Family Labor Supply and Aggregate Saving*

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April 5, 2013

Abstract

I study the impact of idiosyncratic risk on savings and employment in a small open economy populated by two-member families. Families incur a fixed cost of participation when both members are employed. Because of market incompleteness and information asymmetries, this cost coupled with labor market frictions can generate multiple equilibria. In particular, there might be one equilibrium with high employment and low saving and another one with low employment and high saving. The model predicts that aggregate saving and employment rates are negatively correlated across countries. I present empirical evidence that supports the general equilibrium prediction of the model.

JEL Classifications: D52, D90, E13, E21, J21, J22.

Keywords: Saving, Employment, Family labor supply, Multiple equilibria

*I am grateful to Philippe Weil and Alexandre Janiak for their helpful comments and suggestions. I would like to thank seminar participants at ECARES, HEC Montreal, NOVA University, University of Alicante, University of Chile, University of Namur, Warwick University. Any remaining errors are my own. Financial support by the Portuguese Science and Technology Foundation is gratefully acknowledged.

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1 Introduction

The study of aggregate saving has long been central to economics; however, there are persistent differences in saving rates across very similar economies, which traditional models of capital accumulation do not explain well. Together, the life cycle/permanent-income and the precautionary saving/buffer-stock models have been the primary theoretical framework for explaining individual and aggregate saving. Many different versions of the benchmark model of saving have been explored; however, seldom endogenous labor supply decisions are introduced and, in most general equilibrium models of aggregate saving, employment is an exogenous variable.

In this paper I provide an exposition of a model in which aggregate saving and also equilibrium employment is the result of market interaction between firms and a large number of families which face idiosyncratic risk. I model families as two member households, which exhibit prudence, and therefore have a precautionary saving motive. I consider a small open economy, in the sense that the interest rate is exogenous and should be interpreted as the world interest rate. The price which is endogenous in equilibrium is the within household wage-gap between the primary earner and the second family member. In the economy examined, when the wage-gap is low, many families choose to have both members employed in equilibrium. Therefore, the aggregates saving rate is low, because two-earner families are less exposed to idiosyncratic risk than single-earner families. If instead the wage-gap is high in equilibrium, most families are single-earner and the aggregate saving rate is high because agents are vulnerable to idiosyncratic shocks and rely heavily on precautionary saving for self-insurance purposes. In turn, the equilibrium wage-gap and the level of employment are determined by the interaction between firms and workers, in a labor market characterized with frictions and asymmetric information.

The corner-stone of this paper is a model of family labor supply, which captures within household extensive margin labor supply interactions. To introduce a labor market participation choice among household members, I follow Cho and Rogerson (1987) and introduce within family labor supply decisions shaped by symmetric preferences and fixed costs of participation which are incurred when both members of the household are simultaneously in the labor force. Moreover, I pose the problem in such a way that the second member labor supply acts as a source of insurance against within family resources fluctuations. This hypothesis has some empirical support. For example, Dynarski and Gruber (1997) find that families do a good job at smoothing consumption in the face of changes in the head's earnings. Moreover they find that a substantial amount of within family consumption smoothing is achieved through offsetting changes in other sources of family income, including spousal earnings.

Because the value for the firm of hiring a worker depends on the attachment of the worker to the firm, firms have to form rational expectations about the degree of attachment of the worker to the labor force. The first member of the household is always employed, however, the second member enters and exits the labor force in equilibrium. Hence the equilibrium employment rate depends on the labor force exit rate of the second household member, which is endogenous in the rational expectations general equilibrium. Because of market incompleteness and asymmetry of information and given that firms are not willing to pay the same wage to workers with different degrees of attachment to the labor force, allowing for family extensive margin labor supply choices can lead to multiplicity of equilibria. In particular we can have one equilibrium with high employment and low saving and another one with low employment and high saving. The model suggests that a rise in equilibrium employment rates, which translates into an increase in the number of two-earner families, should lead to a lower aggregate saving rate, as the variability of families disposable income decreases. If we interpret high employment rates among married women as a high employment equilibrium in the sense defined above, at the aggregate level, the model predicts a negative correlation between female labor force participation rates and aggregate saving. I present empirical evidence which supports this general equilibrium prediction.

Moreover, the model is not mute about welfare. In the equilibrium with high employment and low aggregate saving, firms are as well off as they would be in the low employment/high saving equilibrium, because expected profits are always zero given the existence of a free entry condition. However, in the high employment equilibrium, households are better off because they solve the same inter-temporal problem but wages are higher because of the lower gender wage-gap. Therefore, the multiple equilibria can be Pareto ranked, and the paper thus offers insights useful for policy-makers.

Not many authors have investigated the impact of idiosyncratic risk on aggregate saving allowing for endogenous labor supply. Notable exceptions are Marcet, Obiols-Homs and Weil (2007) and Pijoan-Mas (2006). These authors find that in a general equilibrium framework, and contrary to models with exogenous labor supply, the presence of uninsurable labor income risk might lead to less aggregate saving than under complete markets. Low (2005) analysis life-cycle labor supply and saving in a partial equilibrium framework. He finds that when labor supply is flexible, consumption is smoother than when work hours are exogenous. Furthermore, he argues that making labor supply flexible has an ambiguous impact on the correlation between precautionary savings and earnings uncertainty, since on the one hand the cost of accumulating precautionary balances is smaller, but on the other hand the value of precautionary wealth holdings is less because households can now adjust labor supply. Attanasio, Low and Sanchéz-Marcos (2005), explore the role of female labor supply as an insurance mechanism against idiosyncratic earnings risk within the family. They find that additional uncertainty increases female participation rates. Another stream of work, pioneered by Chang and Kim (2006), examines the implications of introducing cross-sectional heterogeneity caused by lack of complete insurance markets for the aggregate labor supply elasticity and in particular characterize the resulting reservation-wage distribution.

The remainder of the paper is organized as follows. Section 2 presents the model. In Section 3, I show how to solve numerically for the model's stationary competitive equilibria. Section 4 presents relevant empirical findings which support the model prediction and Section 5 concludes.

2 The Model

2.1 Households and Preferences

The model economy is populated by a continuum (measure one) of infinitely lived families. A family is a partnership between two members, a husband (m) and a wife (f), which make an integrated choice over how much to consume and how many hours each member works. To model the preferences of each household, I follow Cho and Rogerson (1988) and, in particular, it is assumed that a family incurs a fixed cost of participation when both members of the household are simultaneously in the labor force. A family's instantaneous utility is given by

$$u\left(c, \ell_m, \ell_f\right) = \ln\left(c\right) + \Phi(\ell_m, \ell_f)\bar{h}, \tag{2.1}$$

where c is the family's consumption of market goods and $\Phi(\ell_m, \ell_f)\bar{h}$ is the family consumption of home produced goods. Home-produced goods are non-traded and can be used only as consumption. The family either produces (and consumes) $\bar{h}$ home produced goods or none, depending on the choices for ℓ_m and ℓ_f, which are the market hours worked by each respective member of the family. The technology to produce home goods is captured by $\Phi(\ell_m, \ell_f)$:

$$\Phi(\ell_m, \ell_f) = \begin{cases} 1 & if \;\; \ell_m \ell_f = 0 \\ 0 & \text{else} \end{cases} . \tag{2.2}$$

This home goods production function, implies that only families in which at least one member is not supplying market hours are able to consume home produced goods. I abstract from the intensive margin and assume that each household member can either choose to work $\bar{\ell}$ hours or stay at home, in which case $\ell = 0$. Thus, effectively the family incurs a fixed cost when both members of the household are simultaneously employed because the home technology requires one full time worker to yield output. Each period, families have to decide whether to have one or both members employed.

The household member labeled m is the family primary earner and when employed she earns market wage w_m. When employed, the second household member earns wage $\lambda w_m \leq w_m$. The within household wage-gap, λ, is an equilibrium price and is endogenous. Since both family members have identical preferences and make an integrated choice, given the within household wage-gap, the first household member always chooses to be employed. Each family can store wealth, earning a constant risk-free rate r, which should be viewed as the world interest rate in a small open economy. However, borrowing cannot exceed a borrowing limit δ which I normalize to zero. I also impose an upper bound on wealth holdings $\bar{a}$. $\bar{a}$ is chosen to be large enough, so that this additional constraint never binds in equilibrium. I let a_t and $\tilde{e}_t$ denote, respectively, family financial wealth, and an exogenous and idiosyncratic expenditure shock (e.g.: medical expenditure), at time t.

The flow budget constraint of a family choosing to have both members employed is given by

$$c_t + \tilde{e}_t + a_{t+1} = w_m \bar{\ell} + \lambda w_m \bar{\ell} + (1 + r)\, a_t. \tag{2.3}$$

The flow budget constraint of a family choosing to have only the first member employed is

$$c_t + \tilde{e}_t + a_{t+1} = w_m \bar{\ell} + (1+r)\, a_t. \tag{2.4}$$

The exogenous expenditure shock $\tilde{e}_t \in [0, \bar{e}]$ is randomly *i.i.d* distributed across families and time, and is drawn from the cumulative distribution $F(e)$. Crucially, the size of the shock as well as family wealth is private information and cannot be publicly verified. The resources available to the family, net of the expenditure shock are

$$z = (1+r)\, a - e. \tag{2.5}$$

Because shocks are *i.i.d.*, z is the only relevant state variable for the household. From the assumptions made about the support of a and e it follows that z also has a finite support, $[\underline{z}, \bar{z}]$. Finally, notice that because of the extensive margin choice, the family indirect utility function is not concave in general . In particular, let $V_1(z)$ and $V_2(z)$ be, respectively, the value for the family of choosing to have only one member employed or both members employed. Suppose both these functions are concave. The family value function prior to the employment choice is given by $\mathcal{V}(z) = \max[V_1(z), V_2(z)]$. Clearly $\mathcal{V}(z)$ need not and, in general, will not be concave. Thus the solution to the problem solved by families can be improved by using lotteries over wealth[1]. With the purpose of convexifying the problem, the

[1]The need to use lotteries in non-convex economies is an issue raised in a number of papers, notably Phelan and Townsend (1991), Hopenhayn and Nicolini (1997) and Lentz and Tranaes (2005). The last paper discusses in detail how to implement such a mechanism and in this paper

following assumption is made:

Assumption 1 *A family which has available resources z can invest in a complete menu of lotteries yielding return z_L with probability α, and return z_H with probability $1-\alpha$, and all lotteries are fair in the sense that their expected return is zero, or equivalently, $\alpha = \frac{z_H - z}{z_H - z_L}$. The menu of lotteries is complete in the sense that the family can freely choose z_L and z_H, as long as $z_L, z_H \in [\underline{z}, \bar{z}]$.*

The timing of events in the economy with lotteries is specified as follows: The idiosyncratic expenditure shock, $\tilde{e}_t$, is realized; given wealth and the idiosyncratic shock, each family can choose whether to invest in a lottery which yields zero expected return; notice that non-participation is possible, simply by choosing the lottery profile $z_L = z_H = z_t$; finally, the lotteries are realized and each family makes her saving and labor supply decisions. Thus, it is useful to define the value function of a family at the second stage of the family program, after the idiosyncratic shock has been realized, when the family must decide over which lottery to invest in:

$$\mathcal{V}^{\mathcal{L}}(z) = \max_{z_L, z_H \in [\underline{z}, \bar{z}]} \left[\frac{z_H - z}{z_H - z_L} \mathcal{V}(z_L) + \frac{z - z_L}{z_H - z_L} \mathcal{V}(z_H) \right]. \tag{2.6}$$

Notice that $V^{\mathcal{L}}(z)$ is the concave envelope of $\mathcal{V}(z)$. Finally, at the final stage of the household decision process, the two relevant value functions are $V_1(z)$ and $V_2(z)$. The Bellman equation characterizing the problem of a household holding available resources z (after the lottery realization) that chooses to have only the primary earner employed is given by

$$V_1(z) = \max_{a' \in \mathcal{A}} \left\{ u\left(c, \bar{\ell}, 0\right) + \beta \int_0^{\bar{e}} \mathcal{V}^{\mathcal{L}}(z')\, dF(e) \right\}, \tag{2.7}$$

I define a mechanism which resembles strongly their own.

where

$$
\begin{aligned}
c + a' &= w_m \bar{\ell} + z, \\
z' &= (1+r)\, a' - \tilde{e}, \\
a' &\geq -\delta.
\end{aligned}
$$

And the Bellman equation characterizing the problem of a household that chooses to have both members simultaneously employed is given by

$$
V_2(z) = \max_{a' \in \mathcal{A}} \left\{ u\left(c, \bar{\ell}, \bar{\ell}\right) + \beta \int_0^{\bar{e}} \mathcal{V}^{\mathcal{L}}(z')\, dF(e) \right\}, \tag{2.8}
$$

where

$$
\begin{aligned}
c + a' &= w_m \bar{\ell} + \lambda w_m \bar{\ell} + z, \\
z' &= (1+r)\, a' - \tilde{e}, \\
a' &\geq -\delta.
\end{aligned}
$$

Households discount future utility at rate β, and $\beta(1+r) < 1$ which implies that absent uncertainty they would want to borrow against future consumption to finance current consumption. Consequently, the support of the household wealth distribution has a finite endogenous upper bound (see Aiyagari [1994]).

The solution to the household problem is a threshold level

$$
z^T : V_1\left(z^T\right) = V_2\left(z^T\right), \tag{2.9}
$$

above which the household decides to have only one member, the primary earner, employed and a pair of asset demand functions conditional on the number of family laborers

$$\hat{a_1}' = A_1 (z; r, w, \lambda, \delta) \qquad z > z^T, \tag{2.10}$$
$$\hat{a_2}' = A_2 (z; r, w, \lambda, \delta) \qquad z \leq z^T.$$

2.2 Firms and Labor Demand

Before characterizing equilibrium, I have first to model labor demand, coupled with a zero profits/free entry condition. I assume that labor is the only factor of production and that there are constant returns to scale. This allows me to model a firm as a match between an employer and an employee. Firms compete for workers à la Bertrand and, given the equilibrium wage, workers choose whether to work or stay at home. The wage rate is kept constant for as long as the match lasts and in particular is not allowed to vary with worker tenure.

There are no search frictions and no barriers to entry. However, when a new match of a worker and a firm occurs, the worker is less productive in the first period of the match. The marginal productivity of the worker is $y - \kappa$ in the first period of the match and in the following periods and for as long as the match is kept, the marginal productivity is y. This can be interpreted as firm specific human capital, in an otherwise perfectly competitive labor market, which is entirely accumulated in the first period of the match.[2]

[2]The absence of tenure-linked wage rules and the assumption that the productivity of workers is less in the first period of the match are the labor-market frictions leading to wage-discrimination.

The problem of the firm is characterized by the following Bellman equations in discrete time

$$
\begin{aligned}
rJ^n &= \bar{\ell}(y-\kappa-w)+(1-p)(J^s-J^n), && (2.11)\\
rJ^s &= \bar{\ell}(y-w)+p\,(J^n-J^s), && (2.12)
\end{aligned}
$$

where J^n is the value for the firm of creating a vacancy and J^s is the value for the firm of remaining in operation with the same worker as in the period before. The wage rate is given by w and $0<p<1$ is the probability of separation of the match between the firm and the worker, which is endogenous in the rational expectations general equilibrium. From (2.11) and (2.12) we obtain

$$J^s-J^n=\frac{\bar{\ell}\,\kappa}{1+r} \tag{2.13}$$

and the value of a vacancy can be written as

$$rJ^n=\bar{\ell}\left(y-\frac{p+r}{1+r}\,\kappa-w\right). \tag{2.14}$$

The free entry condition implies that in equilibrium J^n, the value of creating a vacancy, must be zero and therefore the equilibrium wage rate is

$$w=y-\frac{p+r}{1+r}\,\kappa. \tag{2.15}$$

Consequently, in this economy there are two wage rules, one for workers of type m and another one for workers of type f because p, the probability of a match being destroyed, differs according to the worker type. Importantly, the worker type is assumed to be publicly observed (in other words, firms know if the worker is the family primary earner). Moreover the primary earner is always part of the labor force and therefore firms forming rational expectations about p_j, $j \in \{m, f\}$, set $p_m = 0$. However, $p_f = P\left(\ell'_f = 0 \mid \ell_f > 0\right)$ is not zero. Since the decision of the second household on the extensive margin is not a trivial one, because of the presence of the fixed cost $\bar{h}$, the firms anticipate this when setting the wage rate. It follows that in equilibrium, the wage-gap across the two types (the within household wage gap) is

$$\lambda = \frac{w_f}{w_m} = \frac{y\,(1+r) - (p_f + r)\,\kappa}{y\,(1+r) - r\kappa}. \tag{2.16}$$

Finally, it is convenient to normalize y/κ and thus by setting $y/\kappa = 1$, we obtain

$$\lambda = 1 - p_f. \tag{2.17}$$

2.3 General Equilibrium

Characterization of a recursive competitive equilibrium for a dynamic heterogeneous agent model would require that we keep track of the wealth distribution because the equilibrium prices depend on the distribution of wealth and the forecast of agents about future prices depends on the law of motion for wealth. However, if the solution of the households' problem at given constant prices induces a stationary distribution of wealth, then a stationary equilibrium exists, because in our model economy there is no aggregate uncertainty and therefore, given a stationary distribution of asset holdings, prices are constant. Following Huggett (1993), it can be shown that a stationary wealth distribution exists provided that: families' wealth holdings are distributed over a compact set; the policy functions are monotonic; the monotone mixing condition is satisfied, which implies that every household has a positive probability of visiting the entire wealth distribution in either direction. All three conditions are satisfied.

Existence of a stationary distribution ensures the existence of constant prices, however this is not sufficient to establish the existence of a competitive equilibrium because the free entry condition might generate prices which are not consistent with household optimal behavior. However, by an appropriate choice of $\bar{h}$, it is possible to make sure that at least one stationary competitive equilibrium exists. To see this, notice that by choosing $\bar{h}$ sufficiently small so that, when $\lambda = 1$, households always find it optimal to have both members employed, we obtain at least one equilibrium; this is because if all households always have both members employed, $p_f = 0$, but then the free entry condition will also be satisfied since $\lambda = 1 - p_f = 1$.

Household wealth is private information which firms cannot observe. Hence, the rational expectations forecast of firms about p_f depends only on the expectations about next period prices and on the equilibrium law of motion of the wealth distribution. In the stationary equilibrium this is time invariant. The rational expectation of firms over p_f is

$$p_f = \int_{-(\delta+\bar{e})}^{z^T(\lambda)} P\left[z' \geq z^T(\lambda) \mid z = z^*\right] \times \frac{\gamma_\lambda(z)}{\Gamma(z)} dz^*, \tag{2.18}$$

where $\gamma_\lambda(z)$ is the stationary probability density function of z, which depends on

λ, and $\Gamma(z)$ is the corresponding stationary cumulative density function.

On the households side, there is a continuum (measure one) of two member families, indexed by $i \in \mathcal{I}$, that have identical preferences and are subject to idiosyncratic expenditure shocks e^i. A stationary competitive equilibrium relies on household behaving optimally given there wealth and prices (r, w_m, λ), firms forming a rational expectation about p_f and a stationary wealth distribution $\Gamma(z)$.

Definition 1 *A stationary competitive equilibrium is defined by the pair* (λ, p_f)*, a threshold level of resources* $z^T(\lambda)$*, and a stationary distribution* $\Gamma(z)$ *for which*

1. *The policy functions*

$$\begin{aligned} \hat{a_1}' &= A_1(z; r, w, \lambda, \delta) \qquad z \le z^T, \\ \hat{a_2}' &= A_2(z; r, w, \lambda, \delta) \qquad z > z^T, \end{aligned}$$

   solve the households' optimum problem;

2. *There is free entry of firms:* $\lambda = 1 - p_f$*;*

3. *Firms are forming rational expectations about* p_f.

Figure 1 illustrates , for a given interest rate r, a possible shape for the locus defined by equation (2.18), which I will call the p_f-locus for ease of exposure. Notice that for low values of λ the p_f-locus is not defined since no household has both members employed and therefore the set of individuals of type f employed has zero measure. However, for completeness, I define $p_f = 1$ to be the out-of equilibrium belief firms form about p_f when λ is too small for any type f individual to be employed. Thus, I make the following assumption

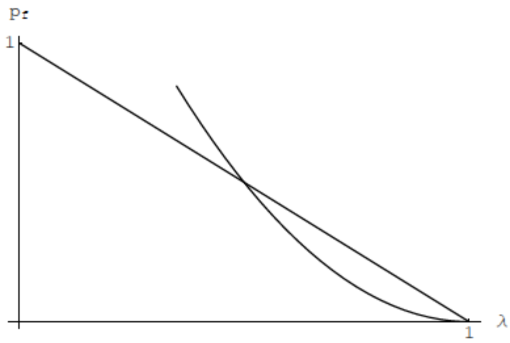

Figure 1: Determination of equilibrium λ and p_f

Assumption 2 *Let* λ^* *be such that* $\Gamma(z^T(\lambda^*)) = 0$. *Then a firm who finds a worker of type* f *willing to work at wage* $\lambda^* w_m$ *forms expectation* $p_f(\lambda^*) = 1$. *This is called an out-of-equilibrium belief.*

An analytical characterization of the p_f-locus is not feasible and therefore characterization of the stationary competitive equilibrium is a numerical exercise, which I describe in the next section. The downward linear slope, which I will call λ-locus, corresponds to equation (2.17). Clearly there may exist more than one equilibrium.[3] Moreover, as mentioned before, $\bar{h}$ is a free parameter which can be chosen in such way to always ensure the existence of an equilibrium. In particular, because $\beta(1+r) < 1$, the individual wealth holdings is bounded and the wealth distribution has finite support, and there exists a $z^*(\lambda)$ such that for all $z \geq z^*(\lambda)$, $z' \leq z^*(\lambda)$ with probability one (Aiyagari [1994]).

The following existence result can therefore be established:

[3] Strictly speaking a situation with $p_f = 1$ and $\lambda = 0$ is a rational expectations equilibrium in this economy however it is not an interesting one as it implies that no family chooses to have both members employed. I therefore ignore this particular equilibrium in what follows.

Proposition 1 *Given an appropriate choice of $\bar{h}$, such that $\ell_f(z^*(1)) = 1$, there exist always an equilibrium with full employment and a zero within family wage gap, that is $\lambda = 1$. I call this equilibrium the non-discriminating equilibrium.*

Proof: if $\bar{h}$ is such that $\ell_f(z^*(1)) = 1$ then, because participation is decreasing in available resources, $\ell_f(z'(1)) = 1$ with probability one and hence the rational expectation of p_f is zero and thus this is an equilibrium.

Finally, it is easy to verify that provided the non-discriminating equilibrium exists and if $\bar{h}$ is not too small, then there will often be a second equilibrium, a discriminating equilibrium, for which $0 < \lambda < 1$. Where $\bar{h}$ not too small requires $\bar{h}$ such that for an arbitrarily small $\lambda > 0$ no family chooses to have both members employed. In particular, I show through numerical simulation that for many plausible parameterizations, two equilibriums exist, one corresponding to a low λ and a weak attachment of workers of type f to the labor force (high p_f) and another one with a strong attachment and a high λ.

These two equilibriums obtained can be Pareto ranked. In the equilibrium with high employment and low aggregate savings, firms are as well off as they would be in the low employment high savings equilibrium, because of the free entry condition. However, in the high employment equilibrium, households are better off because they solve the same inter-temporal problem but facing a looser budget constraint. Before discussing further how to solve for the model equilibrium, I investigate what are the implications for employment and aggregate saving of varying λ, the within family wage gap.

Table 1: Aggregate Statistics Varying the Wage Gap

wage-gap λ	separation rate p_f	participation rate	$\frac{wealth}{income}$
0.05	1.00	0.00	0.43
0.15	0.94	0.01	0.40
0.25	0.85	0.06	0.29
0.35	0.68	0.29	0.13
0.45	0.49	0.56	0.10
0.55	0.28	0.76	0.10
0.65	0.13	0.88	0.10
0.75	0.04	0.96	0.09
0.85	0.01	0.99	0.09
0.95	0.00	1.00	0.08

Note: Parameters used in the simulations are $\beta = 0.95$, $r = 3\%$, $y = 100$, $\mu = 50$, $\sigma_e = 20$ and $\bar{h} = 0.6$.

2.4 λ and Aggregate Saving

The non-convexity introduced by the fixed cost coupled with market incompleteness and household private information about wealth holdings makes possible the existence of multiple equilibria. In particular, often there are two equilibria: one equilibrium where the wage gap is low (high λ) and the participation rates among second household earners is high and another one with low λ and low participation.

Table 2.4 illustrates this point for a given parameterization of the model. The table shows what is the effect of varying λ, for the participation rate of the second household earner as well as for the wealth/income ratio. This exercise clarifies what are the partial equilibrium adjustments, as we move along the p_f-locus. The main point to note is that the separation rate p_f decreases monotonically with λ, suggesting a monotonically decreasing p_f-locus. This finding is robust over wide range of parameterizations. Moreover, the wealth/income ratio decreases as λ increases and, in particular, the wealth/income ratio is lowest when λ approaches one.[4] Thus, assuming that the non-discriminating equilibrium exists and that there exists another equilibrium (the discriminating equilibrium), the wealth/income ratio in the discriminating equilibrium is higher than in the non-discriminating equilibrium. Consider a group of economies which have all identical fundamentals; however, suppose that for some reason they coordinate on different stationary competitive equilibria: some economies coordinate in the discriminating equilibrium and the remaining coordinate in the non-discriminating equilibrium. Employment will be high and aggregate savings will be low in the economies which coordinate in the non-discriminating equilibrium. It is in this sense that the model predicts that saving and employment are negatively correlated across countries.

3 Solving the Model

To characterize equilibrium numerically, assuming that a stationary wealth distribution exists, I solve the household problem for a set of values for the gender wage-gap λ. I solve the dynamic programming problem of the household via value function iteration. The iteration is performed separately over the two conditional value functions, $V_1(z)$ and $V_2(z)$ given a guess for $\mathcal{V}^{\mathcal{L}}$. To accelerate convergence, I follow the suggestion in Aruoba, Villaverde and Rubio-Ramíres (2006). I start

[4] The wealth/income ratio corresponds to

$$\frac{\int_{[\underline{Z},z_T]} A_2\left(z;r,w,\lambda\delta\right)d\Gamma(z) + \int_{(z_T,\bar{z}]} A_1\left(z;r,w,\lambda\delta\right)d\Gamma(z)}{y\left(1+E(1-p_f)\right)},$$

where E is the employment among workers of type f.

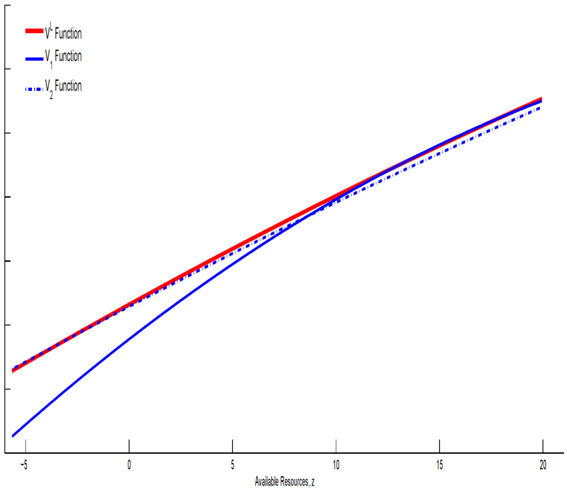

Figure 2: Value Functions Conditional on Employment Status

iterating on a small grid. Then, after convergence, I add more points to the grid, and recompute the Bellman operator using the previously found value function as an initial guess (with linear interpolation to fill the unknown values in the new grid points). Iterating with this grid refinement, I move from an initial five-hundred-point grid into a final one with five thousand points. The continuous distribution for the random expenditure shock is replaced by a discrete distribution. The household can hold a single asset $a_t \in [-\delta, \bar{a}]$. After each iteration j for the conditional value functions, I evaluate V_1^j and V_2^j over a dense grid z_G of points and I obtain $\mathcal{V}^{\mathcal{L}j+1}$ by obtaining the concave envelope of $\max[V_1^j(z_G), V_2^j(z_G)]$. Figure 2 plots the two conditional value functions and the corresponding concave envelope, for a given parametrization.

Once I have obtained both conditional value functions, I determine the threshold level of resources z^T above which families only have one member employed and I obtain the pair of policy functions. I simulate a history for each of a large number of families (typically 10000). For each family I use the following procedure: in any period, I begin with the level of assets from the previous period and multiply by $(1 + r)$. I draw random realizations for the expenditure shock $\tilde{e}$ from the corresponding distribution to obtain the available resources z. I apply the policy rules. This procedure yields end of period assets. Having obtained histories for a large number of families, I obtain the aggregate wealth/income ratio and the equilibrium separation rate by averaging over the last cross-section. Unfortunately, although I allow for lotteries when solving for the optimal policy functions (by obtaining the concave envelope of $\mathcal{V}$) I do not implement it during the simulation stage. This may cause a bias in the computation of the aggregate statistics however, provided that there is enough uncertainty, the bias will be small.[5] Once I have characterized numerically the p_f-locus I find the λ's which satisfy the condition $\lambda = 1 - p_f$.

Figure 3 illustrates with a numerical example the possible multiple equilibria. The net real rate of interest, r, is assumed to be 3% and in the benchmark parametrization of the model I assume that the discount factor, β, is 0.95. The expenditure shock, $\tilde{e}$ is assumed to be normally distributed, with mean μ and variance σ_e^2.[6] Finally, $\bar{\ell}$ is set equal to one so that an employed worker of type m earns labor income w_m and an employed worker of type f earns labor income λw_m, and y is chosen to be 100. $\bar{h}$ is chosen appropriately, to guarantee the existence of the non-dicriminating equilibrium. In the example shown there are two equilibria: the

[5]See Gomes, Greenwood and Rebelo, 2001.

[6]Effectivelly, $\bar{e}$, the exependiture shock upper bound, will be the largest point in the support of the discrete approximation to the shock's distribution and $\underline{e} > 0$.

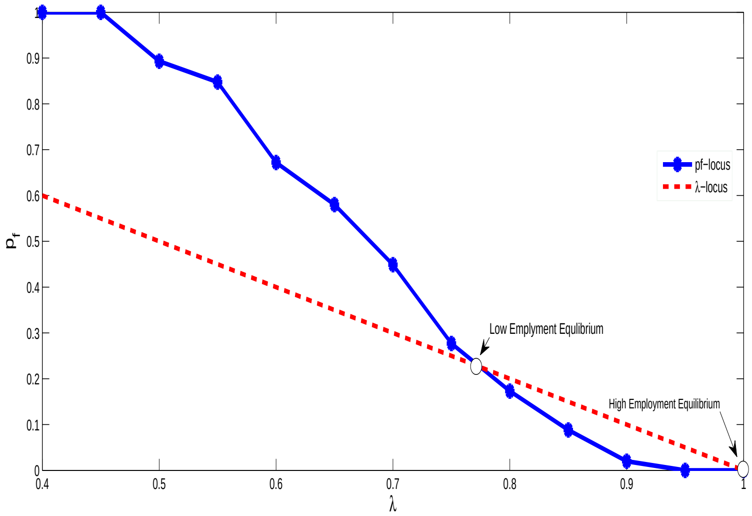

Figure 3: Equilibria: Numerical Example

non-discriminating equilibria, in which the saving rate is low and employment high; and a discriminating equilibria in which the saving rate is high and employment is low. For low values of λ (off-equilibrium) the rational expectation of firms about workers of type f quickly rises to one, when no family ever chooses two have both members employed. Although I am unable to prove that the number of equilibria is at most two, for all parametrizations considered the number of equilibria was at most two.

4 Some Empirical Evidence

A natural interpretation of the model examined in this paper is that one reason why there are persistent differences in saving rates across countries is that the equilibrium levels of employment differ across them. Moreover, these differences in employment are to a large extent accounted for differences in female labor force participation across countries. In this section I test whether these differences can help explaining cross-country variations in national saving rates, as predicted by the model developed in the paper.

I estimate panel regressions over twenty-two countries including all the major OECD countries,[7] between 1980 and 2006. The dependent variable is net national saving divided by net national product. Unlike private saving, national saving is invariant with respect to inflation-induced transfers between the private and the public sector. The baseline specification includes the per capita gross domestic product (GDP) level and growth rate, the inflation rate measured using the GDP deflator, the dependency ratio and finally the female labor force participation rate. Moreover, in all specifications I have included an autoregressive term to capture the dynamics of saving and time dummies, to control for time-specific effects.

The panel regression coefficient's estimates are shown in table 2. Column (i) shows the results from the OLS regression. The coefficient of GDP is positive and significant. The coefficient of GDP growth is positive and significant, consistent with the standard growth theory. The coefficient of the dependency ratio is small and not significantly different from zero. The coefficient of the female participation ratio is negative and precisely estimated, supporting the main prediction of the model. Column (ii) is identical to column (i) except for the inclusion of fixed country effects. The results are appreciably different, however. In particular, the coefficient of the female participation ratio is estimated to be -0.0368 with a t statistic of

[7] see table 3 in appendix for the list of countries.

Table 2: Baseline regressions

Variable	i. OLS	ii. Fixed Effect	iii. Arellano-Bond
Female Participation	-0.0459^{**}	-0.0368	-0.3335^{*}
	(-3.40)	(-0.80)	(-2.10)
log GDP	2.5666^{**}	7.3222^{**}	18.2312^{**}
	(3.33)	(2.85)	(3.67)
GDP growth	0.3938^{**}	0.4979^{**}	0.4453^{**}
	(5.85)	(6.75)	(5.39)
GDP Deflator	0.0010	$0.0163^{\dagger}$	-0.1094^{*}
	(0.43)	(1.83)	(2.52)
Dependency Ratio	0.0145	$0.1285^{\dagger}$	0.4514^{**}
	(0.43)	(7.45)	(4.29)
Lagged Net Saving	0.7864^{**}	0.5911^{**}	0.5117^{**}
	(28.01)	(16.70)	(12.90)
R^2:	0.49	0.51	-
Number of observations:	537	537	537
Number of countries:	22	22	22
Significance levels : $\dagger$: 10% $*$: 5% $**$: 1%			

Note: The dependent variable is net national savings as a % of gross national income. In parenthesis are t-values. The specification includes an intercept and timme dummies.

-0.80; thus it is statistically insignificant although preserving the sign predicted by the model and that found in the pooled OLS regression of column (i).

However, in the regression in column (ii) because the lagged saving rate is mechanically correlated with the error term, the standard fixed-effect estimation is not consistent in panels with a short time dimension. To deal with this problem, in column (iii) I use the generalized method-of-moments estimator (GMM) developed by Arellano and Bond (1991). The estimate for the coefficient of female participation is again negative, as predicted by the model, and statistically significant at the five percent level.

5 Conclusion

This paper investigates the interaction between equilibrium employment and aggregate saving in an environment in which families can choose to have one or both members employed. The main result is that because of information asymmetries and the presence of fixed costs, there can be multiple equilibria. In particular we may have one equilibrium with low employment and high aggregate saving and another one with high employment and low aggregate saving.

The intuition for this result is simple. In the equilibrium with high employment many families have both members employed and moreover the within-family wage gap is smaller; consequently, families are less exposed to idiosyncratic risk and accumulate less precautionary balances. Hence aggregate saving is low. In contrast, in the low employment equilibrium, families are very exposed to idiosyncratic risk and, consequently, aggregate saving is high.

In the equilibrium with high employment and low aggregate savings, firms are as well off as they would be in the low employment/high savings equilibrium, because of the free entry condition. However, in the high employment equilibrium, households are better off because they solve the same inter-temporal problem but wages are higher because of the lower gender wage gap. Therefore, the multiple equilibria can be Pareto ranked, and the paper thus offers insights useful for policy-makers. In light of this result, one promising avenue for research concerns issues of optimal taxation in contexts where labor markets are characterized with frictions.

Finally, one natural interpretation of the model is that in countries where female labor force participation is low the aggregate saving rate is high. This prediction is empirically tested using panel regressions including all the major OECD countries and the empirical evidence supports the prediction of the model. Thus, the results in this paper suggest that to understand persistent international differences in aggregate saving it is important to consider the interaction between the labor market equilibrium and families' saving behavior.

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A Appendix

Table 3: OECD Net Saving and Female Participation

Country	**Net Saving**	**Female Participation**
Australia	5.913	60.526
Austria	8.330	57.437
Belgium	8.133	50.148
Canada	7.244	67.088
Denmark	5.799	75.066
Finland	7.026	70.959
France	7.307	58.740
Germany	5.770	63.627
Greece	12.993	45.600
Iceland	4.300	78.618
Ireland	13.180	46.881
Italy	8.249	44.067
Japan	13.724	57.430
Netherlands	10.758	57.252
New Zealand	4.467	65.300
Norway	13.018	71.281
Portugal	5.265	60.393
Spain	8.127	44.444
Sweden	8.459	77.804
Switzerland	15.263	64.657
United Kingdom	3.737	66.193
United States	4.510	67.214

Source: World Bank Development Indicators. Averages for the sample period 1980-2006.