 6 | 9 | 2 | 3 | 0 | 4 | C |
| 83 | 30/09/1940 | 21 | 47 | 6 | 5 | 5 | 4 | L |
| 84 | 01/10/1940 | 7 | 9 | 4 | 1 | 0 | 4 | L |
| 85 | 02/10/1940 | 2 | 18 | 0 | 1 | 1 | 4 | L |
| 86 | 03/10/1940 | 1 | 9 | 1 | 1 | 0 | 4 | C |
| 87 | 04/10/1940 | 1 | 15 | 1 | 0 | 0 | 4 | L |
| 88 | 05/10/1940 | 7 | 14 | 2 | 3 | 1 | 4 | L |
| 89 | 06/10/1940 | 2 | 9 | 2 | 0 | 0 | 4 | L |
| 90 | 07/10/1940 | 17 | 19 | 9 | 2 | 4 | 4 | C |
| 91 | 08/10/1940 | 8 | 17 | 7 | 0 | 0 | 4 | L |
| 92 | 09/10/1940 | 3 | 9 | 3 | 0 | 0 | 4 | L |
| 93 | 10/10/1940 | 8 | 12 | 6 | 1 | 0 | 4 | L |
| 94 | 11/10/1940 | 9 | 10 | 4 | 4 | 1 | 4 | C |
| 95 | 12/10/1940 | 11 | 13 | 5 | 2 | 3 | 4 | C |
| 96 | 13/10/1940 | 4 | 6 | 1 | 3 | 0 | 4 | L |
| 97 | 14/10/1940 | 1 | 4 | 1 | 0 | 0 | 4 | R |
| 98 | 15/10/1940 | 15 | 16 | 6 | 5 | 2 | 4 | L |
| 99 | 16/10/1940 | 3 | 15 | 3 | 0 | 0 | 4 | (sweeps) |
| 100 | 17/10/1940 | 5 | 16 | 5 | 1 | 0 | 4 | L |
| 101 | 18/10/1940 | 6 | 14 | 5 | 0 | 0 | 4 | R |
| 102 | 19/10/1940 | 1 | 6 | 2 | 0 | 1 | 4 | L |
| 103 | 20/10/1940 | 5 | 11 | 3 | 1 | 0 | 4 | L |
| 104 | 21/10/1940 | 2 | 7 | 2 | 0 | 0 | 4 | L |
| 105 | 22/10/1940 | 6 | 12 | 4 | 1 | 0 | 4 | L |
| 106 | 23/10/1940 | 1 | 4 | 1 | 0 | 0 | 4 | R |
| 107 | 24/10/1940 | 3 | 12 | 3 | 0 | 0 | 4 | L |
| 108 | 25/10/1940 | 14 | 24 | 6 | 5 | 1 | 4 | L |
| 109 | 26/10/1940 | 8 | 10 | 5 | 0 | 0 | 4 | L |
| 110 | 27/10/1940 | 14 | 16 | 6 | 1 | 1 | 4 | L |
| 111 | 28/10/1940 | 0 | 14 | 0 | 0 | 0 | 4 | L |
| 112 | 29/10/1940 | 12 | 28 | 5 | 1 | 2 | 4 | L |
| 113 | 30/10/1940 | 9 | 8 | 6 | 1 | 2 | 4 | L |
| 114 | 31/10/1940 | 0 | 2 | 0 | 0 | 0 | 4 | R |

Table 2: Sortie and weather data. Entries are NA: not available, for which no estimate exists, C: clear, O: overcast or cloudy and R: rainy.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Day** | **Date** | **British Sortie** | **German Sortie** | **Channel & Coast Weather** | **London, Kent, and Estuary Weather** | **Midlands & North Weather** |
| 1 | 10/07/1940 | 609 | *NA* | R | R | R |
| 2 | 11/07/1940 | 452 | *NA* | O | O | R |
| 3 | 12/07/1940 | 670 | *NA* | O | R | R |
| 4 | 13/07/1940 | 449 | *NA* | O | O | O |
| 5 | 14/07/1940 | 593 | *NA* | C | C | C |
| 6 | 15/07/1940 | 470 | *NA* | O | O | O |
| 7 | 16/07/1940 | 313 | *NA* | O | O | O |
| 8 | 17/07/1940 | 253 | *NA* | O | O | O |
| 9 | 18/07/1940 | 549 | 100 | O | R | O |
| 10 | 19/07/1940 | 701 | 150 | O | O | O |
| 11 | 20/07/1940 | 611 | 100 | O | R | R |
| 12 | 21/07/1940 | 571 | *NA* | C | C | C |
| 13 | 22/07/1940 | 611 | 100 | O | O | R |
| 14 | 23/07/1940 | 470 | *NA* | O | O | R |
| 15 | 24/07/1940 | 561 | *NA* | O | O | R |
| 16 | 25/07/1940 | 641 | *NA* | C | C | O |
| 17 | 26/07/1940 | 581 | *NA* | R | R | R |
| 18 | 27/07/1940 | 496 | *NA* | C | O | R |
| 19 | 28/07/1940 | 794 | *NA* | C | C | C |
| 20 | 29/07/1940 | 758 | *NA* | C | C | C |
| 21 | 30/07/1940 | 688 | *NA* | R | R | R |
| 22 | 31/07/1940 | 395 | *NA* | O | C | C |
| 23 | 01/08/1940 | 659 | 100 | O | O | C |
| 24 | 02/08/1940 | 477 | 100 | O | R | C |
| 25 | 03/08/1940 | 425 | 50 | O | O | O |
| 26 | 04/08/1940 | 261 | 80 | C | C | C |
| 27 | 05/08/1940 | 402 | 110 | O | C | C |
| 28 | 06/08/1940 | 416 | 60 | O | O | O |
| 29 | 07/08/1940 | 393 | 70 | C | O | C |
| 30 | 08/08/1940 | 621 | 280 | C | R | R |
| 31 | 09/08/1940 | 409 | 110 | O | O | O |
| 32 | 10/08/1940 | 336 | 80 | O | R | R |
| 33 | 11/08/1940 | 679 | 370 | O | O | O |
| 34 | 12/08/1940 | 732 | 440 | C | C | C |
| 35 | 13/08/1940 | 700 | 450 | O | C | C |
| 36 | 14/08/1940 | 494 | 600 | C | O | O |
| 37 | 15/08/1940 | 974 | 650 | C | C | C |
| 38 | 16/08/1940 | 776 | 800 | O | C | C |
| 39 | 17/08/1940 | 288 | 50 | C | C | C |
| 40 | 18/08/1940 | 755 | 560 | O | O | O |
| 41 | 19/08/1940 | 383 | 400 | O | O | O |
| 42 | 20/08/1940 | 453 | 200 | O | O | R |
| 43 | 21/08/1940 | 589 | 170 | O | O | O |
| 44 | 22/08/1940 | 509 | 220 | O | O | O |
| 45 | 23/08/1940 | 482 | 270 | O | O | R |
| 46 | 24/08/1940 | 936 | 550 | C | C | R |
| 47 | 25/08/1940 | 480 | 325 | O | O | O |
| 48 | 26/08/1940 | 787 | 440 | C | C | O |
| 49 | 27/08/1940 | 288 | 50 | O | O | R |
| 50 | 28/08/1940 | 739 | 400 | C | C | C |
| 51 | 29/08/1940 | 498 | 390 | O | R | R |
| 52 | 30/08/1940 | 1054 | 600 | C | C | C |
| 53 | 31/08/1940 | 978 | 800 | O | C | C |
| 54 | 01/09/1940 | 661 | 490 | C | C | C |
| 55 | 02/09/1940 | 751 | 750 | C | C | C |
| 56 | 03/09/1940 | 711 | 550 | O | C | R |
| 57 | 04/09/1940 | 678 | 550 | O | C | R |
| 58 | 05/09/1940 | 662 | 460 | O | C | C |
| 59 | 06/09/1940 | 987 | 730 | C | C | C |
| 60 | 07/09/1940 | 817 | 700 | O | O | O |
| 61 | 08/09/1940 | 305 | 200 | O | O | O |
| 62 | 09/09/1940 | 466 | 430 | C | R | R |
| 63 | 10/09/1940 | 224 | 50 | O | O | O |
| 64 | 11/09/1940 | 678 | 500 | O | C | C |
| 65 | 12/09/1940 | 247 | 80 | R | R | R |
| 66 | 13/09/1940 | 209 | 130 | R | R | R |
| 67 | 14/09/1940 | 860 | 400 | O | R | R |
| 68 | 15/09/1940 | 705 | 600 | C | C | C |
| 69 | 16/09/1940 | 428 | 250 | R | R | R |
| 70 | 17/09/1940 | 544 | 350 | R | R | R |
| 71 | 18/09/1940 | 1165 | 800 | C | C | C |
| 72 | 19/09/1940 | 237 | 75 | R | R | R |
| 73 | 20/09/1940 | 540 | 150 | O | O | O |
| 74 | 21/09/1940 | 563 | 260 | C | C | C |
| 75 | 22/09/1940 | 158 | 140 | O | O | O |
| 76 | 23/09/1940 | 710 | 300 | C | C | C |
| 77 | 24/09/1940 | 880 | 500 | O | O | O |
| 78 | 25/09/1940 | 668 | 290 | C | C | C |
| 79 | 26/09/1940 | 417 | 220 | C | C | C |
| 80 | 27/09/1940 | 939 | 850 | C | R | R |
| 81 | 28/09/1940 | 770 | 300 | O | O | C |
| 82 | 29/09/1940 | 441 | 180 | C | C | C |
| 83 | 30/09/1940 | 1173 | 650 | C | C | C |
| 84 | 01/10/1940 | *NA* | *NA* | O | O | O |
| 85 | 02/10/1940 | *NA* | *NA* | C | C | C |
| 86 | 03/10/1940 | *NA* | *NA* | R | R | R |
| 87 | 04/10/1940 | *NA* | *NA* | R | R | R |
| 88 | 05/10/1940 | *NA* | *NA* | R | R | R |
| 89 | 06/10/1940 | *NA* | *NA* | R | R | R |
| 90 | 07/10/1940 | *NA* | *NA* | O | O | O |
| 91 | 08/10/1940 | *NA* | *NA* | O | O | O |
| 92 | 09/10/1940 | *NA* | *NA* | R | O | O |
| 93 | 10/10/1940 | *NA* | *NA* | R | R | R |
| 94 | 11/10/1940 | *NA* | *NA* | C | R | C |
| 95 | 12/10/1940 | *NA* | *NA* | O | O | O |
| 96 | 13/10/1940 | *NA* | *NA* | O | O | O |
| 97 | 14/10/1940 | *NA* | *NA* | O | R | O |
| 98 | 15/10/1940 | *NA* | *NA* | O | C | C |
| 99 | 16/10/1940 | *NA* | *NA* | O | O | O |
| 100 | 17/10/1940 | *NA* | *NA* | R | R | R |
| 101 | 18/10/1940 | *NA* | *NA* | O | O | O |
| 102 | 19/10/1940 | *NA* | *NA* | O | O | O |
| 103 | 20/10/1940 | *NA* | *NA* | O | O | O |
| 104 | 21/10/1940 | *NA* | *NA* | O | O | O |
| 105 | 22/10/1940 | *NA* | *NA* | O | O | O |
| 106 | 23/10/1940 | *NA* | *NA* | R | R | R |
| 107 | 24/10/1940 | *NA* | *NA* | O | C | C |
| 108 | 25/10/1940 | *NA* | *NA* | O | O | O |
| 109 | 26/10/1940 | *NA* | *NA* | O | O | R |
| 110 | 27/10/1940 | *NA* | *NA* | O | O | O |
| 111 | 28/10/1940 | *NA* | *NA* | O | O | O |
| 112 | 29/10/1940 | *NA* | *NA* | O | O | O |
| 113 | 30/10/1940 | *NA* | *NA* | R | R | R |
| 114 | 31/10/1940 | *NA* | *NA* | R | O | O |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Scenario | Summary | Starting Date | Bootstrap Reweighting  (Days of Target or Phase) | | | |
| Airfield  P1 | Coast  P2 | London  P3 | Recon  P4 |
| Real | Actual Battle | 10th July | 16  29 | 47  11 | 36  19 | 13  30 |
| CF1 | No switch to London | 10th July | 29 | 11 | 49 | 0 |
| CF2 | Early Start | 16th June | 32 | 18.7 | 32.3 | 30 |
| CF3 | No London and early start | 16th June | 32 | 29.7 | 51.3 | 0 |
| CF4 | Target airfields | 10th July | 43 | 33 | 0 | 13 |
| CF5 | Target airfields and early start | 16th June | 54 | 42 | 0 | 17 |

Table 3: Summary of counterfactual scenarios.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Scenario | Summary | Probabilities of British Victory by Threshold | | |
|  |  |  |
| Real | Actual Battle |  |  |  |
| CF1 | No switch to London |  |  |  |
| CF2 | Early Start |  |  |  |
| CF3 | No London and early start |  |  |  |
| CF4 | Target airfields |  |  |  |
| CF5 | Target airfields and early start |  |  |  |

Table 4: Summary of counterfactual scenario outcomes.

1. For the literature see MacKay, Niall, and Christopher Price, "Safety in Numbers: Ideas of concentration in Royal Air Force Fighter Defence from Lanchester to the battle of Britain." *History* 96, no. 323 (2011): 304-325. [↑](#footnote-ref-1)
2. Bungay, Stephen, *The Most Dangerous Enemy: a History of the Battle of Britain*, Aurum, 2000, p.33; Deighton, Len, *Fighter: The True Story of the Battle of Britain*, Jonathan Cape, 1977, p.51. [↑](#footnote-ref-2)
3. Kershaw, Ian, *Fateful choices: ten decisions that changed the world, 1940-1941*. Penguin, 2008. [↑](#footnote-ref-3)
4. Cox, Richard, *Operation Sealion*. Thornton Cox, 1974. Kieser, Egbert, *Hitler on the Doorstep: Operation “Sea Lion”: the German Plan to Invade Britain, 1940*. Naval Institute Press, 1997. Evans, Martin Marix, and Angus Mcgeoch, *Invasion! Operation Sea Lion, 1940*. Routledge, 2004. Forczyk, Robert, *We March Against England: Operation Sea Lion, 1940–41*. Bloomsbury, 2016. [↑](#footnote-ref-4)
5. Forester, Cecil Scott, “If Hitler had invaded England”. In *Gold from Crete*. Pan, 1971. Macksey, Kenneth, *Invasion: The German Invasion of England, July 1940*. Greenhill, 1980. Messenger, Charles, “The Battle of Britain 1940: Triumph of the Luftwaffe”. In *Third Reich Victorious: Alternative Histories of World War II,* edited by Peter G. Tsouras, 65-96. Greenhill, 2002. [↑](#footnote-ref-5)
6. The possibility of protracted indirect war on British seaborne trade and strategic interests in the Mediterranean lies outside the scope of this article. Forczyk, *We March Against England.* [↑](#footnote-ref-6)
7. Originally introduced by Bradley Efron in 1979, the bootstrap is described in Tibshirani, Robert J., and B. Efron, “An introduction to the bootstrap.” *Monographs on Statistics and Applied Probability* 57 (1993): 1-436. [↑](#footnote-ref-7)
8. For example, we make no use of Lanchester’s scaling laws for attrition, stochastic duels, or any other assumptions about *how* casualties are produced or models for attrition. Washburn, Alan R., and Moshe Kress. *Combat Modeling*. Springer, 2009. [↑](#footnote-ref-8)
9. For an excellent recent summary see Bungay, *Most Dangerous Enemy*. The 2015 75th anniversary edition of *Royal Air Force* *Air Power Review* (18 no.2, Summer 2015) provides a useful collection of original RAF sources with commentary. [↑](#footnote-ref-9)
10. Grinnell-Milne, Duncan W., *The Silent Victory: September 1940*. Bodley Head, 1958. [↑](#footnote-ref-10)
11. Cumming, Anthony. J., *The Royal Navy and the Battle of Britain*, Naval Institute Press, 2010. [↑](#footnote-ref-11)
12. James, Brian, “Pie in the Sky?” *History Today*, September 2006, p.38. [↑](#footnote-ref-12)
13. Goulter, Christine, Andrew Gordon and Gary Sheffield, “The Royal Navy did not win the ‘Battle of Britain’”. *Journal of the Royal United Services Institute* 151 no.5 (2006):66-67. This exchange is discussed in Mackay and Price 2011, p.305. [↑](#footnote-ref-13)
14. Bungay, at one extreme, argues that the Luftwaffe “never came close” to their desired strategic result and that in terms of the important Sector stations, closing Biggin Hill for a few hours on one day was the extent of their success in damaging 11 Group’s infrastructure. Bungay, *Most Dangerous Enemy*, p.368-9. [↑](#footnote-ref-14)
15. A fictionalized account of the 1974 RMA wargame of Sea Lion is given by Cox, *Operation Sealion*. Details of German preparations are in Schenk, Peter*, Invasion of England 1940: The Planning of Operation Sealion*, Oberbaum, 1987; Engl. trans. Conway 1990. A summary of the strategic planning is *German Plans for the Invasion of England: Operation Sealion, 1940*, Declassified Central Intelligence Agency report, published by Digital Publications 2017. [↑](#footnote-ref-15)
16. Johnson, Ian R. and Niall J. MacKay, “Lanchester models and the Battle of Britain”, *Naval Research Logistics* 58 (2011):210-222; MacKay and Price 2011; MacKay, Niall J., ”Is air combat Lanchestrian?” *Phalanx: the Bulletin of Military Operations Research* 44 (2011):12-14; Horwood, Ian, Niall MacKay and Christopher Price, “Concentration and Asymmetry in Air Combat: Lessons for the defensive employment of air power”, *RAF Air Power Review* 17 no.2 (2014):68-91. [↑](#footnote-ref-16)
17. Johnson and MacKay 2011, Figure 1. [↑](#footnote-ref-17)
18. Bungay, *Most Dangerous Enemy*, Ch.30. [↑](#footnote-ref-18)
19. Spaatz, Carl, "Strategic Air Power: Fulfillment of a Concept." *Foreign Affairs* 24, no. 3 (1946): 385-396. Townsend, Peter, *Duel of Eagles*. Cassell, 1970. Churchill, W.S., *Their Finest Hour: The Second World War.* Vol. 2. 2nd ed. Cassell, 1950. [↑](#footnote-ref-19)
20. Warden, John A., *The Air Campaign: planning for combat*. Brassey’s, 1989. p.103 [↑](#footnote-ref-20)
21. For example, Macksey, Kenneth. *Military Errors of World War Two.* p.45. [↑](#footnote-ref-21)
22. A discussion of German attacks on UK coastal towns in 1914 is given by Friedman, Norman, *Fighting the Great War at Sea: Strategy, Tactics and Technology.* Naval Institute Press, 2014. [↑](#footnote-ref-22)
23. Holwell, Sue, and Peter Checkland, “An information system won the war.” *IEE Proceedings-Software* 145, no. 4 (1998): 95-99. [↑](#footnote-ref-23)
24. Dempster, Derek D., and Derek Wood, *The Narrow Margin: the Battle of Britain and the rise of air power 1930-1940.* Hutchinson, 1961. [↑](#footnote-ref-24)
25. MacKay and Price, 2011; Bungay, *Most Dangerous Enemy*; Deighton, Len, *Fighter: The True Story of the Battle of Britain.* Jonathan Cape, 1977. [↑](#footnote-ref-25)
26. Dowding, Hugh, “Battle of Britain despatch”, reprinted in *Royal Air Force Air Power Review* 18 no.2 (2015), items 181-193. [↑](#footnote-ref-26)
27. Townsend, Peter, *Duel of Eagles*. [↑](#footnote-ref-27)
28. See, for example, Warden, *The Air Campaign*, for whom this is a matter of airpower theory, and the correct concentration of force. That airfield attacks were a winning tactic for the Luftwaffe is, as noted above, contested by Bungay, *Most Dangerous Enemy*. [↑](#footnote-ref-28)
29. *German Plans* 2017; Grinnell-Milne, *Silent Victory*, Ch.7: “Time and the Tides”. [↑](#footnote-ref-29)
30. Airframe losses were compiled from Ramsey, Winston G., *The Battle of Britain: Then and Now.* London: Battle of Britain prints international, 5th ed., 1989. British pilot casualties were also compiled from Ramsey, *Battle of Britain*, and were cross-checked with the RAF Battle of Britain Honour Roll [online: raf.mod.uk/campaign/battle-of-britain-75th/the-few/battle-of-britain-roll-of-honour/, accessed 19 September 2017] and with Wynn, Kenneth G., *Men of the Battle of Britain: A Biographical Dictionary of The Few.* Frontline, 2015. Targets are from Dowding, Hugh, *Enemy Air Offensive Against Great Britain*, 1941-1947. Held as AIR 2/7771, UK National Archives, Kew. Weather is from the RAF Campaign Diaries [online: raf.mod.uk/history/campaign diaries.cfm, accessed: 10 April 2017] Sortie numbers are from James, T. C. G., *The Battle of Britain (Royal Air Force Official Histories: Air Defence of Great Britain, vol.2).* New ed. Routledge, ed. S. Cox, 2000. German sortie numbers are not generally available before 1st August. [↑](#footnote-ref-30)
31. James, *Battle of Britain*. [↑](#footnote-ref-31)
32. Deighton *Fighter, p.216*; Bungay, *Most Dangerous Enemy*, p.208, 221*.* [↑](#footnote-ref-32)
33. Dempster and Wood, *Narrow Margin,* p.104. [↑](#footnote-ref-33)
34. Bungay, *Most Dangerous Enemy*, p.373. [↑](#footnote-ref-34)
35. Dempster and Wood, *Narrow Margin*, Appendix 11. [↑](#footnote-ref-35)
36. That is, to the extent to which they believe that additional days’ fighting would have followed the pattern of actual days’ results. Since it uses only the actual days of fighting, what the bootstrap cannot do is to include any of the unlikely things that one could argue might have happened, but did not. There are no “black swans” in bootstrapped counterfactuals. Taleb, Nassim Nicholas*. The Black Swan: The impact of the highly improbable.* Random House, 2007. [↑](#footnote-ref-36)
37. Forester, *Gold from Crete*; Macksey, *Invasion*; Messenger, *Battle of Britain*. [↑](#footnote-ref-37)
38. Macksey*, Invasion,* p.13. [↑](#footnote-ref-38)
39. Townsend, *Duel of Eagles*, p.325. Townsend himself is quite clear: “Goering made a crucial error. For Fighter Command was more vulnerable on the ground than in the air” (p.379). [↑](#footnote-ref-39)
40. Townsend, *Duel of Eagles*, p.360. [↑](#footnote-ref-40)
41. To be clear: we are not imposing a normal distribution on the number of pilots evaluated on a single day, nor on one neap cycle’s possible launch dates. Instead, we are imposing a normal distribution on the lowest number of pilots during any of the three possible neap windows. In this way, we ensure that the probabilities correspond to whether or not Germany launches an invasion at all during a given trial, and prevent over-counting trials where the number of pilots remains low multiple times. [↑](#footnote-ref-41)
42. Note that fractional values are possible due to our assigning less value to new pilots in . [↑](#footnote-ref-42)
43. A box-and-whisker plot shows the spread of data by way of five values – the median, quartiles and 5th and 95th centiles – supplemented by outliers. The middle value is the median, above and below which 50% of the data lie. The lower (respectively, upper) edges of the boxes correspond to the 1st (resp. 3rd) quartiles, which divide the data into the lowest 25% (resp. 75%) and highest 75% (resp. 25%). The distance between the 1st and 3rd quartiles is then used to compute the locations of the whiskers, beyond which all points are considered outliers. As we should expect, the number of pilots from day to day in the actual battle matches well with the trends of the bootstrap, due to ordering the days of our trials in order of phase. [↑](#footnote-ref-43)
44. We used the 1929-1979 record of sunlight hours at the Meteorological Office’s Oxford weather station [online: www.metoffice.gov.uk/pub/data/weather/uk/climate/stationdata/oxforddata.txt, accessed 24 July 2017] to give us typical ranges of sunlight during July to October, including 1940, and applied this to the more qualitative Meteorological Office 1940 Daily Weather Reports [online: digital.nmla.metoffice.gov.uk/archive/sdb%3AdeliverableUnit%7Ceda9f47f-c326-4991-ada4-a3e4c8bd11d9/, accessed 22 July 2017]. [↑](#footnote-ref-44)
45. Wood and Dempster, *Narrow Margin*, p.212. [↑](#footnote-ref-45)
46. As Macksey notes, “The basic reason for Germany’s failure to invade Britain in 1940 is … the lack of any preconceived will or intention to do so.” Macksey, *Military Errors*. [↑](#footnote-ref-46)
47. Alexander, Martin S. "After Dunkirk: The French Army's Performance against Case Red, 25 May to 25 June 1940." *War in History* 14, no. 2 (2007): 219-264. [↑](#footnote-ref-47)
48. Macksey, *Invasion*. [↑](#footnote-ref-48)
49. Alexander 2007. [↑](#footnote-ref-49)
50. Guderian, Heinz. *Panzer Leader*. Penguin, 2009, p.109. [↑](#footnote-ref-50)
51. Bungay, *Most Dangerous Enemy*, p.104-105. [↑](#footnote-ref-51)
52. These are “cotenability problems” in the sense of Tetlock, Philip E., and Aaron Belkin, eds. *Counterfactual thought experiments in world politics: Logical, methodological, and psychological perspectives.* Princeton University Press, 1996. [↑](#footnote-ref-52)