Trade and Synchronization in a Multi-Country Economy

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Abstract

Countries with strong trade linkages have more synchronized business cycles. However, the standard international business cycle framework cannot replicate this finding, uncovering the trade-comovement puzzle. Modeling trade using more sophisticated micro-level assumptions does not help resolve the puzzle. This happens because for a large class of trade models, under certain macro-level conditions, output comovement is determined by the same factor structure. We show that in such models comovement can be explained by three factors: (i) the correlation between each country’s TFP; (ii) the correlation between each country’s share of expenditure on domestic goods; and (iii) the correlation between each country’s TFP and the partner’s share of expenditure on domestic goods. An empirical investigation of the link between trade and each of the three factors shows that the trade-comovement relation is in large part explained by the second factor while in the theoretical model this factor reacts counterfactually to changes in trade costs.

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1 Introduction

Substantial empirical evidence suggests that countries or regions with stronger trade linkages have more correlated business cycles. Frankel and Rose (1998), Clark and van Wincoop (2001), Calderon, Chong, and Stein (2007), Baxter and Kouparsas (2004), and Imbs (2004), among others, show that pairs of countries that trade with each other exhibit a high degree of business cycle comovement. These findings have been interpreted as evidence that greater trade integration leads to business cycle synchronization. However, from a theoretical perspective the standard international real business cycle model (IRBC), based on Backus, Kehoe, and Kydland (1994), has difficulties in replicating this empirical fact (see Kose and Yi, 2001 and 2006). In the latter paper, the authors’ baseline model explains only one-tenth of the responsiveness of comovement to trade intensity. This has given rise to the so-called trade-comovement puzzle: standard models are unable to generate high output correlations arising from high bilateral trade intensity.

In the conventional IRBC framework, trade is modeled using the Armington specification, which imposes an exogenous trade specialization pattern. In the Armington framework trade adjustments are only at the intensive margin. By contrast, in the new trade theory, trade shares also adjust at the extensive margin: when a country’s relative efficiency declines it exports a narrower range of goods. We show that modeling trade linkages using more sophisticated micro-level assumptions does not help resolve the trade-comovement puzzle. This happens because output comovement in an important class of trade models is determined by the same factor structure.

The main contribution of this paper is to show that under certain macro-level assumptions, in a large class of trade models, output correlation between country $i$ and country $j$, denoted $\text{cor} \left( \tilde{\mathcal{Y}}_i, \tilde{\mathcal{Y}}_j \right)$, is explained by three factors: (i) the correlation between each country’s total factor productivity (TFP), denoted $\text{cor} \left( \tilde{\mathcal{A}}_i, \tilde{\mathcal{A}}_j \right)$; (ii) the correlation between each country’s share of expenditure on domestic goods, $\text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right)$; and (iii) the correlation between a country’s share of expenditure on domestic goods and the partner’s TFP, $\text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\mathcal{A}}_j \right)$. This relation is summarized by the following equation:

$$\text{cor} \left( \tilde{\mathcal{Y}}_i, \tilde{\mathcal{Y}}_j \right) = \beta_1 \text{cor} \left( \tilde{\mathcal{A}}_i, \tilde{\mathcal{A}}_j \right) + \beta_2 \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) + \beta_3 \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\mathcal{A}}_j \right).$$

(1)

Therefore, the ability of a model to generate higher output synchronization arising from increased
bilateral trade depends on the extent to which trade integration affects each of these three factors. This result relates to recent work by Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008) and Arkolakis, Costinot, and Rodriguez-Clare (2012) concerning the welfare gains from trade. These authors show that the real wage (which determines the welfare gains from trade) can be computed as a function of the import penetration ratio and an elasticity parameter that, depending on the particular micro-level assumptions, relates either to preferences or technology. In particular, Arkolakis et al. (2012) show that the gains from trade have the same form in a large class of trade models including the Armington model, Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), Krugman (1980), and multiple versions of Melitz (2003).

In the context of the IRBC model that concerns us, the labor supply responds to changes in the real wage (which is a function of the share of expenditure on domestic goods). It follows that, in the absence of short-run wealth effects on the labor supply, the share of expenditure on domestic goods, a parameter related to the labor supply elasticity and the trade elasticity are the only determinants of employment and output fluctuations in response to foreign shocks.

Using data on bilateral trade in manufacturing, TFP and output correlations for a panel of 21 OECD countries we show that the two factors that explain the trade-comovement relation empirically are the correlation between each country-pair’s TFP and the correlation between each country-pair’s share of expenditure on domestic goods.

To investigate the quantitative implications of this result, our starting point is a multi-country model of international trade with endogenous specialization inspired by Eaton and Kortum (2002). We embed it into an IRBC framework by including country specific fluctuations in TFP, allowing for endogenous labor supply, inter-temporal substitution in consumption and complete financial markets. We show that in this framework the trade-comovement puzzle arises because higher trade integration counterfactually reduces the correlation between each country’s share of expenditure on domestic goods and fails to substantially increase the correlation between each country’s share of expenditure on domestic goods and the trade-partner’s TFP.

When we allow the correlation of TFP shocks to increase with trade, as observed in the data, the model is more successful in replicating the link between trade and comovement. However, we show
that even in the presence of correlated shocks, the model still implies a counterfactual relation between bilateral trade and each country pair’s correlation of the share of expenditure on domestic goods. The reason for this result is twofold. First, a TFP shock implies a deterioration in the terms-of-trade. Second, the elasticity of the share of expenditure on domestic goods to changes in the terms-of-trade is increasing in the level of bilateral trade. Therefore, following a TFP shock at Home, the Foreign share of expenditure on domestic goods will fall by more, the higher the level of trade integration between Home and Foreign.

In our model, Home TFP improvements are transmitted positively to the trade-partners through a deterioration of Home’s terms-of-trade. This is a common feature in most IRBC models. However, this feature is shown to be counterfactual for many countries. For instance, Corsetti, Dedola and Leduc (2008 and 2014) show that the terms-of-trade improve conditional on positive TFP shocks, in particular in large countries and relatively less open economies. Raffo (2008) shows that, contrary to what is implied by the standard IRBC model, the terms-of-trade do not have a strong cyclical pattern. We show that with countercyclical terms-of-trade it is not possible for trade integration to raise the correlations of the share of expenditure in domestic goods. Therefore, this provides more evidence in favor of models with acyclical or procyclical terms-of-trade. Ghironi and Melitz (2005) and also Corsetti et al. (2008) develop models that imply procyclical terms-of-trade conditional on positive TFP shocks.

Our paper is related to a strand of the literature that extends the IRBC model by changing the micro-level assumptions about trade to improve the model’s ability to explain the empirical association between trade and business cycle synchronization. Burstein, Kurz, and Tesar (2008) highlight the role of vertical specialization and show that countries with tighter links in the chain of production exhibit higher bilateral manufacturing output correlations. Arkolakis and Ramanarayanan (2009) develop a two-country international business cycle model augmented with vertical specialization. They conclude that vertical specialization alone is insufficient to solve the trade-comovement puzzle and suggest that allowing for variable markups may be helpful.

In a related study, di Giovanni and Levchenko (2010) emphasize the empirical relevance of vertical linkages in production to explain the effect of bilateral trade on business cycle synchronization. However, Johnson (2014) finds evidence against the idea that input trade is the missing link to
understand the trade-comovement relation, as comovement in value added is not greatly increased with trade integration. Finally, Liao and Santacreu (2014) build a model based on Ghironi and Melitz (2005), where the dynamics of TFP are driven by extensive margin changes in the number and average productivity of domestic and foreign intermediate producers. There model is able to produce endogenous TFP correlations consistent with the data.

The rest of the paper is organized as follows. Section 2 describes our equilibrium model of trade and the business cycle used to analyze the relation between trade integration and business cycle synchronization. In Section 3, we study in depth the channels through which trade affects business cycle synchronization and, in particular, derive equation (1) that represents the factor structure of GDP comovement. Section 4 presents the main empirical and quantitative findings of the paper. Finally, Section 5 concludes.

2 The Theoretical Economy

In this Section, we develop a simple model of the world economy to study the link between trade integration and business cycle synchronization. The setup of the model builds on Eaton and Kortum (EK, 2002) and Alvarez and Lucas (2007), but it incorporates intertemporal substitution in consumption and endogenous labor supply. The world economy consists of \( n \) countries. Each country is populated with a continuum (unit measure) of identical households. At each date \( t \), the world economy experiences one of finitely many states, or events, \( s_t \in \mathcal{S} \equiv [s(1), \ldots, s(k)] \). We denote by \( s^t = (s_0, \ldots, s_t) \) the history of events through period \( t \). The probability of any particular event \( s_{t+1} \) conditional on history \( s^t \) is \( \pi(s_{t+1}|s^t) \). The initial realization \( s_0 \) is given.

2.1 Technology and Market Structure

Each country has two sectors: a traded intermediate good sector and a non-traded final good sector. The intermediate good sector produces a continuum of traded differentiated inputs using labor. The final good sector is represented by a stand-in firm that produces a non-traded good that is a composite of the intermediate varieties. All markets are perfectly competitive. In what follows we describe each sector in greater detail.
Final Good Sector

The stand-in final good firm in country $i$ makes use of a continuum of differentiated manufactured intermediate commodities indexed by $v \in [0,1]$ that are combined as follows:

$$Y_i(s^t) = A_i(s^t) \left[ \int_0^1 y_i(v,s^t) \phi dv \right]^{1/\phi},$$

(2)

where $y_i(v,s^t)$ is the input of the differentiated intermediate commodity of type $v$. The parameter $\phi \in (0,1)$ relates to the elasticity of substitution across differentiated intermediate commodities, given by $\sigma = 1/(1-\phi)$. The productivity level $A_i(s^t)$ follows a mean one, serially correlated discrete Markov process and is independent across countries. From (2), it follows that the demand in country $i$ for intermediate variety $v$ satisfies the relation

$$y_i(v,s^t) = \left[ \frac{p_i(v,s^t)}{P_i(s^t)} \right]^{-\sigma} \frac{Y_i(s^t)}{A_i(s^t)},$$

(3)

where $P_i(s^t) = \left[ \int_0^1 p_i(v,s^t) 1-\sigma dv \right]^{1/(1-\sigma)}$ is the ideal price index in country $i$ of the composite of intermediate commodities, with $p_i(v,s^t)$ the price of intermediate variety $v$ in country $i$. The upshot is that the price of the final good in country $i$ is given by

$$\mathcal{P}_i(s^t) = P_i(s^t)/A_i(s^t).$$

(4)

Intermediate Good Sector

The structure of the intermediate-good sector is as in EK where technology differences are modeled using a probabilistic approach. Countries have differential access to technology, so efficiency varies across commodities and countries. The intermediate commodities are subject to trade barriers that take the form of an iceberg cost: to successfully deliver in country $j$ one unit of any differentiated intermediate commodity produced in country $i$, $\tau_{ji} \geq 1$ units need to be shipped, with $\tau_{ii} = 1$.

Producing one unit of the intermediate commodity $v$ in country $i$ requires $\varphi^{-1}_{i,v}$ units of labor. Therefore, the cost for intermediate firms in country $i$ to deliver one unit of intermediate commodity
The ideal price index in country $j$ is
\[ P_j (s^t) = \kappa \Phi_j (s^t)^{-1/\theta}, \] (8)
where $\kappa > 0$ is a constant and the random variable and
\[ \Phi_j (s^t) = \sum_{i=1}^{n} \left[ W_i (s^t) \tau_{ji} \right]^{-\theta}, \] (9)
determines the distribution of prices.$^1$

\section*{Bilateral Trade Flows}

The probability $\lambda_{ji}$ that country $i$ is the lowest-cost supplier to $j$ for any particular intermediate commodity is given by$^2$
\[ \lambda_{ji} (s^t) = \left[ \frac{\kappa W_i (s^t) \tau_{ji}}{P_j (s^t)} \right]^{-\theta}. \] (10)

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$^1\kappa = \left[ \Gamma \left( \frac{1-\sigma+\theta}{\sigma} \right) \right]^{1/(1-\sigma)}$, where $\Gamma(.)$ is the Gamma function.

$^2$This probability is obtained by calculating $\lambda_{ji} (s^t) = \text{Prob} \left( p_{ji} (v, s^t) \leq \min \left[ p_{jl} (v, s^t); l \neq i \right] \right)$. 

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$v$ to country $j$ is
\[ p_{ji} (v, s^t) = \left[ \frac{W_i (s^t)}{\varphi_{i,v}} \right] \tau_{ji}, \] (5)
where $W_i (s^t)$ is the wage rate in country $i$. There is perfect competition, so country $i$ firms potentially sell the commodity $v$ to country $j$ at price $p_{ji} (v, s^t)$. The commodity is purchased from the lowest-cost supplier; hence, the price of commodity $v$ in country $j$ is given by
\[ p_j (v, s^t) = \min_{i=1,\ldots,n} \left[ p_{ji} (v, s^t) \right]. \] (6)

The variable $\varphi_{i,v}$ determines the efficiency of country $i$ to produce input $v$. We follow EK and model firms’ efficiency using a probabilistic approach: it is assumed that country $i$’s efficiency in producing commodity $v$ is the realization of a random variable $\varphi$, which is drawn independently for each $v$. Country $i$’s efficiency follows a Fréchet distribution:
\[ F_i (\varphi) = \exp \left( -\varphi^{-\theta} \right), \] (7)
where $0 \leq \varphi$. The parameter $\theta > 1$ controls the degree of heterogeneity across firms, with higher $\theta$ implying less heterogeneity. The upshot is that in equilibrium the ideal price index in country $j$ is
Since there are a continuum of intermediate goods, the probability $\lambda_{ji}$ also corresponds to country $j$’s expenditure on country $i$’s differentiated intermediate goods as a fraction of country $j$’s total expenditure on differentiated intermediate goods. This measure of bilateral trade intensity is closely linked to one of the measures proposed by Frankel and Rose (FR, 1998), which is the sum of a country’s bilateral exports divided by the sum of each country’s aggregate net income.

2.2 Preferences and Financial Markets

The stand-in household in country $i$ has preferences represented by a utility function of the form introduced by Greenwood et al. (1988), given by

$$u(C_i, L_i; s^t) = \ln \left[ \frac{C_i(s^t) - L_i(s^t)\left(\frac{1+1/\sigma_n}{1+1/\sigma_n}\right)}{1+1/\sigma_n} \right],$$

(11)

where $C_i(s^t)$ and $L_i(s^t)$ are, respectively, consumption and time spent working by the stand-in household. The parameter $\sigma_n$ corresponds to the Frisch elasticity of labor supply. The choice of preferences does not have wealth effects and therefore excludes intertemporal substitution in the labor choice.\(^3\)

There are complete financial markets. In particular, we assume the existence of a complete set of Arrow securities denominated in units of the numéraire. Thus, the budget constraint of the stand-in agent in country $i$ is

$$\mathcal{P}_i(s^t) C_i(s^t) + \sum_{s \in S} Q(s^t, s) B_i(s^t, s) = W_i(s^t) L_i(s^t) + B_i(s^{t-1}, s_t),$$

(12)

where $B_i(s^t, s)$ denotes the quantity of bonds purchased by the stand-in household in country $i$ following history $s^t$, that entitles the holder to receive a payment worth one unit of the numéraire in period $t + 1$ if the state of the economy at that date turns out to be $s \in S$. The price of this state contingent security in units of the numéraire is denoted $Q(s^t, s)$. The Bellman equation

\(^3\)Jaimovich and Rebelo (2009) find evidence of a weak wealth effect in labor supply choices. Raffo (2008) shows that this choice of preferences in IRBC models helps reproducing the cyclical properties of quantities, including the correlation and volatility of trade variables with respect to output.
characterizing the stand-in household optimal behavior is

\[ V_i(s^t) = \max_{C_i, L_i} \left[ u(C_i, L_i; s^t) + \beta \sum_{s_{t+1} \in S} \pi(s_{t+1} | s^t) V_i(s^{t+1}) \right], \tag{13} \]

with \( \beta \in (0, 1) \) and subject to the budget constraint (12).

### 2.3 Equilibrium Conditions

The first-order conditions that characterize the solution to the problem of the stand-in household in country \( i \) are given by

\[ Q(s^t, s_{t+1}) \mu_i(s^t) = \beta \pi(s_{t+1} | s^t) \mu_i(s^{t+1}) \quad \forall s_{t+1} \in S, \tag{14} \]

\[ \mu_i(s^t) = \beta R_t \sum_{s_{t+1} \in S} \pi(s_{t+1} | s^t) \mu_i(s^{t+1}), \tag{15} \]

\[ L_i(s^t) = \left[ \frac{W_i(s^t)}{\mathcal{P}_i(s^t)} \right]^{\sigma_n}, \tag{16} \]

with \( R_t = \left[ \sum_{s \in S} Q(s^t, s) \right]^{-1} \) and where

\[ \mu_i(s^t) = \frac{1}{\mathcal{P}_i(s^t)} \left[ C_i(s^t) - \frac{L_i(s^t)^{1+1/\sigma_n}}{1 + 1/\sigma_n} \right]^{-1}, \tag{17} \]

represents the marginal utility of current per capita wealth in units of the numéraire for the stand-in household in country \( i \) following history \( s^t \).

By combining equations (4) and (10), the real wage in country \( i \) can be expressed in terms of the technology shock \( A_i \) and the share of expenditure on domestic goods \( \lambda_{ii} \), as follows

\[ \frac{W_i(s^t)}{\mathcal{P}_i(s^t)} = \frac{A_i(s^t)}{\kappa} \left[ \frac{1}{\lambda_{ii}(s^t)} \right]^{1/\theta}. \tag{18} \]

Substituting for the real wage in (16) using (18), it follows that equilibrium hours worked can be

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expressed as

\[ L_i \left( s^t \right) = \left[ \frac{A_i \left( s^t \right)}{\kappa} \right]^\sigma_n \left[ \frac{1}{\lambda_{ii} \left( s^t \right)} \right]^{\sigma_n / \theta}. \tag{19} \]

The market clearing condition in financial markets requires

\[ \sum_{i=1}^n B_i \left( s^t, s \right) = 0, \quad \forall s \in S. \tag{20} \]

Finally, equilibrium in the market for produced goods in each country \( i \) requires total domestic labor income \( W_i \left( s^t \right) L_i \left( s^t \right) \) to equal world spending on domestically produced goods, so that

\[ W_i \left( s^t \right) L_i \left( s^t \right) = \sum_{j=1}^n \lambda_{ji} \left( s^t \right) \left[ W_j \left( s^t \right) L_j \left( s^t \right) + D_j \left( s^t \right) \right], \tag{21} \]

where

\[ D_j \left( s^t \right) = B_j \left( s^{t-1}, s_t \right) - \sum_{s \in S} Q \left( s^t, s \right) B_j \left( s^t, s \right), \tag{22} \]

is country \( j \)'s trade deficit. Of course, the market clearing condition (20) implies \( \sum_{i=0}^n D_i \left( s^t \right) = 0 \).

Finally, using (22) to substitute in (12) we have

\[ \mathcal{P}_i \left( s^t \right) C_i \left( s^t \right) = W_i \left( s^t \right) L_i \left( s^t \right) + D_j \left( s^t \right). \tag{23} \]

**Sequential Competitive Equilibrium**

Define the \( n \times 1 \) wage vector \( \mathcal{W}_t = [W_1 \left( s^t \right), \ldots, W_n \left( s^t \right)]' \), the \( k \times 1 \) security price vector \( Q_t = [Q_1 \left( s^t, s \left( 1 \right) \right), \ldots, Q_k \left( s^t, s \left( k \right) \right)]' \), the \( n \times 1 \) consumption vector \( C_t = [C_1 \left( s^t \right), \ldots, C_n \left( s^t \right)]' \), the \( n \times 1 \) employment vector \( \mathbb{L}_t = [L_1 \left( s^t \right), \ldots, L_n \left( s^t \right)]' \), and the \( n \times k \) matrix that describes the security holdings \( \mathbb{B}_t = [\mathcal{B}_1 \left( s^t \right), \ldots, \mathcal{B}_n \left( s^t \right)]' \), where \( \mathcal{B}_i \left( s^t \right) = [B_i \left( s^t, s \left( 1 \right) \right), \ldots, B_i \left( s^t, s \left( k \right) \right)]. \)

A sequential competitive equilibrium in the world economy is an allocation \( \{[C_t, \mathbb{L}_t, \mathbb{B}_t]\}_{t=0}^{\infty} \), a wage sequence \( \{\mathcal{W}_t\}_{t=0}^{\infty} \) and a price sequence for the Arrow securities \( \{Q_t\}_{t=0}^{\infty} \) such that at every date \( t \):

- the stand-in household in each country behaves optimally given wages and prices; condition (20) is satisfied, so that financial markets clear; condition (21) is satisfied, so that product markets clear.

In the sequel, we consider a symmetric world economy with ex-ante identical countries, so that we
have \( \mu_i(s^0) = \bar{\mu} \) for all \( i = 1, \ldots, n \). The upshot is that, with complete markets, equation (14) implies that, for each history \( s^t \), \( \mu_i(s^t) = \mu_t \) for all \( i, \ldots, n \). This corresponds to the familiar risk-sharing condition conditional on trade costs: the relative marginal utility of consumption between country \( i \) and country \( j \) is equal to their consumption based real exchange rate, as follows\(^4\)

\[
\frac{u'_c(C_i, \ell_i; s^t)}{u'_c(C_j, \ell_j; s^t)} = \frac{P_i(s^t)}{P_j(s^t)}.
\] (24)

Finally, from (15), \( \mu_t \) must satisfy the familiar Euler equation

\[
\mu_t = \beta R_t E_t (\mu_{t+1}),
\] (25)

with \( E_t (\mu_{t+1}) = \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) \mu_{t+1} \), the conditional expectation of \( \mu_{t+1} \) at date \( t \).

Following standard steps, the individual optimality conditions and the market clearing conditions are log-linearized around the symmetric perfect-foresight steady state equilibrium and combined so as to characterize the equilibrium dynamics.\(^5\)

### 3 Trade and the Channels of Synchronization

In this Section we use the equilibrium conditions obtained in Section 2 to better understand the nature of the trade-comovement puzzle. We begin by constructing a measure of real GDP that is consistent with the methodology used by the National Accounts. Then, we show that comovement in real GDP can be explained by three factors: the correlation between each country’s TFP; the correlation between each country’s share of expenditure on domestic goods; and the correlation between each country’s TFP and the partner’s share of expenditure on domestic goods.

\(^4\)Fitzgerald (2012) tests and fails to reject the hypothesis of optimal consumption risk-sharing conditional on trade costs among developed countries in the context of a gravity model with intertemporal trade, similar to the one developed here.

\(^5\)Alvarez and Lucas (2007) establish a very generic result for existence and uniqueness of equilibrium with balanced trade in a model with the EK structure. They also develop a tatonnement algorithm to numerically solve for the equilibrium. In our framework, thanks to the symmetry assumption, it is possible to solve analytically for the steady state equilibrium. See Appendix A for details. The log-linear model is described in Appendix B.
3.1 GDP at Constant Prices

The construction of a measure of real GDP consistent with the National Accounts requires the computation of value added at constant prices. Arkolakis and Ramanarayanan (2009) show that measuring real GDP correctly implies that comovement in total factor productivity (TFP) across countries is unrelated to the degree of trade integration. Their result has important implications for our analysis as we show below.

Using the fact that the value added by each intermediate good firm evaluated at current prices is equal to that firm’s wage bill, so that \( p(v, s^t) y_i (v, s^t) = W_i (s^t) L_i (v, s^t) \), GDP at constant prices in country \( i \) is given by

\[
Y_i (s^t) = \int_{\Omega_i (s^t)} \left[ \frac{A_i (s^t) W_i (s^0)}{A_i (s^0) W_i (s^t) / \varphi_{i,n}} \right] W_i (s^t) L_i (v, s^t) dv,
\]

where \( \Omega_i (s^t) \) is the set of differentiated intermediate goods produced in country \( i \) in period \( t \), \( L_i (v, s^t) \) is employment in industry \( v \) and \( p_i (v, s^0) \) is the price of the intermediate variety \( v \) in period 0, which is chosen as the base year to compute GDP at constant prices.

Of course, some goods produced by country \( i \) in period \( t \) may not have been produced in period 0 and therefore \( \Omega_i (s^t) \neq \Omega_i (s^0) \). For such goods, \( p_i (v, s^0) \) is undefined. One way to resolve this problem is to use the price at which country \( i \) would have been able to sell the good to itself, had it not imported it, \( p_{ii} (v, s^0) \). This interpretation seems the most consistent with the recommendation by the UN System of National Accounts 1993 (SNA 93), which suggests to use the average price changes of similar products as a proxy for the change in price of the new good. Following this convention and making use of (5), equation (26) can be written as follows

\[
Y_i (s^t) = A_i (s^t) \int_{\Omega_i (s^t)} L_i (v, s^t) dv,
\]

\[
= A_i (s^t) L_i (s^t),
\]

\[
\text{(27)}
\]

\[\text{\footnotesize{6}}\]Kehoe and Ruhl (2008) demonstrate a similar result.

\[\text{\footnotesize{7}}\]Following Arkolakis and Ramanarayanan (2009), an alternative would be to use the price at which the good was imported at date 0. The two approaches yield the same conclusion about the muted role of trade in measured TFP.
where we use the labor market clearing condition and set \( W_i(s^0) / A_i(s^0) = 1 \), without loss of generality. The upshot is that measured total factor productivity is given by

\[
\text{TFP}_{i,t} = \frac{\mathcal{Y}_i(s^t)}{L_i(s^t)} = A_i(s^t),
\]

and, hence, it is independent from changes in the terms-of-trade or the level of trade integration.

Finally, combining (27) with (19) we obtain

\[
\mathcal{Y}_i(s^t) = \kappa - \sigma_n A_i(s^t)^{1+\sigma_n} \left[ \frac{1}{\lambda_{ii}(s^t)} \right]^{\sigma_n/\theta}.
\]

(29)

### 3.2 Factor Structure of GDP Comovement

From equation (29) it follows that real GDP fluctuations in country \( i \) (in log deviations from steady state) are given by

\[
\tilde{Y}_{i,t} = (1 + \sigma_n) \tilde{A}_{i,t} - \left( \frac{\sigma_n}{\theta} \right) \tilde{\lambda}_{ii,t}.
\]

(30)

Expression (30) implies that the degree of comovement between any country-pair depends on the correlation between each country’s productivity levels \( \tilde{A}_{i,t} \) and on the correlation between each country’s share of expenditures on domestic goods \( \tilde{\lambda}_{ii,t} \) (which, in log-deviation from steady state, is equal to the negative of the import penetration ratio). It turns out that expression (30) holds for a large class of trade models, as we establish in Result 1:

**Result 1** Suppose the following macro-level assumptions are satisfied:

1. Profits are a constant share of revenue;
2. The import demand system exhibits constant elasticity of substitution (CES);
3. Labor supply choices are independent of wealth.

It follows that, irrespectively of the micro-level assumptions about trade, output fluctuations are given by equation (30).

See appendix C for proof.
This result builds on the work of Arkolakis et al. (2012), who show that the predictions of a large class of trade models concerning the change in real income associated with any foreign shock only depend on the import penetration ratio and the trade elasticity. The relevant class of models is large and includes many well-known trade models such as the Armington model, Eaton and Kortum (2002), Bernard et al. (2003) extension of EK to imperfect competition, Krugman (1980), and multiple versions of Melitz (2003).

From equation (30), we obtain the following three-factor model for the output correlation between countries $i$ and $j$

\[
\text{cor} \left( \tilde{Y}_i, \tilde{Y}_j \right) = \beta_1 \text{cor} \left( \tilde{A}_i, \tilde{A}_j \right) + \beta_2 \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) + \beta_3 \text{cor} \left( \tilde{\lambda}_{ii}, A_j \right),
\]

(31)

where \( \text{cor} \left( x, z \right) \) denotes the correlation between two variables \( x \) and \( z \). The coefficients \( \beta_1 \) and \( \beta_2 \) are positive while \( \beta_3 \) is negative.\(^8\) The equation implies three channels through which trade can increase business cycle synchronization, summarized in the following result:

**Result 2** The output correlation for each country-pair may be expressed as the sum of three factors, as in equation (31). It follows that there are three channels through which an increase in bilateral trade may increase business cycle synchronization: (i) increased bilateral trade resulting in a higher correlation between each country’s technology shocks; (ii) increased bilateral trade resulting in a higher correlation between each country’s share of expenditure on domestic goods; and (iii) increased bilateral trade raising the correlation between the share of expenditure on domestic goods and foreign technology shocks.

See appendix D for proof.

\(^8\)The factor loadings are given by

\[
\beta_1 \equiv (1 + \sigma_n)^2 \left[ \frac{\text{var} \left( \tilde{A}_i \right)}{\text{var} \left( \tilde{Y}_i \right)} \right], \quad \beta_2 \equiv \left( \frac{\sigma_n}{\theta} \right)^2 \left[ \frac{\text{var} \left( \tilde{\lambda}_{ii} \right)}{\text{var} \left( \tilde{Y}_i \right)} \right] \quad \text{and}
\]

\[
\beta_3 \equiv -2(1 + \sigma_n) \left( \frac{\sigma_n}{\theta} \right) \left[ \frac{\text{std} \left( \tilde{A}_i \right) \text{std} \left( \tilde{\lambda}_{ii} \right)}{\text{var} \left( \tilde{Y}_i \right)} \right].
\]

where \( \text{var} \) and \( \text{std} \) denote, respectively, the variance and standard deviation of a variable \( x \). See Appendix D for detailed derivations.
Equation (31) and Result 2 provide the basis for the empirical analysis that follows. We use bilateral trade data on manufactures, manufacturing output and employment for a panel of 21 OECD countries over the period 1988 – 2007. The share of expenditure on domestic goods \( \lambda_{ii} \) (equivalently, one minus the import penetration ratio) can be calculated from bilateral trade data. We compute TFP for each country as the ratio between real manufacturing GDP and employment in manufacturing, calculate the correlations for each term of equation (31) and estimate the three-factor structure. We evaluate our model by testing if the factor loadings that we estimate from the data have the expected sign and are statistically significant, and by judging the model’s goodness of fit.

If the fit of the empirical model is judged to be good, we can dissect the trade-comovement puzzle by examining carefully the channels through which trade leads to business cycle synchronization in the theoretical model and contrast it with the empirical results. In particular, by inspecting how bilateral trade intensity affects each of the three factors we are able to identify if country-pairs that trade more experience stronger output comovement because of: (i) higher correlation between each country’s TFP; (ii) higher correlation between each country’s share of expenditure on domestic goods; or (iii) higher correlation between the share of expenditure on domestic goods and foreign TFP.

Finally, notice that the three factors are not independent. In particular, an increased correlation between each country’s TFP may imply an increased correlation between each country’s share of expenditure on domestic goods. We investigate this possibility using the theoretical model and allowing for TFP correlation to vary with the level of trade integration in a way that is consistent with the data. The next Section describes the quantitative results.

4 Quantitative Results

This Section examines the ability of our model to replicate the trade-comovement relation by confronting the model with the data. We first estimate a regression in the spirit of FR using both empirical data and simulated data from the theoretical model. To this aim, we solve the model for a world economy where the number of countries N is set equal to four, implying six distinct
Table 1: Descriptive statistics (empirical data)

<table>
<thead>
<tr>
<th></th>
<th>Bilateral trade</th>
<th>Detrended GDP correlation</th>
<th>Detrended TFP correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.001</td>
<td>−0.557</td>
<td>−0.636</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.388</td>
<td>0.991</td>
<td>0.900</td>
</tr>
<tr>
<td>Median</td>
<td>0.015</td>
<td>0.645</td>
<td>0.350</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.042</td>
<td>0.390</td>
<td>0.377</td>
</tr>
</tbody>
</table>

The number of observations is 210 country-pairs. See Appendix E for details. GDP and TFP are linearly detrended.

country-pairs. This allows us to construct a symmetric world economy matching the minimum, maximum and median bilateral trade flows observed in the data and reported in Table 1. In the baseline model, the correlation of the TFP shocks is fixed for all country pairs at a level that matches the median correlation observed in the empirical data (the details about the calibration are explained in Appendix F).

After establishing that our model with uncorrelated technology shocks is unable to replicate the link between trade and comovement observed in the data we proceed in two steps. First, we dissect the relation between trade and business cycle synchronization by analyzing the three-factor model of equation (31) and assessing how trade affects each factor. Second, we allow for the presence of endogenous TFP correlation and show that although the model is now more successful to explain the trade-comovement relation, it still implies a counterfactual relation between trade integration and \( \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) \).

4.1 The FR Regression

As a starting point we estimate by ordinary least squares (OLS) the following regression in the spirit of FR using both empirical and simulated data:

\[
\text{cor} \left( \tilde{Y}_i, \tilde{Y}_j \right) = b_0 + b_1 \log (\text{Bilateral Trade})_{ij} + \varepsilon_{ij},
\]  

(32)
Table 2: Frankel and Rose regressions (data and model)

<table>
<thead>
<tr>
<th>dependent variable: cor($\tilde{Y}_i, \tilde{Y}_j$)</th>
<th>empirical data</th>
<th>model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.</td>
<td>2.</td>
</tr>
<tr>
<td>log(Bilateral Trade)</td>
<td>0.103***</td>
<td>0.074***</td>
</tr>
<tr>
<td></td>
<td>(4.87)</td>
<td>(3.79)</td>
</tr>
<tr>
<td>constant</td>
<td>0.963***</td>
<td>1.391***</td>
</tr>
<tr>
<td></td>
<td>(10.36)</td>
<td>(4.64)</td>
</tr>
<tr>
<td>country dummies</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.10</td>
<td>0.72</td>
</tr>
<tr>
<td>observations</td>
<td>210</td>
<td>210</td>
</tr>
<tr>
<td>Wald test:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{1,\text{data}} = b_{1,\text{baseline model}}$</td>
<td>20.60 (0.00)</td>
<td>11.80 (0.00)</td>
</tr>
<tr>
<td>$b_{1,\text{data}} = b_{1,\text{endogenous TFP cor}}$</td>
<td>3.23 (0.07)</td>
<td>0.22 (0.64)</td>
</tr>
</tbody>
</table>

*p<0.10, ** p<0.05, *** p<0.01.
Coefficient estimates reported with $t$−stat in brackets.
Wald test reported with $p$−value in brackets.

where cor($\tilde{Y}_i, \tilde{Y}_j$) is the correlation between detrended log GDP in country $i$ and country $j$, and log(Bilateral Trade)$_{ij}$ is log of the country-pair’s bilateral trade intensity, defined as the sum of bilateral manufacturing imports from country $i$ to country $j$ and from country $j$ to country $i$, as a fraction of the two countries’ total manufacturing output averaged over the entire period.

We estimate equation (32) using manufacturing data, obtained from the OECD STAN database over the 1988 – 2007 period. We focus on manufacturing data because our empirical work requires panel data on bilateral trade flows and the use of manufacturing data allows us to extend the time-series length of the panel substantially. However, the FR result concerning trade and business cycle synchronization is robust across alternative measures of output and trade. Details about the data are included in Appendix E.

The first two columns of Table 2 show the OLS estimates of the FR regression. The first column only includes the bilateral trade covariate and shows that an increase in trade intensity leads to
Table 3: The factor structure of GDP comovement (empirical data)

<table>
<thead>
<tr>
<th></th>
<th>dependent variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cor ((\bar{Y}_i, \bar{Y}_j))</td>
</tr>
<tr>
<td>1. cor ((\bar{A}_i, \bar{A}_j))</td>
<td>0.135*</td>
</tr>
<tr>
<td>2. cor ((\bar{\lambda}<em>{ii}, \bar{\lambda}</em>{jj}))</td>
<td>0.320***</td>
</tr>
<tr>
<td>3. cor ((\bar{\lambda}_{ii}, \bar{A}_j))</td>
<td>-0.282*</td>
</tr>
<tr>
<td>4. log ((\text{Bilateral Trade})_{ij})</td>
<td>0.061***</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5. constant</td>
<td>0.266***</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td>0.17</td>
</tr>
<tr>
<td>observations</td>
<td>210</td>
</tr>
</tbody>
</table>

*p<0.10, ** p<0.05, *** p<0.01.
Coefficient estimates reported with \(t\)-stat in brackets.

higher business cycle synchronization. The estimated \(b_1\) coefficient is statistically significant and equal to 0.103. In terms of magnitude, the slope coefficient estimate implies that a country pair with twice the trade intensity as another country pair will have a 0.071 higher GDP correlation. However, in this regression the \(R^2\) is small, indicating that bilateral trade only explains 10% of the cross-section variation in business cycle comovement. Therefore, in the second column we also include fixed effects for each individual country.\(^9\) The estimated \(b_1\) coefficient in the second

\(^9\)As the unit of observation is the country-pair, we identify a fixed effect for each individual country. In particular, fixed effects regression that we estimate is

\[
\text{cor} \left( \bar{Y}_i, \bar{Y}_j \right) = b_0 + b_1 \log \left( \text{Bilateral Trade} \right)_{ij} + f_i + f_j + \varepsilon_{ij},
\]

with \(f_i\) and \(f_j\) the country fixed effects.
regression remains positive and significant with a value of 0.074, indicating that there are no severe problems with omitted variable bias. Reassuringly, the adjusted $R^2$ is greatly increased at 73%, suggesting that the fixed effects capture most of the variation due to unobserved covariates.

The third column in Table 2 reports the slope coefficient implied by the baseline model. In this case $b_1$ is equal to 0.007, which is at least an order of magnitude smaller than the slope coefficient from the data. The Wald test is performed to test if the coefficient estimated using empirical data is the same as the implied coefficient obtained from the model. The null hypothesis is, of course, overwhelmingly rejected. This constitutes the trade-comovement puzzle documented in Kose and Yi (2006): the theoretical model is not able to replicate the empirical relation between trade and business cycle synchronization. Thus, our first finding is that a model that represents trade using the more sophisticated EK structure is not more successful at explaining the relationship between trade and business cycle comovement than the standard IRBC model.
4.2 Dissecting the Trade-comovement Puzzle

We use the factor structure of GDP comovement obtained in equation (31) to dissect the sources of the trade-comovement puzzle. We begin by examining if the factor structure derived for the theoretical economy holds empirically. The results in the first column of Table 3 are encouraging. Each factor loading has the expected sign and the three factors explain 17% of the variation in comovement observed in the data. Thus, the factor structure implied by equation (31) provides a satisfactory empirical model of GDP comovement and, hence, we can use it to inspect the sources of the trade comovement puzzle.

Having established that the fit of the three-factor model is good and consistent with theory, the next step is to study how each factor responds to changes in bilateral trade intensity. This allows us to study empirically the channels through which trade leads to higher business cycle synchronization, and contrast these results with the theoretical predictions (within the framework of Result 2).
do this, we estimate by OLS the following three equations:

\[
\text{cor} (\tilde{A}_i, \tilde{A}_j) = b_0 + b_1 \log (\text{Bilateral Trade})_{ij} + \varepsilon_{ij}, \tag{33}
\]

\[
\text{cor} (\tilde{\lambda}_{ii}, \tilde{\lambda}_{jj}) = b_0 + b_1 \log (\text{Bilateral Trade})_{ij} + \varepsilon_{ij}, \tag{34}
\]

\[
\text{cor} (\tilde{\lambda}_{ii}, \tilde{A}_j) = b_0 + b_1 \log (\text{Bilateral Trade})_{ij} + \varepsilon_{ij}, \tag{35}
\]

The last three columns of Table 3 report the estimation results. The results suggest that higher bilateral trade intensity is associated with (i) a higher correlation between each country’s technology shocks, and (ii) higher correlation between each country’s share of expenditure on domestic goods. By contrast, there is no significant relation between trade and the correlation between the share of expenditure on domestic goods and foreign TFP. Therefore, we conclude that empirically the first and second factors are responsible for the positive association between trade and business cycle comovement. In particular, the second factor is the most important: countries with strong trade linkages exhibit a higher correlation of the share of expenditures on domestic goods.

These findings are in sharp contrast to what is implied by the theoretical economy. This is illustrated in Figure 1. The world economy consists of four countries, so that each country has three trade partners and the calibration of trade costs is done such that the bilateral trade intensities correspond to the minimum, the maximum and the median observed in the empirical data. The first panel of the figure shows the relation between GDP correlation and bilateral trade implied by the model. In turn, panels two, three and four look at each factor in isolation.

The first panel illustrates the trade-comovement puzzle and the three subsequent panels dissect the puzzle exploiting the factor structure of GDP comovement. Looking at the second panel, there is no association between trade and the correlation of the TFP shocks. This is by design, as in the baseline model the TFP correlation is fixed. Looking at the third panel, the link between trade and the correlation between each country’s share of expenditure on domestic goods is counterfactually negative. This happens because, with uncorrelated shocks, a positive TFP shock in country \(i\) leads to an increase in both its share of expenditure on domestic goods (\(\lambda_{ii}\)) and the foreign country...
Table 4: Trade and business cycle synchronization (endogenous TFP correlation)

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>empirical data</th>
<th>model: endogenous TFP correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{cor} \left( \tilde{Y}_i, \tilde{Y}_j \right) )</td>
<td>( \text{log (Bilateral Trade)} )</td>
<td>( \text{log (Bilateral Trade)} )</td>
</tr>
<tr>
<td>1.</td>
<td>0.103***</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td>(4.87)</td>
<td>(4.87)</td>
</tr>
<tr>
<td>( \text{cor} \left( \tilde{A}_i, \tilde{A}_j \right) )</td>
<td>constant</td>
<td>constant</td>
</tr>
<tr>
<td>2.</td>
<td>0.061***</td>
<td>0.560</td>
</tr>
<tr>
<td></td>
<td>(2.87)</td>
<td>(5.61)</td>
</tr>
<tr>
<td>3.</td>
<td>0.074***</td>
<td>0.937</td>
</tr>
<tr>
<td></td>
<td>(3.25)</td>
<td>(9.42)</td>
</tr>
<tr>
<td>4.</td>
<td>-0.014</td>
<td>-0.035</td>
</tr>
</tbody>
</table>

Wald test:

| \( b_{1,\text{data}} = b_{1,\text{model}} \) | 3.23 (0.07) | 0.00 (1.00) | 69.44 (0.00) | 4.78 (0.03) |

*\( p<0.10 \), ** \( p<0.05 \), *** \( p<0.01 \).
Coefficient estimates reported with \( t \)-stat in brackets.
Wald test reported with \( p \)-value in brackets.

import penetration ratio (so that \( \lambda_{jj} \) falls). Finally, looking at the fourth panel, the small positive association between trade and comovement is (counterfactually) driven by the third component: an increase in trade is associated with a lower correlation between a country’s TFP and the foreign country’s share of expenditure on domestic goods.\(^{10}\)

### 4.3 Endogenous TFP Correlation

In the baseline model, the correlation of the TFP shocks is fixed for all country pairs at a level that matches the median correlation in the data. However, this is counterfactual as there is evidence

\(^{10}\)Recall that the third factor of equation (31) is not significant empirically.
that countries with stronger trade linkages also have more correlated TFP shocks.\textsuperscript{11} Therefore, we now depart from the baseline model and allow the correlation of the TFP shocks to vary with trade. We do this by using the estimated regression equation (33) to predict the level of TFP correlation given the corresponding level of bilateral trade and calibrating the covariance matrix of the TFP shocks accordingly.

The results are shown in Table 4. The table includes three panels: the first panel reports again the empirical regression between GDP correlation and bilateral trade, and also the regressions studying the association between trade intensity and each separate factor; the second panel reports the same regression coefficients implied by the theoretical economy with endogenous TFP correlation; finally, the third panel reports the results of the Wald test for the hypothesis that the coefficient estimated using empirical data is equal to the coefficient implied by the theoretical model with endogenous TFP correlation.

Once we allow for endogenous TFP correlations, the model is able to better replicate the trade-comovement relation. The implied coefficient in the regression between GDP correlation and trade intensity is 0.065, which is roughly 65\% of the empirical counterpart. In fact, at the 5\% level, we are unable to reject the hypothesis that the estimated coefficient is equal to the value implied by the model.\textsuperscript{12} Thus, allowing for endogenous TFP correlation helps to resolve the trade-comovement puzzle. This finding is consistent with those by Kose and Yi (2006). However, given the factor structure of GDP comovement derived earlier, we can inspect the mechanism further and look at how each of the three factors that explain GDP comovement is affected by bilateral trade.

Thus, we now focus on columns two, three and four of Table 4. The results in column two simply clarify the calibration strategy: in the regression between TFP correlation and trade, the model implied coefficients are the same as the empirical coefficients by construction. Our main finding is reported in column three: the model with endogenous TFP correlations still implies a negative association between bilateral trade and the correlation between each country’s share of expenditure on domestic goods, with a coefficient equal to $-0.115$.\textsuperscript{13} This is counterfactual since in the data we

\textsuperscript{11}See, for instance, Kose and Yi (2006) and Liao and Santacreu (2014).

\textsuperscript{12}The $p$–value for this null hypothesis is 7\%. If we consider the estimated coefficient from the fixed effect regression, the $p$–value is even higher, at 64\%. This is reported in Table 2.

\textsuperscript{13}Interestingly, the relation between bilateral trade and cor $\left(\tilde{\lambda}_{it}, \tilde{\lambda}_{jt}\right)$ implied by the model is not monotonic. In
find that for country-pairs with high bilateral trade intensity, the correlation between each country’s share of expenditure on domestic goods is positive and large, with a regression coefficient equal to 0.074. Finally, the behavior of the third factor is more consistent with the empirical data: higher trade intensity lowers the correlation between foreign TFP shocks and the share of expenditure on domestic goods.\textsuperscript{14}

The main empirical result is well illustrated in Figure 2. Although allowing for TFP correlation to increase with bilateral trade as in the data helps the model to account for the trade comovement relationship (shown in the first graph), it leads to a strong counterfactual negative association between the correlation of the share of expenditure on domestic goods and bilateral trade (shown in the third graph). Our findings suggest that both empirical research and theoretical models should examine further the positive association between trade and the comovement in the share of expenditure on domestic goods. Empirically, this channel is the most important to explain why trade leads to business cycle synchronization. Therefore, we argue that this should constitute a litmus test for theoretical models: stronger trade linkages should lead to synchronization in the share of expenditure on domestic goods.

4.4 Inspecting the Mechanism: bilateral trade and \( \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) \)

The relation between bilateral trade and \( \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) \) implied by the model with endogenous TFP correlations is not monotonic. As shown in the third graph of Figure 2, as we move from low levels of bilateral trade to intermediate levels, \( \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) \) increases. After that, as we move to high levels of bilateral trade, it falls again. Therefore, it is important to inspect the mechanism more carefully. We do this by looking at impulse response functions (IRF) conditional on a TFP shock, for each component of \( \tilde{\lambda}_{ii} \).\textsuperscript{15} In particular, we start from equation (10) to obtain

\[
\frac{\lambda_{j1} (s)}{\lambda_{jj} (s')} = \left[ \frac{W_{1} (s)}{W_{j} (s')} \right]^{-\theta} \tau_{j1}^{\theta}.
\] (36)
In log-deviations from steady state equation (36) takes the form

\[
\tilde{\lambda}_{jj,t} = \theta \left( \tilde{W}_{1,t} - \tilde{W}_{j,t} \right) + \tilde{\lambda}_{j1,t},
\]

where an increase in \( \left( \tilde{W}_{1,t} - \tilde{W}_{j,t} \right) \) in equation (37) represents an improvement in the terms-of-trade of country \( i \) vis-à-vis country \( j \), while \( \tilde{\lambda}_{j1,t} \) corresponds to country \( j \)’s import ratio from country 1.\(^\text{16}\) A positive technology shock in country 1 (hereafter, the Home country) raises Home’s share of expenditure on domestic goods, \( \tilde{\lambda}_{11,t} \). We now ask, under what circumstances does the share of expenditure on domestic goods also increase in other countries?

From equation (37) we see that, for the share of expenditure on domestic goods to increase in countries \( j = 2, 3, 4 \) (henceforth, the Foreign countries) conditional on a positive TFP shock at Home, either the Home terms-of-trade must improve or the Foreign’s import ratio must increase. But the IRFs in Figure 3 show that these two terms move in opposite directions: a positive TFP

\(^{16}\text{From equation (5), changes in the price of country } i \text{ exports are given by } \tilde{p}_{ji,t} = \tilde{W}_{i,t}, \text{ and so } \left( \tilde{W}_{i,t} - \tilde{W}_{j,t} \right) \text{ captures changes in the terms-of-trade.} \)
shock at Home worsens Home’s terms-of-trade while increasing Foreign’s import ratio. Thus, the overall effect on Foreign’s share of expenditure on domestic goods is ambiguous.

From the IRFs in the bottom-right panel of Figure 3 we see that for the Foreign countries with high and median trade costs the two effects almost cancel out and, hence, Home’s TFP shock has almost no effect on those countries’ share of expenditure on domestic goods. Instead, for the Foreign country with low trade costs the deterioration in the Home’s terms-of-trade dominates and, hence, Foreign’s share of expenditure on domestic goods falls. In Appendix B we show that the direct impact of changes in the terms-of-trade on the Foreign import ratio from Home depends on the bilateral trade flow in the following way:

\[ \frac{\partial \tilde{\lambda}_{j1,t}}{\partial (\tilde{W}_{1,t} - \tilde{W}_{j,t})} = -\theta \left(1 - \tilde{\lambda}_{j1}\right), \quad (38) \]

and, combining (37) with (38), we obtain

\[ \frac{\partial \tilde{\lambda}_{jj,t}}{\partial (\tilde{W}_{1,t} - \tilde{W}_{j,t})} = \theta \tilde{\lambda}_{j1}. \quad (39) \]

Therefore, the deterioration in Home’s terms-of-trade implies counterfactually that, conditional on a positive Home TFP shock, the correlation between the Home and the Foreign’s share of expenditure in domestic goods falls with increased trade integration (measured by \( \tilde{\lambda}_{j1} \)). The strength of this effect depends on \( \theta \): with higher \( \theta \), implying less firm heterogeneity, the counterfactual effect is stronger.\(^{17}\)

The parameter \( \theta \) is calibrated to 3.60 following empirical evidence in Bernard et al. (2003). The upshot is that given the median bilateral trade flow of 1.5\%, the elasticity of the domestic expenditure share to changes in the terms-of-trade is only 5.4\%. Instead, for the maximum bilateral trade flow observed in the data, which is 39\%, the same elasticity is 140.4\%. Hence, the fall in the share of expenditure on domestic goods is much higher for the country with low bilateral trade costs. This happens despite the deterioration in Home’s terms-of-trade vis-à-vis the Foreign economies with median and high trade costs being greater than that vis-à-vis the Foreign economy with low.

\(^{17}\)It is interesting to note that this elasticity does not depend on the elasticity of substitution across varieties \( \sigma \).
trade costs. Given such a large difference in the relative elasticities, allowing for correlated TFP shocks is not enough to overthrow this result.

The inability of the model to generate the observed relation between trade and the synchronization of the share of expenditure on domestic goods follows from the behavior of the terms-of-trade. Indeed, our model possesses the feature, common to most IRBC models, that Home TFP gains are transmitted positively to the trade partners through a deterioration of Home’s terms-of-trade.\(^\text{18}\) This property of the IRBC models is often counterfactual as shown in Corsetti et al. (2008) and, using an alternative identification scheme, in Corsetti et al. (2014). However, the elasticity of the share of expenditure on domestic goods to changes in the terms-of-trade is increasing in the level of trade integration. The upshot is that the only mechanism compatible with increased comovement in the share of expenditure on domestic goods following greater trade integration requires procyclical terms-of-trade.

5 Conclusion

Substantial empirical evidence suggests that countries or regions with stronger trade linkages have more correlated business cycles. However, from a theoretical perspective the IRBC model has difficulties in replicating this empirical fact. This has given rise to the so-called trade-comovement puzzle: standard models are unable to generate high output correlations arising from high bilateral trade intensity.

In this paper, we examine the sources of the trade-comovement puzzle. We show that within a large class of trade models (that includes the Armington model, Eaton and Kortum 2002, Bernard et al. 2003, Krugman 1980, and multiple versions of Melitz 2003), there are three channels through which bilateral trade may increase business cycle synchronization: (i) if trade raises the correlation between each country’s technology shocks; (ii) if trade leads to higher correlation between each country’s share of expenditure on domestic goods; and (iii) if trade raises the correlation between the share of expenditure on domestic goods and foreign TFP. Using bilateral trade data in manufactures

\(^{18}\) An exception is Ghironi and Melitz (2005) who construct a model where the entry of new producers and varieties in the Home economy following a TFP shock may dampen or even reverse the deterioration in the terms-of-trade.
for a panel of 21 OECD countries we find that the first and second channels are supported by the data: higher bilateral trade intensity is associated with higher correlation between each country’s TFP and with a higher correlation between each country’s share of expenditure on domestic goods. Consistent with the results of Kose and Yi (2006), we find that allowing for correlated TFP shocks is part of the solution to the trade-comovement puzzle. However, we show that even allowing for correlated shocks, the model still implies a counterfactual relation between bilateral trade and each country-pair correlation of the share of expenditure on domestic goods. Therefore, simply allowing for correlated shocks is not a complete solution to the trade-comovement puzzle. Instead, our findings suggests that both the empirical and theoretical research should examine the positive association between trade and the comovement in the share of expenditure on domestic goods. This invites further research to uncover new trade related transmission mechanisms of productivity shocks. This will require the development of richer micro foundations concerning the relation between trade and business cycle fluctuations. In particular, we have shown that the elasticity of the share of expenditure on domestic goods to changes in the terms-of-trade is greater for country-pairs with large bilateral trade flows. This suggests that for trade to increase the correlation between the share of expenditure on domestic goods, the terms-of-trade must improve following a positive TFP shock. Investigating this possibility is one promising avenue for future research.
References


APPENDIX

A Steady State Equilibrium

Let $\bar{X}$ denote the steady state level for the variable $X$. We consider a symmetric perfect-foresight steady state equilibrium with, in particular, $\bar{\lambda}_{ii}$, $\bar{W}_i$, $\bar{\mathcal{P}}_i$, $\bar{C}_i$ and $\bar{L}_i$ equal for all $i$. Moreover, in the symmetric equilibrium there are no trade deficits, so that $\bar{D}_i = 0$ for all $i$. The symmetry assumption makes it possible to solve analytically for the steady state. We measure everything in units of labor and, hence, $\bar{W} = 1$. Combining equations (8), (9) and (10) yields that in a symmetric steady state equilibrium, $\bar{\lambda}_{ii}$ is given by

$$\bar{\lambda} = \left( \sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{-1} .$$

(A.1)

Substituting in (19) for $\bar{\lambda}$ using (A.1) yields

$$\bar{L} = \kappa^{-\sigma_n} \left( \sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{\sigma_n/\theta} .$$

(A.2)

Making use of (10) to solve for the price level yields

$$\bar{\mathcal{P}} = \kappa \left( \sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{-1/\theta} .$$

(A.3)

Using the fact that in the steady state equilibrium there are no trade deficits so that $\bar{\mathcal{P}} \bar{C} = \bar{W} \bar{L}$, yields the steady state level of consumption

$$\bar{C} = \kappa^{-(1+\sigma_n)} \left( \sum_{j=1}^{n} \tau_{ij}^{-\theta} \right)^{(1+\sigma_n)/\theta} .$$

(A.4)

Finally, equation (17) yields the steady state marginal utility of wealth

$$\bar{\mu} = \left[ \bar{\mathcal{P}} \left( \bar{C} - \frac{\bar{L}^{1+1/\sigma_n}}{1+1/\sigma_n} \right) \right]^{-1} .$$

(A.5)
B Log-linear Equilibrium Conditions

Following standard steps, the individual optimality conditions and the market clearing conditions are log-linearized around the symmetric perfect-foresight steady state equilibrium and combined so as to characterize the equilibrium dynamics. We consider all the variables in log-deviation except for the trade deficit which is considered in levels since the trade deficit must be zero in steady state. Let \( \tilde{X}_{i,t} \) denote the variable \( X_i \) in steady state and \( \mathcal{D}_{i,t} = (\tilde{C} \bar{C})^{-1} D \) denote the ratio between the trade deficit at date \( t \) and the steady state income. The numéraire is taken to be labor in country 1, so that \( \tilde{W}_{1,t} = 0 \) for all \( t \). The log-linear equilibrium conditions are the following

\[
\begin{align*}
\tilde{\mu}_t + \tilde{\rho}_{i,t} &= -\tilde{\rho} \tilde{\mu} \left( \tilde{C} \tilde{C}_{i,t} - \tilde{L}_{1+1/\sigma_n} \tilde{L}_{i,t} \right), \quad \forall i = 1, \ldots, n, \\
\tilde{\mu}_t &= \tilde{R}_t + E_t (\tilde{\mu}_{t+1}), \\
\tilde{\rho}_{i,t} + \tilde{A}_{i,t} &= \tilde{\lambda} \sum_{j=1}^{n} \tilde{W}_{j,t} \tau_{ij}^{-\theta}, \\
\tilde{\lambda}_{ji,t} &= -\theta \left( \tilde{W}_{i,t} - \tilde{\rho}_{j,t} - \tilde{A}_{j,t} \right), \\
\tilde{L}_{i,t} &= \sigma_n \left( \tilde{W}_{i,t} - \tilde{\rho}_{i,t} \right), \\
\tilde{W}_{i,t} + \tilde{L}_{i,t} &= \sum_{j=1}^{n} \tilde{\lambda}_{ji} \left( \tilde{\lambda}_{ji,t} + \tilde{W}_{j,t} + \tilde{L}_{j,t} + \mathcal{D}_{j,t} \right), \\
\tilde{\rho}_{i,t} + \tilde{C}_{i,t} &= \tilde{W}_{i,t} + \tilde{L}_{i,t} + \mathcal{D}_{i,t}, \\
\sum_{i=0}^{n} \mathcal{D}_i \left( s^t \right) &= 0.
\end{align*}
\]
Finally, in Section 4 we make use of equation (38). To obtain this equation, combine (B.3) and (B.4) to obtain
\[
\tilde{\lambda}_{ji,t} = -\theta \left( \tilde{W}_{i,t} - \sum_{l=1}^{n} \tilde{W}_{l,t} \tau_{ji}^{\theta} \right)
\]
\[
= -\theta \left[ (\tilde{W}_{i,t} - \tilde{W}_{j,t}) + \bar{\lambda} \tilde{W}_{j} \sum_{l=1}^{n} \tau_{ji}^{\theta} - \bar{\lambda} \sum_{l=1}^{n} \tilde{W}_{l,t} \tau_{ji}^{\theta} \right] 
\]
\[
= -\theta \left[ (\tilde{W}_{i,t} - \tilde{W}_{j,t}) - \bar{\lambda} \sum_{l=1}^{n} (\tilde{W}_{l,t} - \tilde{W}_{j,t}) \tau_{ji}^{\theta} \right], 
\]
where we have used the fact that \( \bar{\lambda} = \left( \sum_{l=1}^{n} \tau_{ji}^{\theta} \right)^{-1} \).

The upshot is that
\[
\frac{\partial \tilde{\lambda}_{j1,t}}{\partial \left( \tilde{W}_{i,t} - \tilde{W}_{j,t} \right)} = -\theta \left( 1 - \bar{\lambda} \tau_{ji}^{\theta} \right)
\]
\[
= -\theta \left( 1 - \tilde{\lambda}_{ji} \right), 
\]
where we have used the fact that \( \tilde{\lambda}_{ji} = \bar{\lambda} \tau_{ji}^{-\theta} \). This corresponds to equation (38) in the main text.

C Proof of Result 1

If assumptions A1 and A2 are satisfied, it follows from the results in Arkolakis et al. (2012) that the following gravity equation holds
\[
\lambda_{ji} (s^t) = \left[ \kappa W_{i} (s^t) \frac{\tau_{ji}}{A_{j} (s^t) \mathcal{P}_{j} (s^t)} \right]^{-\theta},
\]
where \( \kappa \) is a constant parameter and \( \mathcal{P}_{j} (s^t) = A (s^t)^{-1} \left[ \int_{0}^{1} p_{j} (n, s^t) 1^{-\sigma} \, dn \right]^{1/(1-\sigma)} \) is the aggregate price level. Solving for the real wage we obtain
\[
\frac{W_{i} (s^t)}{\mathcal{P}_{j} (s^t)} = A (s^t)^{\frac{1}{\kappa}} \left[ \frac{1}{\lambda_{jj} (s^t)} \right]^{1/\theta}.
\]
From assumption A3, it follows that the labor supply is a function only of the real wage, so that
\[
L_{i} (s^t) = \left[ \frac{W_{i} (s^t)}{\mathcal{P}_{j} (s^t)} \right]^{\sigma_n},
\]
where $\sigma_n$ is the labor supply elasticity. The value added by each intermediate firm is equal to the sum of that firm’s wage bill and profits, so that

$$p_i(v, s^t) y_i(v, s^t) = W_i(s^t) L_i(v, s^t) + \Pi_i(v, s^t), \quad (C.4)$$

where $\Pi_i(v, s^t)$ are profits for the intermediate firm producing variety $v$. From assumption A2 profits are a constant share $\gamma$ of revenues, so that $\Pi_j(v, s^t) = \gamma p_i(v, s^t) y_i(v, s^t)$. It follows that

$$S_i(s^t) \equiv \int_{\Omega_i(s^t)} p_i(v, s^0) y_i(v, s^t) \, dv = (1 - \gamma)^{-1} \int_{\Omega_i(s^t)} \left[ \frac{p_i(v, s^0)}{p_i(v, s^t)} \right] W_i(s^t) L_i(v, s^t) \, dv. \quad (C.5)$$

Making use of (C.1) and (C.2) yields

$$S_i(s^t) = \kappa - \sigma_n A_i(s^t) \left[ 1 + \frac{1}{\lambda_{ii}(s^t)} \right]^{\sigma_n/\theta}, \quad (C.6)$$

where we have substituted for prices following the same steps described in Section 3.1.

The upshot is that output in log-deviations from steady state is given by equation (30) as had to be shown.

**D Proof of Result 2**

From equation (30), GDP fluctuations in country $i$ (in log-deviations from steady state) are given by

$$\tilde{Y}_{i,t} = (1 + \sigma_n) \tilde{A}_{i,t} - \left( \frac{\sigma_n}{\theta} \right) \tilde{\lambda}_{ii,t}. \quad (D.1)$$

It follows that the covariance between the logarithm of output in country $i$ and in country $j$ is given by

$$\text{cov} \left( \tilde{Y}_i, \tilde{Y}_j \right) = \vartheta_1 \text{cov} \left( \tilde{A}_i, \tilde{A}_j \right) + \vartheta_2 \text{cov} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) + \vartheta_3 \left[ \text{cov} \left( \tilde{\lambda}_{ii}, A_j \right) + \text{cov} \left( \tilde{\lambda}_{jj}, A_j \right) \right], \quad (D.2)$$

where $\vartheta_1 = (1 + \sigma_n)^2$, $\vartheta_2 = (\sigma_n/\theta)^2$ and $\vartheta_3 = -(1 + \sigma_n) (\sigma_n/\theta)$. In a symmetric world economy $\text{var} (\tilde{Y}_i)$, $\text{var} (\tilde{A}_i)$ and $\text{var} (\tilde{\lambda}_{ii})$ are the same for all countries and $\text{cov} \left( \tilde{\lambda}_{ii}, A_j \right) = \text{cov} \left( \tilde{\lambda}_{jj}, A_j \right)$.

The upshot is that by dividing each side of equation (D.2) by $\text{var} (\tilde{Y}_i)$, and dividing and multiplying \( ^{19}\text{Note that in our theoretical framework profits are zero because we are considering a model of perfect competition.} \)
the first term of the RHS by \( \text{var} \left( \tilde{A}_i \right) \), the second term by \( \text{var} \left( \tilde{\lambda}_{ii} \right) \) and the third term by \( \text{std} \left( A_i \right) \times \text{std} \left( \tilde{\lambda}_{ii} \right) \) yields the equation

\[
\text{cor} \left( \tilde{Y}_i, \tilde{Y}_j \right) = \beta_1 \text{cor} \left( \tilde{A}_i, \tilde{A}_j \right) + \beta_2 \text{cor} \left( \tilde{\lambda}_{ii}, \tilde{\lambda}_{jj} \right) + \beta_3 \text{cor} \left( \tilde{\lambda}_{ii}, A_j \right),
\]

where the factor loadings are given by

\[
\beta_1 \equiv (1 + \sigma_n)^2 \left[ \frac{\text{var} \left( \tilde{A}_i \right)}{\text{var} \left( \tilde{Y}_i \right)} \right], \\
\beta_2 \equiv (\frac{\sigma_n}{\theta})^2 \left[ \frac{\text{var} \left( \tilde{\lambda}_{ii} \right)}{\text{var} \left( \tilde{Y}_i \right)} \right] \\
\beta_3 \equiv -2 (1 + \sigma_n) \left( \frac{\sigma_n}{\theta} \right) \left[ \frac{\text{std} \left( \tilde{A}_i \right) \text{std} \left( \tilde{\lambda}_{ii} \right)}{\text{var} \left( \tilde{Y}_i \right)} \right].
\]

Result 2 follows immediately from equation (D.3), where the factors (i), (ii) and (iii) are as follows: (i) the correlation between each country’s TFP; (ii) the correlation between each country’s share of expenditure on domestic goods; (iii) the correlation between the country’s share of expenditure on domestic goods and the country-pair’s TFP (equal to the negative of the correlation between the country’s import penetration ratio and the country-pair’s technology shocks).

E Data

We consider a sample of 21 OECD countries composed of Austria (AT), Belgium (BE), Denmark (DK), France (FR), Germany (DE), Italy (IT), Netherlands (NL), Norway (NO), Sweden (SW), Switzerland (CH), Canada (CA), Japan (JP), Finland (FI), Greece (GR), Ireland (IR), Korea (KO), Portugal (PT), Spain (SP), and New Zealand (NZ) United Kingdom (UK) and the United States (US), over the period 1988–2007.

The variables definitions and data sources are as follows:

Real output

Real output is measured using manufacturing GDP and CPI data from the OECD STAN database. Bilateral correlations are calculated using the linearly detrended log real output.

Country’s share of expenditure on domestic goods
The measure $\lambda_{ii}$ captures the fraction of total expenditure on country $i$ on goods made in country $i$. Expenditure on goods made at home is measured as the gross manufacturing output (converted to dollars using current exchange rates) less total manufacturing exports. Total expenditure is measured as the sum of expenditure on goods made at home and expenditure on total imports. All data are from the OECD STAN database. Bilateral correlations between $\lambda_{ii}$ and $\lambda_{jj}$ are calculated from the raw series of $\lambda$’s.

**Trade intensity**

We measure trade intensity between each country pair $i$ and $j$, labeled (Bilateral Trade)$_{ij}$, by normalizing bilateral trade—that is, the sum of each country’s manufacturing imports from the other—by the sum of nominal manufacturing GDP in the two countries, averaged over the entire period. Manufacturing imports data, denominated in dollars, is taken from the OECD STAN database. We normalize trade by nominal GDP (converted in dollars using current exchange rates), also from the STAN database.

**Total factor productivity (TFP)**

We measure TFP as the ratio between real manufacturing output and manufacturing employment obtained from the STAN database.

**F  Calibration of the Model**

The number of countries $N$ is set equal to 4 to replicate, implying 6 distinct country-pairs. Having 6 distinct country-pairs allows us to construct a symmetric world economy matching the minimum, maximum and median bilateral trade flows observed in the data.

The list of technology parameters that have to be determined includes the following: the elasticity of substitution between intermediate inputs $\sigma$; the parameter that controls the level of heterogeneity in productive efficiencies $\theta$; and the 6 trade-cost parameters $\tau_{ij}$ for each distinct country pair.\(^{20}\)

\(^{20}\)We assume $\tau_{ij} = \tau_{ji}$ and $\tau_{ii} = 1$. 

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plant-level data. The parameter $\theta$ is chosen to match the productivity advantage of exporters, and the parameter $\sigma$ corresponds to the price elasticity of demand for differentiated intermediate commodities and therefore relates to the size advantage of exporting establishments. The values estimated by Bernard et al. (2003) for $\theta$ and $\sigma$ are, respectively, 3.60 and 3.79.

The trade-cost parameters $\tau_{ij}$ are chosen to match the minimum, maximum and median bilateral trade flows observed in the OECD Structural Analysis (STAN) database. To construct a symmetric world economy, the matrix of trade costs must be symmetric, of the form

$$T = \begin{bmatrix}
1 & \tau_L & \tau_M & \tau_H \\
\tau_L & 1 & \tau_M & \tau_H \\
\tau_M & \tau_H & 1 & \tau_L \\
\tau_H & \tau_M & \tau_L & 1
\end{bmatrix}, \quad (F.1)
$$

with $1 < \tau_L < \tau_M < \tau_H$. We choose $\tau_L$ and $\tau_H$ to match, respectively, the maximum and minimum bilateral trade flow observed in our data and $\tau_M$ to match the median trade flow. In a symmetric world economy the bilateral trade flow between two countries $i$ and $j$ is given by $\lambda_{ij}$. Let $\lambda_{\text{min}}$, $\lambda_{\text{max}}$ and $\lambda_{\text{med}}$ denote the minimum, maximum and median bilateral trade flows. Then, from the manipulation of equation (10) we obtain, in the symmetric world economy with $N = 4$ and $\bar{W} = 1$ (labor is the numéraire) the following relations

$$1 - \lambda_{\text{min}} - \lambda_{\text{max}} - \lambda_{\text{med}} = \left(1 + \tau_L^{-\theta} + \tau_M^{-\theta} + \tau_H^{-\theta}\right)^{-1},$$

$$\tau_L = \left(\frac{\lambda_{\text{max}}}{1 - \lambda_{\text{min}} - \lambda_{\text{max}} - \lambda_{\text{med}}}\right)^{-1/\theta},$$

$$\tau_H = \left(\frac{\lambda_{\text{min}}}{1 - \lambda_{\text{min}} - \lambda_{\text{max}} - \lambda_{\text{med}}}\right)^{-1/\theta},$$

$$\tau_M = \left(\frac{\lambda_{\text{med}}}{1 - \lambda_{\text{min}} - \lambda_{\text{max}} - \lambda_{\text{med}}}\right)^{-1/\theta}. \quad (F.2)$$

Thus, given targets for $\lambda_{\text{min}}$, $\lambda_{\text{max}}$ and $\lambda_{\text{med}}$ it becomes possible to set values for each of the three trade costs $\tau_L$, $\tau_H$ and $\tau_M$. The resulting matrix of trade costs is given by

$$T = \begin{bmatrix}
1.000 & 1.127 & 2.801 & 5.948 \\
1.127 & 1.000 & 5.948 & 2.801 \\
2.801 & 5.948 & 1.000 & 1.127 \\
5.948 & 2.801 & 1.127 & 1.000
\end{bmatrix}. \quad (F.3)$$
The remaining technology parameters that need to be chosen are the parameters of the stochastic process for the technology shocks, $\tilde{A}_{i,t}$, which follows a correlated discrete Markov process. In particular, we use a finite-state Markov process with states and transition probabilities set to approximate the continuous autoregressive model given by (up to a constant)

$$\tilde{A}_{i,t} = \rho \tilde{A}_{i,t-1} + \tilde{E}_t,$$

where $\tilde{E}_t$ is a normally distributed and zero-mean shock with standard deviation $\sigma$. We allow the shocks across countries to be correlated across countries but in the baseline model set the correlation coefficient to be the same for all country-pairs. This correlation coefficient is chosen to match the median bilateral correlation of TFP observed in our sample. In one of our additional quantitative experiments we allow the correlation of TFP shocks across countries to vary with the level of trade between each country-pair. We choose values for $\sigma$ and $\rho$ to match the standard deviation and the autocorrelation of output fluctuations in the US.

The only preference parameter that must be chosen is the labor supply elasticity $\sigma_n$. In our baseline calibration we set $\sigma_n = 1$ consistent with the estimates of the labor supply elasticity using macro data.