# Adaptive Randomized Distributed Space-Time Coding for Cooperative MIMO Relaying Systems

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Abstract—An adaptive randomized matrix optimization algorithm for cooperative MIMO networks with linear minimum mean square error (MMSE) receivers which employ an amplifyand-forward (AF) strategy and space-time coding schemes at the relay nodes is proposed. The randomized matrix multiplies the space-time coded matrix at the relay node and can be regarded as a part of the space-time coding scheme. We derive the upper bound of the error probability of a cooperative MIMO system employing the randomized space-time coding scheme first, and then, present an optimization algorithm based on the MMSE criterion. A stochastic gradient algorithm is derived in the calculation of the adaptive optimization in order to release the receiver from the massive calculation complexity. The simulation results indicate that the proposed algorithm obtains gains compared to the existing schemes.

#### I. INTRODUCTION

Multiple-input and multiple-output (MIMO) communication systems employ multiple collocated antennas at both the source node and the destination node in order to obtain the diversity gain and combat multi-path fading in wireless links. The different methods of space-time coding (STC) schemes, which can provide a higher diversity gain and coding gain compared to an uncoded system, are also utilized in MIMO wireless systems for different numbers of antennas at the transmitter and different conditions of the channel. Cooperative MIMO systems, which employ multiple relay nodes with antennas between the source node and the destination node as a distributed antenna array, apply distributed diversity gain and provide copies of the transmitted signals to improve the reliability of wireless communication systems [1]. Among the links between the relay nodes and the destination node, cooperation strategies, such as Amplify-and-Forward (AF), Decode-and-Forward (DF), and Compress-and-Forward (CF) [2] and various distributed STC (DSTC) schemes in [5], [6] and [19] can be employed.

The utilization of a distributed STC (DSTC) at the relay node in a cooperative network, providing more copies of the desired symbols at the destination node, can offer the system diversity gains and coding gains to combat the interference. The recent focus on the DSTC technique lies in the delaytoleration code design and the full-diversity schemes design with the minimum outage probability. In [7], the distributed delay-tolerant version of the Golden code [8] is proposed, which can provide full-diversity gain with a full coding rate. An opportunistic DSTC scheme with the minimum outage probability is designed for a DF cooperative network and compared with the fixed DSTC schemes in [9]. An adaptive distributed-Alamouti (D-Alamouti) STBC design is proposed in [10] for the non-regenerative dual-hop wireless system which achieves the minimum outage probability. DSTC schemes for the AF protocol are discussed in [11]-[12]. In [11],

the GABBA STC scheme is extended to a distributed MIMO network with full-diversity and full-rate, while an optimal algorithm for design of the DSTC scheme based on achieving the optimal diversity and multiplexing tradeoff is derived in [12].

The performance of cooperative networks using different strategies has been widely discussed in the literature. In [13], an exact pairwise error probability of the D-Alamouti STBC scheme is derived according to the position of the relay node. In [14], a bit error rate (BER) analysis of the distributed-Alamouti STBC scheme is proposed. The difference between these two works lies in the different cooperative schemes they consider. A maximum likelihood (ML) detection algorithm for a MIMO relay system with DF protocol is derived in [15] with its performance analysis as well. The symbol error rate and diversity order upper bound for the scalar fixed-gain AF cooperative protocol are given in [16]. The utilization of single-antenna relay nodes and the DF cooperative protocol constitute the main differences between their works and ours. The most similar works compared with our work are [17] and [18]. However, an STC encoding process is implemented at the source node in [17], which decrease the output of the system and increase the calculation complexity at the destination node to decode. In [18], the BER upper bound is given without the STC scheme utilization at the relay node.

In this paper, we propose an adaptive randomized distributed space-time coding scheme and algorithms for cooperative MIMO relaying systems. The upper bound pairwise error probability of the randomized-STC schemes (RSTC) in a cooperative MIMO system which employs multi-antenna relay nodes with the AF protocol is analyzed. We focus on how the randomized matrix affects the DSTC during the encoding and how to optimize the parameters in the matrix, and it is shown that the utilization of a randomized matrix benefits the performance of the system by lowering the upper bound compared to using traditional STC schemes. Then an adaptive optimization algorithm is derived based on the MSE criterion, with the utilization of the Stochastic Gradient (SG) algorithm in order to release the destination node from the high computing complexity of the optimization process. The updated randomized matrix is transmitted to the relay node through a feedback channel that is assumed in this work error free and delay free.

The paper is organized as follows. Section II introduces a two-hop cooperative MIMO system with multiple relays applying the AF strategy and the randomized DSTC scheme. In Section III the proposed optimization algorithm for the randomized matrix is derived, and the analysis of the upper bound of pairwise error probability using the randomized D-STC is shown in Section IV. Section V focus on the results



Fig. 1. Cooperative MIMO System Model with  $n_r$  Relay nodes

of the simulations and Section VI leads to the conclusion.

#### II. COOPERATIVE SYSTEM MODEL

The communication system under consideration, shown in Fig.1, is a cooperative communication system employing multi-antenna relay nodes transmitting through a MIMO channel from the source node to the destination node. The 4-QAM modulation scheme is used in our system to generate the transmitted symbol vector s[i] at the source node. There are  $n_r$  relay nodes with N antennas for transmitting and receiving, applying an AF cooperative strategy as well as a DSTC scheme, between the source node and the destination node. A two-hop communication system that broadcasts symbols from the source to  $n_r$  relay nodes as well as to the destination node in the first phase, followed by transmitting the amplified and re-encoded symbols from each relay node to the destination node in the next phase. We consider only one user at the source node in our system that has N Spatial Multiplexing (SM)organized data symbols contained in each packet. The received symbols at the k-th relay node and the destination node are denoted as  $r_{SR_k}$  and  $r_{SD}$ , respectively, where  $k = 1, 2, ..., n_r$ . The received symbols  $r_{SR_k}$  will be amplified before mapped into an STC matrix. We assume that the synchronization at each node is perfect. The received symbols at the destination node and each relay node can be described as follows

$$\boldsymbol{r}_{SR_k}[i] = \boldsymbol{F}_k[i]\boldsymbol{s}[i] + \boldsymbol{n}_{SR_k}[i], \qquad (1)$$

$$\boldsymbol{r}_{SD}[i] = \boldsymbol{H}[i]\boldsymbol{s}[i] + \boldsymbol{n}_{SD}[i], \qquad (2)$$

$$i = 1, 2, \dots, N, \quad k = 1, 2, \dots n_r,$$

where the  $N \times 1$  vector  $\boldsymbol{n}_{SR_k}[i]$  and  $\boldsymbol{n}_{SD}[i]$  denote the zero mean complex circular symmetric additive white Gaussian noise (AWGN) vector generated at each relay and the destination node with variance  $\sigma^2$ . The transmitted symbol vector  $\boldsymbol{s}[i]$ contains N parameters,  $\boldsymbol{s}[i] = [s_1[i], s_2[i], \dots, s_N[i]]$ , which has a covariance matrix  $E[\boldsymbol{s}[i]\boldsymbol{s}^H[i]] = \sigma_s^2 \boldsymbol{I}$ , where  $E[\cdot]$ stands for expected value,  $(\cdot)^H$  denotes the Hermitian operator,  $\sigma_s^2$  is the signal power which we assume to be equal to 1 and  $\boldsymbol{I}$  is the identity matrix.  $\boldsymbol{F}_k[i]$  and  $\boldsymbol{H}[i]$  are the  $N \times N$  channel gain matrices between the source node and the k - th relay node, and between the source node and the destination node, respectively.

After processing and amplifying the received vector  $\mathbf{r}_{SR_k}[i]$ at the k - th relay node, the signal vector  $\tilde{\mathbf{s}}_{SR_k}[i] = \mathbf{A}_{R_k D}[i](\mathbf{F}_k[i]\mathbf{s}[i] + \mathbf{n}_{SR_k}[i])$  can be obtained and will be forwarded to the destination node. The amplified symbols in  $\tilde{\mathbf{s}}_{SR_k}[i]$  will be re-encoded by a  $N \times T$  DSTC scheme  $\mathbf{M}(\tilde{\mathbf{s}}[i])$ and then multiplied an  $N \times N$  randomized matrix  $\mathfrak{R}[i]$  in [22], then forwarded to the destination node. The relationship between the k - th relay and the destination node can be described as

$$\boldsymbol{R}_{R_kD}[i] = \boldsymbol{G}_k[i]\boldsymbol{\mathfrak{R}}[i]\boldsymbol{M}_{R_kD}[i] + \boldsymbol{N}_{R_kD}[i], \qquad (3)$$
$$k = 1, 2, \dots, n_r,$$

where the  $N \times T$  matrix  $M_{R_kD}[i]$  is the DSTC matrix employed at the relay nodes whose elements are the amplified symbols in  $\tilde{s}_{SR_k}[i]$ . The  $N \times T$  received symbol matrix  $R_{R_kD}[i]$  in (3) can be written as an  $NT \times 1$  vector  $r_{R_kD}[i]$ given by

$$\boldsymbol{r}_{R_kD}[i] = \boldsymbol{\mathfrak{R}}_{eq_k}[i]\boldsymbol{G}_{eq_k}[i]\tilde{\boldsymbol{s}}_{SR_k}[i] + \boldsymbol{n}_{R_kD}[i], \qquad (4)$$

where the block diagonal  $NT \times NT$  matrix  $\mathfrak{R}_{eq_k}[i]$  denotes the equivalent randomized matrix and the  $NT \times N$  matrix  $G_{eq_k}[i]$  stands for the equivalent channel matrix which is the DSTC scheme  $M(\tilde{s}[i])$  combined with the channel matrix  $G_{R_kD}[i]$ . The  $NT \times 1$  equivalent noise vector  $n_{R_kD}[i]$  generated at the destination node contains the noise parameters in  $N_{R_kD}[i]$ . After rewriting  $R_{R_kD}[i]$  we can consider the received symbol vector at the destination node as a  $N(n_r+1)$ vector with 2 parts, one is from the source node and another one is the superposition of the received vectors from each relay node, therefore the received symbol vector for the cooperative MIMO network we considered can be written as

$$\boldsymbol{r}[i] = \begin{bmatrix} \boldsymbol{H}[i]\boldsymbol{s}[i] \\ \sum_{k=1}^{n_r} \boldsymbol{\mathfrak{R}}_{eq_k}[i]\boldsymbol{G}_{eq_k}[i]\tilde{\boldsymbol{s}}_{SR_k}[i] \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_{SD}[i] \\ \boldsymbol{n}_{RD}[i] \end{bmatrix} \\ = \boldsymbol{D}_D[i]\tilde{\boldsymbol{s}}_D[i] + \boldsymbol{n}_D[i],$$
(5)

where the  $(T+1)N \times (n_r+1)N$  block diagonal matrix  $D_D[i]$ denotes the channel gain matrix of all the links in the network which contains the  $N \times N$  channel coefficients matrix H[i]between the source node and the destination node, the  $NT \times N$ equivalent channel matrix  $G_{eq_k}[i]$  for  $k = 1, 2, ..., n_r$  between each relay node and the destination node. The  $(n_r + 1)N \times 1$ noise vector  $n_D[i]$  contains the received noise vector at the destination node and the amplified noise vectors from each relay node, which can be derived as an AWGN with zero mean and covariance matrix  $\sigma^2(1+ || \Re_{eq_k}[i]G_{eq_k}[i]A_{R_kD}[i] ||_F^2)I$ , where  $|| X ||_F = \sqrt{\operatorname{Tr}(X^H \cdot X)} = \sqrt{\operatorname{Tr}(X \cdot X^H)}$  stands for the Frobenius norm.

# III. ADAPTIVE RANDOMIZED STC OPTIMIZATION ALGORITHM

As derived in the previous section, the DSTC scheme used at the relay node will be multiplied by a randomized matrix before being forwarded to the destination node. In this section, we design an adaptive optimization algorithm based on the stochastic gradient (SG) estimation algorithm [20] for determining the optimal randomized matrix.

# A. Optimization Method Based on the MSE Criterion

From (5) we propose the MSE based optimization at the destination node as

$$[\boldsymbol{W}[i], \boldsymbol{\mathfrak{R}}_{eq}[i]] = \arg\min_{\boldsymbol{W}[i], \boldsymbol{\mathfrak{R}}_{eq}[i]} E\left[\|\boldsymbol{s}[i] - \boldsymbol{W}^{H}[i]\boldsymbol{r}[i]\|^{2}\right],$$

where r[i] is the received symbol vector at the destination node which contains the randomized matrix to be optimized. If we only consider the received symbols from the relay node, the received symbol vector at the destination node can be derived as

$$\begin{aligned} \boldsymbol{r}[i] &= \boldsymbol{D}_{D}[i]\tilde{\boldsymbol{s}}_{D}[i] + \boldsymbol{n}_{D}[i] \\ &= \boldsymbol{\mathfrak{R}}_{eq}[i]\boldsymbol{G}_{eq}[i]\boldsymbol{A}[i]\boldsymbol{F}[i]\boldsymbol{s}[i] + \boldsymbol{\mathfrak{R}}_{eq}[i]\boldsymbol{G}_{eq}[i]\boldsymbol{A}[i]\boldsymbol{n}_{SR}[i] \\ &+ \boldsymbol{n}_{RD}[i] \\ &= \boldsymbol{\mathfrak{R}}_{eq}[i]\boldsymbol{C}[i]\boldsymbol{s}[i] + \boldsymbol{n}_{D}[i], \end{aligned}$$

where C[i] is an  $NT \times N$  matrix that contains all the complex channel gains and the amplified matrix assigned to the received vectors at the relay node, and the noise vector  $n_D$  is a Gaussian noise with zero mean and variance  $\sigma^2(1+ || \mathfrak{R}_{eq}[i] G_{eq}[i] A[i] ||_F^2)$ . Therefore, we can rewrite the MSE cost function as

$$\begin{bmatrix} \boldsymbol{W}[i], \boldsymbol{\mathfrak{R}}_{eq}[i] \end{bmatrix} = \arg \min_{\boldsymbol{W}[i], \boldsymbol{\mathfrak{R}}_{eq}[i]} E\left[ \|\boldsymbol{s}[i] - \boldsymbol{W}^{H}[i](\boldsymbol{\mathfrak{R}}_{eq}[i]\boldsymbol{C}[i]\boldsymbol{s}[i] + \boldsymbol{n}_{D}[i])\|^{2} \right]$$
(7)

By expanding the righthand side of (7) and taking the gradient with respect to  $W^*[i]$  and equaling the terms to zero, we can obtain the MMSE filter vector equalling

$$\boldsymbol{W}[i] = \left( E\left[\boldsymbol{r}[i]\boldsymbol{r}^{H}[i]\right] \right)^{-1} E\left[\boldsymbol{r}[i]\boldsymbol{s}^{H}[i]\right], \quad (8)$$

where the first term denotes the inverse of the auto-correlation matrix and the second one is the cross-correlation matrix. Define  $\tilde{r} = C[i]s[i] + C[i]n_{SR}$ , then the randomized matrix can be calculated by taking the gradient with respect to  $\mathfrak{R}^*[i]$  and equating the term to zero, resulting in

$$\boldsymbol{\mathfrak{R}}[i] = \left( \boldsymbol{W}^{H}[i] (E\left[\tilde{\boldsymbol{r}}[i]\tilde{\boldsymbol{r}}^{H}[i]\right]) \boldsymbol{W}[i] \right)^{-1} E\left[\boldsymbol{s}[i]\tilde{\boldsymbol{r}}^{H}[i]\right] \boldsymbol{W}[i],$$
(9)

where  $E\left[\tilde{r}[i]\tilde{r}^{H}[i]\right]$  is the auto-correlation of the spacetime coded received symbol vector at the relay node, and  $E\left[s[i]\tilde{r}^{H}[i]\right]$  is the cross-correlation. The optimization method requires an inversion calculation with a high computational complexity.

#### B. Adaptive Randomized Matrix Optimization Algorithm

In order to reduce the computational complexity and achieve the optimal performance, an adaptive randomized matrix optimization (ARMO) algorithm based on an estimation algorithm is designed. The MMSE problem is derived in (7), and the MMSE filter matrix can be calculated by (8) first during the optimization process. The simple ARMO algorithm can be achieved by taking the instantaneous gradient term of (7) with respect to the randomized matrix  $\Re_{eq}^{*}[i]$ , which is

$$\nabla \mathcal{L}_{\mathfrak{R}_{eq}^{*}[i]} = \nabla E \left[ \| \boldsymbol{s}[i] - \boldsymbol{W}^{H}[i](\mathfrak{R}_{eq}[i]\boldsymbol{C}[i]\boldsymbol{s}[i] + \boldsymbol{n}_{D}[i]) \|^{2} \right]_{\mathfrak{R}_{eq}^{*}[i]}$$
  
=  $-(\boldsymbol{s}[i] - \boldsymbol{W}^{H}[i]\boldsymbol{r}[i])\boldsymbol{s}^{H}[i]\boldsymbol{C}^{H}[i]\boldsymbol{W}[i]$   
=  $-\boldsymbol{e}[i]\boldsymbol{s}^{H}[i]\boldsymbol{C}^{H}[i]\boldsymbol{W}[i],$  (10)

where e[i] stands for the detected error vector. After we obtain (10) the ARMO algorithm can be obtained by introducing a step size into a gradient optimization algorithm to update the result until the convergence is reached, and it is given by

$$\mathfrak{R}[i+1] = \mathfrak{R}[i] + \mu(\boldsymbol{e}[i]\boldsymbol{s}^{H}[i]\boldsymbol{C}^{H}[i]\boldsymbol{W}[i]), \quad (11)$$

where  $\mu$  stands for the estimation step size. The complexity of calculating the randomized matrix is O(2*N*), which is much

less than that of the calculation method derived in (9). As mentioned in Section I, the randomized matrix will be sent back to the relay nodes via a feedback channel which is assumed to be error-free in the simulation, however in the practical circumstances, the errors caused by the broadcasting and the diversification of the feedback channel with time changes will affect the accuracy of the received randomized matrix at the relay node.

# IV. PROBABILITY OF ERROR ANALYSIS

In this section, the upper bound of the pairwise error probability of the system employing the randomized DSTC will be derived. As we mentioned in the first section, the randomized matrix will be considered in the derivation and it affects the performance by reducing the upper bound of the pairwise error probability. For the sake of simplicity, we consider a 2 by 2 MIMO system with 1 relay node, and the direct link is ignored in order to prominent the effect of the randomized matrix. The expression of the upper bound is also stable for the increase of the system size and the number of relay nodes.

Consider an  $N \times N$  STC scheme we use at the relay node with L codewords. The codeword  $C^1$  is transmitted and decoded to another codeword  $C^i$  at the destination node, where i = 1, 2, ..., L. According to [23], the probability of error can be upper bounded by the sum of all the probabilities of incorrect decoding, which is given by

$$\mathbf{P}_{\mathbf{e}} \le \sum_{i=2}^{L} \mathbf{P}(\boldsymbol{C}^{1} \to \boldsymbol{C}^{i}).$$
(12)

Assuming the codeword  $C^2$  is decoded at the destination node and we know the channel information perfectly at the destination node, we can derive the pairwise error probability as

$$P(\boldsymbol{C}^{1} \rightarrow \boldsymbol{C}^{2} \mid \boldsymbol{\Re})$$

$$= P(\parallel \boldsymbol{R}^{1} - \boldsymbol{G}\boldsymbol{\Re}\boldsymbol{C}^{1} \parallel_{F}^{2} - \parallel \boldsymbol{R}^{1} - \boldsymbol{G}\boldsymbol{\Re}\boldsymbol{C}^{2} \parallel_{F}^{2} > 0 \mid \boldsymbol{\Re}_{eq})$$

$$= P(\parallel \boldsymbol{r}^{1} - \boldsymbol{\Re}_{eq}\boldsymbol{G}_{eq}\boldsymbol{F}\boldsymbol{s}^{1} \parallel_{F}^{2} - \parallel \boldsymbol{r}^{1} - \boldsymbol{\Re}_{eq}\boldsymbol{G}_{eq}\boldsymbol{F}\boldsymbol{s}^{2} \parallel_{F}^{2} > 0 \mid \boldsymbol{\Re}_{eq}), \qquad (13)$$

where F and  $G_{eq}$  stand for the channel coefficient matrix between the source node and the relay node, and between the relay node and the destination node, respectively. The randomized matrix is denoted by  $\Re_{eq}$ . Define  $H = G_{eq}F$ , which stands for the total channel coefficients matrix. After the calculation, we can transfer the pairwise error probability expression in (13) to

$$P(\boldsymbol{C}^1 \to \boldsymbol{C}^2 \mid \boldsymbol{\Re}_{eq}) = P(\parallel \boldsymbol{\Re}_{eq} \boldsymbol{H}(\boldsymbol{s}^1 - \boldsymbol{s}^2) \parallel_F^2 < \boldsymbol{Y}), (14)$$

where  $\mathbf{Y} = \text{Tr}(\mathbf{n}^{1^{H}} \mathfrak{R}_{eq} \mathbf{H}(\mathbf{s}^{1} - \mathbf{s}^{2}) + (\mathfrak{R}_{eq} \mathbf{H}(\mathbf{s}^{1} - \mathbf{s}^{2}))^{H} \mathbf{n}^{1})$ , and  $\mathbf{n}^{1}$  denotes the noise vector at the destination node with zero mean and covariance matrix  $\sigma^{2}(1 + || \mathfrak{R}_{eq} \mathbf{G}_{eq} ||_{F}^{2})\mathbf{I}$ . By making use of the Q function, we can derive the error probability function as

$$P(\boldsymbol{C}^{1} \to \boldsymbol{C}^{2} \mid \boldsymbol{\mathfrak{R}}_{eq}) = Q\left(\sqrt{\frac{\gamma}{2}} \parallel \boldsymbol{\mathfrak{R}}_{eq}\boldsymbol{H}(\boldsymbol{s}^{1} - \boldsymbol{s}^{2}) \parallel_{F}\right),$$
(15)

where

$$Q = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{u^2}{2}\right) du,$$
 (16)

and  $\gamma$  is the received SNR at the destination node assuming the transmit power is equal to 1.

In order to obtain the upper bound of  $P(C^1 \to C^2 | \Re_{eq})$ we expand the formula  $|| \Re_{eq}H(s^1-s^2) ||_F^2$ . Let  $U^H \Lambda_s U$  be the eigenvalue decomposition of  $(s^1 - s^2)^H(s^1 - s^2)$ , where U is a Hermitian matrix and  $\Lambda_s$  contains all the eigenvalues of the difference between two different codewords  $s^1$  and  $s^2$ . Let  $V^H \Lambda_{\Re} V$  stand for the eigenvalue decomposition of  $(\Re_{eq}HU)^H \Re_{eq}HU$ , where V is a random Hermitian matrix and  $\Lambda_{\Re}$  is the ordered diagonal eigenvalue matrix. Therefore, the probability of error can be written as

$$P(\boldsymbol{C}^{1} \to \boldsymbol{C}^{2} \mid \boldsymbol{\mathfrak{R}}_{eq}) = Q\left(\sqrt{\frac{\gamma}{2} \sum_{m=1}^{NT} \sum_{n=1}^{N} \lambda_{\boldsymbol{\mathfrak{R}}_{m}} \lambda_{s_{n}} |\xi_{n,m}|^{2}}\right),$$
(17)

where  $\xi_{n,m}$  is the (n,m) - th element in V, and  $\lambda_{\mathfrak{R}_m}$  and  $\lambda_{s_n}$  are eigenvalues in  $\Lambda_{\mathfrak{R}}$  and  $\Lambda_s$ , respectively. According to [23], a good upper bound assumption of the Q function is given by

$$Q(x) \le \frac{1}{2}e^{\frac{-x^2}{2}}.$$
 (18)

Thus, we can derive the upper bound of pairwise error probability for a randomized STC scheme as

$$P(\boldsymbol{C}^{1} \to \boldsymbol{C}^{2} \mid \boldsymbol{\mathfrak{R}}_{eq}) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4} \sum_{m=1}^{NT} \sum_{n=1}^{N} \lambda_{\boldsymbol{\mathfrak{R}}_{m}} \lambda_{s_{n}} |\xi_{n,m}|^{2}\right)$$
(19)

while the upper bound of the error probability expression for a traditional STC is given by

$$P(\boldsymbol{C}^{1} \to \boldsymbol{C}^{2} \mid \boldsymbol{H}_{eq}) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4} \sum_{m=1}^{NT} \sum_{n=1}^{N} \lambda_{s_{n}} |\xi_{n,m}|^{2}\right).$$
(20)

With comparison of (19) and (20), it is obvious to note that the eigenvalue of the randomized matrix is the difference, which suggests that employing a randomized matrix for a STC scheme at the relay node can provide an improvement in BER performance.

# V. SIMULATIONS

The simulation results are provided in this section to assess the proposed scheme and algorithm. The system we considered is an AF cooperative MIMO system with the Alamouti STBC scheme [23] using QPSK modulation in quasi-static block fading channel with AWGN, as derived in Section II. The bit error ratio (BER) performance of the ARMO algorithm is assessed. The simulation system with 1 relay node and each transmitting and receiving node employs 2 antennas. In the simulation we define both the symbol power at the source node and the noise variance  $\sigma^2$  for each link to be equal to 1.

The upper bound of the D-Alamouti and the randomized D-Alamouti we derived in the pervious section are shown in Fig. 2. The theoretical pairwise error probabilities provide the largest decoding errors of the two different coding schemes and as shown in the figure, by employing a randomized matrix at the relay node decreases the decoding error upper bound. The comparison of the simulation results in BER performance of the R-Alamouti and the D-Alamouti indicates the advantage of using the randomized matrix.

The proposed ARMO algorithm is compared with the SM scheme and the traditional RSTC algorithm using the



Fig. 2. BER performance v.s.  $E_b/N_0$  for the upper bound of the R-Alamouti scheme without the Direct Link



Fig. 3. BER performance v.s.  $E_b/N_0$  for ARMO Algorithm with and without the Direct Link

distributed-Alamouti (D-Alamouti) STBC scheme in [19] with  $n_r = 1$  relay nodes in Fig. 3. The number of antennas N = 2 at each node and the effect of the direct link is considered. The results illustrate that without the direct link, by making use of the STC or the RSTC technique, a significant performance improvement can be achieved compared to the spatial multiplexing system, and the RSTC algorithm is outperforms the STC-AF system, while the ARMO algorithm can improve the performance by about 3dB as compared to the RSTC algorithm. With the consideration of the direct link, the results indicate that the cooperative diversity order can be increased, and using the ARMO algorithm achieves the optimal performance with 2dB gains compared to employing the RSTC algorithm and 3dB gains compared to employing the traditional STC-AF algorithm.

The simulation results shown in Fig. 4 illustrate the convergence property of the ARMO algorithm. The SM, D-



Fig. 4. BER performance v.s. Number of Samples for ARMO Algorithm without the Direct Link

Alamouti and the randomized D-Alamouti algorithms obtain nearly flat performance in BER as the utilization of fixed STC scheme and the randomized matrix. The SM scheme has the worst performance due to the lack of coding gains, while the D-Alamouti scheme can provide a significant performance improvement in terms of the BER improvement, and by employing the randomized matrix at the relay node the BER performance can decrease further when the transmission circumstances are the same as that of the D-Alamouti. The ARMO algorithm shows its advantage in a fast convergence and a lower BER achievement. At the beginning of the optimization process with a small number of samples, the ARMO algorithm achieves the BER level of the D-Alamouti one, but with the increase of the received symbols, the ARMO algorithm achieves a better BER performance.

### VI. CONCLUSION

We have proposed an adaptive randomized matrix optimization (ARMO) algorithm for the randomized DSTC using a linear MMSE receive filter at the destination node. The pairwise error probability of introducing the randomized DSTC in a cooperative MIMO network with the AF protocol is derived. The simulation results illustrate the advantage of the proposed ARMO algorithm by comparing it with the cooperative network employing the traditional DSTC scheme and the fixed randomized STC scheme. The proposed algorithm can be utilized with different distributed STC schemes using the AF strategy, extended to DF cooperation protocols and non-linear detectors.

#### References

- [1] A. Scaglione, D. L. Goeckel, and J. N. Laneman, "Cooperative Communications in Mobile Ad Hoc Networks", IEEE Signal Process. Magazine, pp. 1829, September 2006.
- J. N. Laneman and G. W. Wornell, "Cooperative Diversity in Wireless [2] Networks: Efficient Protocols and Outage Behaviour", IEEE Trans. Inf. *Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004. P. Clarke, R. C. de Lamare, "Joint Transmit Diversity Optimization and
- [3] Relay Selection for Multi-Relay Cooperative MIMO Systems Using Discrete Stochastic Algorithms",IEEE Communications Letters, vol. 15, no. 10, 2011, pp. 1035 - 1037.

- [4] P. Clarke, R. C. de Lamare, "Transmit Diversity and Relay Selection Algorithms for Multirelay Cooperative MIMO Systems", IEEE Trans. on Vehicular Technology, vol. 61, no. 3, 2012, pp. 1084 - 1098. J. N. Laneman and G. W. Wornell, "Distributed Space-Time-Coded
- Protocols for Exploiting Cooperative Diversity in Wireless Networks", IEEE Trans. Inf. Theory, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.
- S. Yiu, R. Schober, L. Lampe, "Distributed Space-Time Block Coding" IEEE Trans. on Wireless Commun., vol. 54, no. 7, pp. 1195-1206, Jul.
- M. Sarkiss, G.R.-B. Othman, M.O. Damen, J.-C. Belfiore, "Construc-[7] ion of New Delay-Tolerant Space-Time Codes", IEEE Transactions on Information Theory, vol. 57, Issue: 6, pp. 3567 - 3581, June 2011. J.-C. Belfiore, G. Rekaya, E. Viterbo, "The Golden Code: A 22 Full-Rate Space-Time Code with Nonvanishing Determinants", IEEE
- [8] Transactions on Information Theory, vol. 51, Issue: 4, April 2005. Yulong Zou, Yu-Dong Yao, Baoyu Zheng, "Opportunistic Distributed
- [9] Space-Time Coding for Decode-and-Forward Cooperation Systems", IEEE Transactions on Signal Processing, vol. 60, pp. 1766 - 1781, April 2012
- [10] J. Abouei, H. Bagheri, A. Khandani, "An Efficient Adaptive Distributed Space-Time Coding Scheme for Cooperative Relaying", *IEEE Trans.* on Wireless Commun., vol. 8, Issue: 10, pp. 4957-4962, October 2009.
  B. Maham, A. Hjorungnes, G. Abreu, "Distributed GABBA Space-
- [11] B. Maham, A. Hjorungnes, G. Abreu, Time Codes in Amplify-and-Forward Relay Networks", IEEE Trans.
- on Wireless Commun., vol.8, Issuse: 4, pp. 2036 2045, April 2009. [12] Sheng Yang, J.-C. Belfiore, "Optimal Space-Time Codes for the MIMO Amplify-and-Forward Cooperative Channel", IEEE Transactions on Information Theory, vol. 53, Issue: 2, pp. 647-663, Feb. 2007. T.Q. Duong, Ngoc-Tien Nguyen, Viet-Kinh Nguyen, "Exact Pairwise
- [13] Error Probability of Distributed Space-Time Coding in Wireless Relays Networks", International Symposium on Communications and Information Technologies 2007, pp. 279 - 283, 17-19 Oct. 2007. Minchul Ju, Hyoung-Kyu Song, Il-Min Kim, "Exact BER Analysis
- [14] of Distributed Alamouti's Code for Cooperative Diversity Networks", IEEE Trans. on Commun., vol. 57, Issue: 8, pp. 2380 - 2390, Aug. 2009
- [15] G.V.V. Sharma, V. Ganwani, U.B. Desai, S.N. Merchant, "Performance Analysis of Maximum Likelihood Detection for Decode and Forward MIMO Relay Channels in Rayleigh Fading", IEEE Trans. on Wireless Commun., vol. 9, Issue: 9, pp. 2880 - 2889, September 2010. Youngpil Song, Hyundong Shin, Een-Kee Hong, "MIMO Cooperative
- [16] Diversity with Scalar-Gain Amplify-and-Forward Relaying", IEEE Transactions on Communications, vol. 57, Issue: 7, pp. 1932 - 1938, July 2009.
- Liang Yang, Q.T. Zhang, "Performance Analysis of MIMO Relay [17]
- Lang Tang, Q.1. Zhang, Ferformance Anarysis of Armore Reary Wireless Networks With Orthogonal STBC", IEEE Transactions on Vehicular Technology, vol. 59, Issue: 7, pp. 3668 3674, Sept. 2010. T. Unger,A. Klein, "On the Performance of Distributed Space-Time Block Codes in Cooperative Relay Networks", IEEE Communications [18] T. Unger, A. Klein,
- Letters, vol. 11, Issue: 5, pp. 411 413, May 2007.
  [19] R.C. de Lamare, R. Sampaio-Neto, "Blind Adaptive MIMO Receivers for Space-Time Block-Coded DS-CDMA Systems in Multipath Channels Úsing the Constant Modulus Criterion", IEEE Trans. on Commun.,
- vol. 58, no. 1, Jan. 2010 S. Haykin, "Adaptive Filter Theory", 4th ed. Englewood Cliffs, NJ: [20] Prentice- Hall, 2002.
- [21] R. C. de Lamare and R. Sampaio-Neto, "Adaptive Reduced-Rank Processing Based on Joint and Iterative Interpolation, Decimation and Filtering", *IEEE Transactions on Signal Processing*, vol. 57, no. 7, July 2009, pp. 2503 - 2514.
- [22] B. Sirkeci-Mergen, A. Scaglione, "Randomized Space-Time Coding for Distributed Cooperative Communication", IEEE Transactions on Signal Processing, vol. 55, no. 10, Oct. 2007.
- [23] H. Jafarkhani, "Space-Time Coding Theory and Practice", Cambridge University Press, 2005.
- R.C. de Lamare, R. Sampaio-Neto, "Minimum Mean-Squared Error [24]Iterative Successive Parallel Arbitrated Decision Feedback Detectors for DS-CDMA Systems", IEEE Transactions on Communications, vol. 56, no. 5, May 2008, pp. 778 - 789.