Discussion of The Fisher Effect Puzzle: A Case of Non-Linear Relationship

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1. Assessing the Fisher equation

Analysis of the Fisher equation or, equivalently, of the determination of the real interest rate has been the subject of many studies over the years. It provides a significant challenge because the two key variables to be modelled both appear in expectation and so the impact of how expectations are modelled has to be allowed for along with the other aspects of the problem. The Fisher equation is usually understood to connect the ex-ante real interest rate r_t^e to ex-ante (or expected) inflation π_t^e and the observed nominal interest rate i_t (which many authors regard as fixed in advance of the determination of inflation), so $r_t^e = i_t - \pi_t^e$. The difference between the ex-ante and ex-post real interest rate r_t is the (assumed to be) random inflation expectational error:

$$r_t = r_t^e + (\pi_t^e - \pi_t) = r_t^e + \varepsilon_t = i_t - \pi_t \qquad (1)$$

where the expectational error is $\varepsilon_t = \pi_t^e - \pi_t$. The Fisher equation is therefore an accounting identity which defines the ex-post real interest rate. The approach of Hall et al is to focus on the relationship between the nominal interest rate and ex-post inflation $\mathrm{i}_t = \mathrm{a} + \mathrm{b}\pi_t + \mathrm{u}_t$ where the hypothesis of interest is b=1 and the real interest rate is a constant a. As the authors say, analysis of this particular equation, either through cointegration analysis or otherwise has often returned estimates of b substantially (and often significantly) below one. It is impressive that the method of estimation employed in this paper returns a (time varying average) value of b close to one. There is, however, a serious limitation to the approach that the authors adopt in that they assume that the real interest rate is constant, so:

$$r_t = i_t - \pi_t = a + b(\pi_t^e - \pi_t) + u_t = a + w_t \quad (2)$$

As is demonstrated below, it is unlikely in most modelling situations that this will be true. Put differently, the error term u_t is likely to be predictable. An alternative approach is to test for cointegration between i_t and π_t based on the finding that they are integrated I(1), non-stationary, variables. This approach has the appeal that it allows the equilibrium (or long-run) real interest rate to be time varying and stationary and which could be a function of long-run stationary features of the macro economy such as technology growth.

Having said this, Hall et al use a result in Swamy at al (2008) which shows that their estimation method is robust to mispecification of various types. Their estimate of b is therefore reliable and it is interesting that they obtain estimates of b which are close to unity on average over time. The time-varying nature of the estimates of a and b must depend in part on the elements of the determination of the real interest rate which have been omitted. It would have been interesting to see the impact of introducing some of these variables on the estimates of a and b. The authors offer the possibility of asymmetry in the behaviour of inflation in the parameters after 1983 and more could be learnt about this by further structural analysis.

2. Evaluating the persistence and stationarity of the elements of the real interest rate

The time-series properties of inflation and the nominal and real interest rates have occupied numbers of researchers. Representative is the work of Rose (1988) who found that the nominal interest rate and prices appear to both be I(1) implying that inflation is stationary whilst the nominal interest rate is non-stationary. He then goes on to show how this prime facia evidence of non-stationarity in the ex-ante real interest rate is inconsistent with the implications of the consumption-based capital asset pricing model (CCAPM) given the stationarity of consumption growth and the real returns of a number of other financial assets. Subsequently, evidence has been presented that price inflation maybe be non-stationary (see Ball and

Cecchetti (1990), for example). This, however, has not resolved the problems because further analysis, supported by the results in Table 1 of Hall et al, show no evidence in favour of cointegration between inflation and the nominal interest rate and therefore stationarity of the ex- ante real interest rate.

Further evidence, mirroring developments in the modelling of persistence in time series processes, has supported the argument that inflation and the nominal interest rate are not best described as integrated I(1) or stationary I(0) processes but rather as persistent fractionally integrated I(d) processes where 0<d<1. Whether they are fractionally stationary or not depends on estimates of the parameter d. Sun and Phillips (2004) initiated much of this work and found estimates of d≈0.9 for i_t, π_t and r_t for a sample of quarterly data from 1934-1999 implying fractional non-stationarity for all three variables. Their conclusion then followed that the real interest rate or Fisher equation is not a cointegrating long-run relationship. This analysis, however, does not take into account the potential structural breaks and non-linearity in the behaviour of the variables concerned. Recent research suggests that, having accounted for one or more of these factors, the evidence might be that the three variables concerned are, in fact, fractionally stationary. That would mean that they are stationary but persistent which would lead to behaviour of the real interest rate which is more consistent with the implications of economic theory. Charfeddine and Guegan (2007), for example, find breaks in inflation in the US in April 1967, January 1973 and July 1981 using Bai and Perron (1998) sequential testing. Their estimates of d for untransformed inflation are between 0.408 (standard error = 0.05) and 0.89 (0.10). When they remove the structural breaks in inflation from the series and re-estimate d they obtain estimates between 0.046 (0.039) and 0.228 (0.074) which are clearly significantly smaller than 0.5 and border on failing to reject I(0) stationarity against a more persistent alternative. Baillie and Morana (2009) allow for non-linearity in estimation of d. They estimate an adaptive ARFIMA model which allows the constant term to be time varying (they choose a flexible fourier form which can approximate a range of non-linear functions). Their results are that estimation of d produces an estimate of between 0.42 (0.042) and 0.69 (0.067) for unadjusted US CPI inflation from January 1948 to October 2006. The estimate of d for a generalisation of the ARFIMA model which allows for non-linearity is 0.346 (0.04). Therefore allowing for non-linearity and structural breaks reduces the estimated persistence of the inflation rate significantly. If similar results can be found for the nominal interest rate series there would then be the prospect of the three series i_t, π_t and r_t being regarded as mildly persistent stationary series. Whilst further analysis is required, similar results for consumption growth and other asset returns would re-establish the consistency of the time-series behaviour of key variables and the equilibrium properties of standard macroeconomic models such as the CCAPM.

3. The macroeconomic determinants of the real interest rate

In parallel to the investigation of the time series properties of the real interest rate there has been development in analysis of the macroeconomic equilibrium determinants. In a standard monetary model (eg Gali (2008), Ch 2) the ex-ante real interest rate is given by:

$$\mathrm{r}_t = \rho + \sigma \psi_{ya} E_t \left\{ \Delta \mathrm{a}_{t+1} \right\} \tag{3}$$

where ρ=-lnβ the discount rate of utility, σ the coefficient of relative risk aversion (CRRA), ψ_{ya} is the reduced-form parameter relating equilibrium output to technology and $\Delta \mathrm{a}_{t+1}$ is the technology shock. In this equation the real interest rate is determined by the real side of the economy in equilibrium and is stationary for a stationary growth rate in technology. In a stochastic model, Canzoneri and Dellas (1998) show that monetary factors can also play a role in the equilibrium real interest rate. They show that when decomposed into a risk-free rate and a risk premium, the determination of the ex-ante real interest rate can be affected by central bank operating procedures such as the choice between interest rate, money and nominal income targeting. In particular, they show that real interest rates should be lower under money than interest rate targetting for a given set of shocks.

What have we learned about the determination and behaviour of the real interest rate? Many studies have contributed to finding that the real interest rate is a function of a number of, probably stationary, variables including technology growth. In addition, changes in monetary policy regimes are likely to be important source of variation in the real interest rate. These effects might appear as structural change and parameter variation in estimates of the Fisher equation. Hall et al show us that, armed with an estimation method which is robust to these developments, we can obtain an estimate of the slope of the Fisher equation which is much closer to our expectations than when we employ more limited methods.

4. References

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