Multi-Branch MMSE Decision Feedback Detection Algorithms with Error Propagation Mitigation for MIMO Systems

Rodrigo C. de Lamare † and Didier Le Ruyet ‡

† Communications Research Group, University of York, UK

‡ Conservatoire National des Arts et Métiers (CNAM), Paris, France.

Emails : rcdl500@ohm.york.ac.uk, leruyet@cnam.fr

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Introduction

– MIMO systems offer high performance and capacity but also present many design challenges [1].

– Interference in MIMO reduces capacity and performance.

– Mitigation of interference and exploitation of diversity $\rightarrow$ MIMO detectors

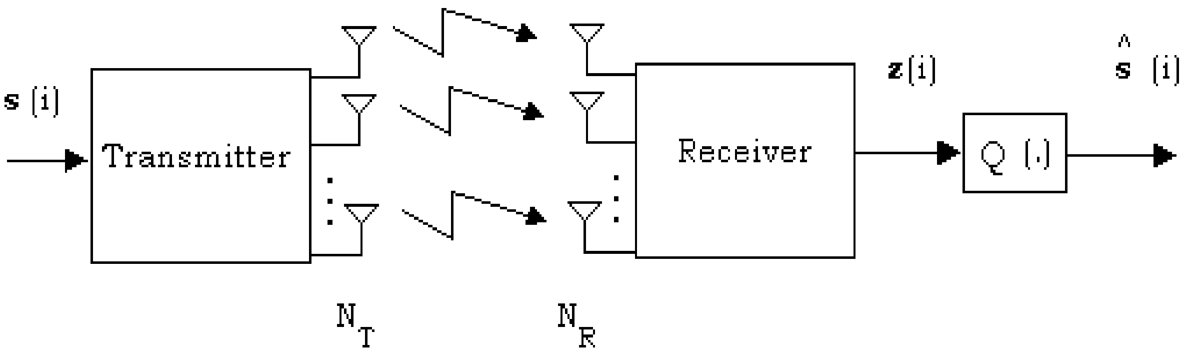
– MIMO detectors with different trade-offs between performance and complexity : ML, Sphere Decoder [3], lattice reduction, VBLAST with ordered successive interference cancellation (SIC) [2], Linear and Decision Feedback (DF) [6].

– Challenge $\rightarrow$ design of detectors with near ML performance and low complexity.

– Contributions : MMSE DF detector with multiple cancellation branches, MMSE design of the filters with shape and magnitude constraints, ordering strategies, and multistage scheme with the proposed detector .

MIMO System Model

– Consider a MIMO system with N_T transmit antennas and N_R receive antennas in a spatial multiplexing configuration.



– The signals are transmitted over single-path channels.

– We assume that the channel is constant during each packet transmission (block fading) and the receiver is perfectly synchronized.

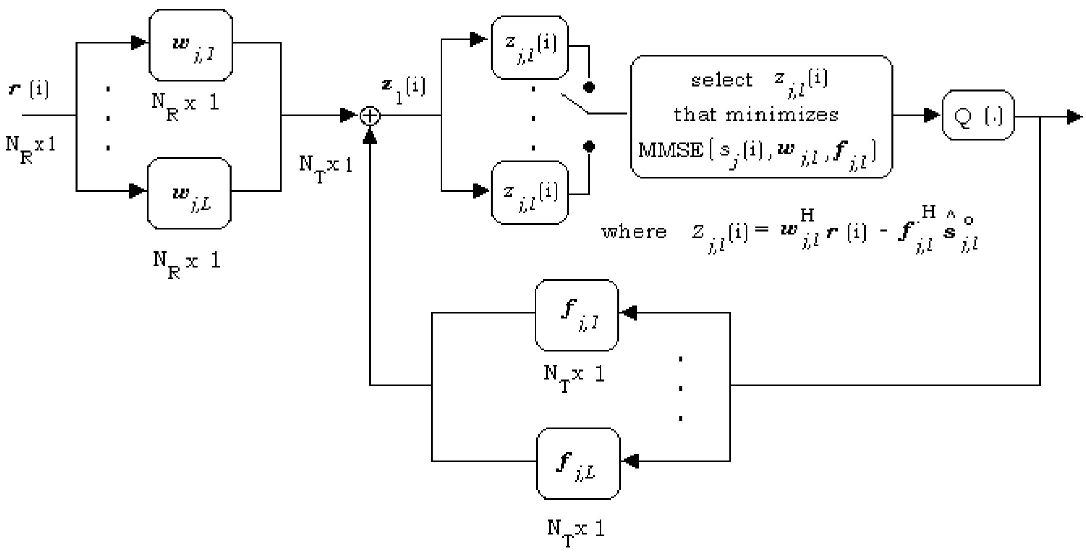
MIMO Data Model

- The received signal is applied to a matched filter, sampled and collected into a $N_R \times 1$ vector $\mathbf{r}[i]$ given by

$$\mathbf{r}[i] = \mathbf{H}\mathbf{s}[i] + \mathbf{n}[i],$$

- The $N_R \times 1$ vector $\mathbf{n}[i]$ is a zero mean complex circular symmetric Gaussian noise vector with $E\Big[\mathbf{n}[i]\mathbf{n}^H[i]\Big]$ $\sigma_n^2\mathbf{I}$, where σ_n^2 is the noise variance.
- The symbol vector $\mathbf{s}[i]$ has mean zero and a covariance matrix $E\Big[\mathbf{s}[i]\mathbf{s}^H[i]\Big] = \sigma_s^2\mathbf{I}$, where σ_s^2 is the signal power.
- The elements h_{n_R,n_T} of the $N_R \times N_T$ channel matrix $\mathbf{H}$ correspond to the complex channel response from the n_Tth transmit antenna to the n_Rth receive antenna.

Proposed Multi-Branch Decision Feedback Detection



– The proposed multi-branch detector considers the following combination of weights :

$$\mathbf{z}_{j,l}[i] = \mathbf{w}_{j,l}^{H}\mathbf{r}[i] - \mathbf{f}_{j,l}^{H}[i]\hat{\mathbf{s}}_{o}[i], \text{ for } l = 1, \ldots, L, \, m = 1,$$

– The $N_R \times 1$ vector $\mathbf{w}_{j,l}$ denotes the feedforward filter, the vector of initial decisions $\hat{\mathbf{s}}_o[i]$ is fed back through the $N_T \times 1$ feedback filter vector $\mathbf{f}_{j,l}[i]$.

Proposed Multi-Branch Decision Feedback Detection (cont.)

– The proposed MB-MMSE-DF detector selects the best branch according to

$$l_{\text{opt}} = \arg \min_{1 \leq l \leq L} \text{MMSE}(s_j[i], \boldsymbol{w}_{j,l}, \boldsymbol{f}_{j,l}),\ j = 1, \ldots, N_T$$

where $\text{MMSE}(s_j[i], \boldsymbol{w}_{j,l}, \boldsymbol{f}_{j,l})$ corresponds to the instantaneous MMSE produced by the pair of filters $\boldsymbol{w}_{j,l}$ and $\boldsymbol{f}_{j,l}$.

– The final detected symbol of the MB-MMSE-DF detector is obtained by :

$$\begin{aligned} \hat{s}_j[i] &= Q\Big[\boldsymbol{z}_{j,l_{\text{opt}}}[i]\Big] \\ &= Q\Big[\boldsymbol{w}^H_{j,l_{\text{opt}}}\boldsymbol{r}[i] - \boldsymbol{f}^H_{j,l_{\text{opt}}}\hat{\boldsymbol{s}}^o_{j,l_{\text{opt}}}[i]\Big],\ j = 1, \ldots, N_T \end{aligned}$$

where $Q(\cdot)$ is a slicing function that makes the decisions about the symbols, which is drawn from an M-PSK or a QAM constellation.

MMSE Filter Design

– The design of the MMSE filters of the proposed MB-MMSE-DF detector must solve the following optimization problem

$$\text{min MSE}(s_j[i], \boldsymbol{w}_{j,l}, \boldsymbol{f}_{j,l}) = E\Big[|s_j[i] - \boldsymbol{w}_{j,l}^H \boldsymbol{r}[i] + \boldsymbol{f}_{j,l}^H$$

$$\text{subject to } \boldsymbol{S}_{j,l}\boldsymbol{f}_{j,l} = \boldsymbol{v}_{j,l} \text{ and } ||\boldsymbol{f}_{j,l}||^2 = \gamma_{j,l}||\boldsymbol{f}_{j,l}^c$$

for $j = 1, \ldots, N_T$ and $l = 1, \ldots, L$,

where

-the $N_T \times N_T$ shape constraint matrix is $\boldsymbol{S}_{j,l}$,

-$\boldsymbol{v}_{j,l}$ is the resulting $N_T \times 1$ constraint vector and

-$\boldsymbol{f}_{j,l}^c$ is a feedback filter without constraints on the magnitude of its squared norm.

MMSE Filter Design (cont.)

– Expressions for $\boldsymbol{w}_{j,l}$ and $\boldsymbol{f}_{j,l}$ obtained after solving the optimization problem :

$$\boldsymbol{w}_{j,l} = \boldsymbol{R}^{-1}(\boldsymbol{p}_j + \boldsymbol{Q}\boldsymbol{f}_{j,l}),$$

$$\boldsymbol{f}_{j,l} = \beta_{j,l}\boldsymbol{\Pi}_{j,l}(\boldsymbol{Q}^H\boldsymbol{w}_{j,l} - \boldsymbol{t}_j) + (\boldsymbol{I} - \boldsymbol{\Pi}_{j,l})\boldsymbol{v}_{j,l},$$

where

$$\boldsymbol{\Pi}_{j,l} = \boldsymbol{I} - \boldsymbol{S}_{j,l}^H(\boldsymbol{S}_{j,l}^H\boldsymbol{S}_{j,l})^{-1}\boldsymbol{S}_{j,l}$$

is a projection matrix that ensures the shape constraint $\boldsymbol{S}_{j,l}$ and $\beta_{j,l} = (1 - \alpha_{j,l})^{-1}$ is a factor that adjusts the magnitude of the feedback, $0 \leq \beta_{j,l} \leq 1$ and $\alpha_{j,l}$ is the Lagrange multiplier.

– The $N_R \times N_R$ covariance matrix of the input data vector is $\boldsymbol{R} = E[\boldsymbol{r}[i]\boldsymbol{r}^H[i]]$, $\boldsymbol{p}_j = E[\boldsymbol{r}[i]s_j^*[i]]$, $\boldsymbol{Q} = E\Big[\boldsymbol{r}[i]\hat{\boldsymbol{s}}_{j,l}^{o,H}[i]\Big]$, and $\boldsymbol{t}_j = E[\hat{\boldsymbol{s}}_{j,l}^o[i]s_j^*[i]]$ is the $N_T \times 1$ vector of correlations between $\hat{\boldsymbol{s}}_{j,l}^o[i]$ and $s_j^*[i]$.

MMSE Filter Design (cont.)

– Simplification of the filter expressions (using the fact that the quantity $\boldsymbol{t}_j = \mathbf{0}$ for interference cancellation, $\boldsymbol{v}_{j,l} = \mathbf{0}$, and assuming perfect feedback ($\boldsymbol{s} = \hat{\boldsymbol{s}}$)) :

$$\boldsymbol{w}_{j,l} = \left(\boldsymbol{H}\boldsymbol{H}^H + \sigma_n^2/\sigma_s^2\boldsymbol{I}\right)^{-1}\boldsymbol{H}(\boldsymbol{\delta}_j + \boldsymbol{f}_{j,l})$$

$$\boldsymbol{f}_{j,l} = \beta_{j,l}\boldsymbol{\Pi}_{j,l}\left(\sigma_s^2\boldsymbol{H}^H\boldsymbol{w}_{j,l}\right),$$

where $\boldsymbol{\delta}_j = [\underbrace{0\ldots0}_{j-1}\ 1\ \underbrace{0\ldots0}_{N_T-j-2}]^T$ is a $N_T \times 1$ vector with a one in the jth element and zeros elsewhere.

– The proposed MB-MMSE-DF detector expressions above require the channel matrix $\boldsymbol{H}$ (in practice an estimate of it) and the noise variance σ_n^2 at the receiver.

MMSE Filter Design (cont.)

– In terms of complexity, it requires for each branch l the inversion of an $N_R \times N_R$ matrix and other operations with complexity $O(N_R^3)$. However, the matrix inversion is identical for all branches and the MB-MMSE-DFE only requires further additions and multiplications of the matrices.

– Moreover, we can verify that the filters $\boldsymbol{w}_{j,l}$ and $\boldsymbol{f}_{j,l}$ are dependent on one another, which means the designer has to iterate them before applying the detector.

– The MMSE associated with the pair of filters $\boldsymbol{w}_{j,l}$ and $\boldsymbol{f}_{j,l}$ and the statistics of data symbols $s_j[i]$ is given by

$$\text{MMSE}(s_j[i], \boldsymbol{w}_{j,l}, \boldsymbol{f}_{j,l}) = \sigma_s^2 - \boldsymbol{w}_{j,l}^H \boldsymbol{R} \boldsymbol{w}_{j,l} + \boldsymbol{f}_{j,l}^H \boldsymbol{f}_{j,l}$$

where $\sigma_s^2 = E[|s_j[i]|^2]$ is the variance of the desired symbol.

Design of Cancellation Patterns

- Design of the shape constraint matrices $\boldsymbol{S}_{j,l}$ and vectors $\boldsymbol{v}_{j,l}$: pre-stored patterns at the receiver for the N_T data streams and for the L branches.
- Basic idea : to shape the filters $\boldsymbol{f}_{j,l}$ for the N_T data streams and the L branches with the matrices $\boldsymbol{S}_{j,l}$ such that resulting constraint vectors $\boldsymbol{v}_{j,l}$ are null vectors.
- For the first branch of detection ($l = 1$), we can use the SIC approach and

$$\boldsymbol{S}_{j,l}\boldsymbol{f}_{j,l} = \mathbf{0}, \quad l = 1$$

$$\boldsymbol{S}_{j,l} = \begin{bmatrix} \mathbf{0}_{j-1} & \mathbf{0}_{j-1,N_T-j+1} \\ \mathbf{0}_{N_T-j+1,j-1} & \boldsymbol{I}_{N_T-j+1} \end{bmatrix}, \quad j = 1, \ldots$$

where $\mathbf{0}_{m,n}$ denotes an $m \times n$-dimensional matrix full of zeros, and $\boldsymbol{I}_m$ denotes an m-dimensional identity matrix.

Design of Cancellation Patterns (cont.)

– For the remaining branches, we adopt an approach based on permutations of the structure of the matrices $\boldsymbol{S}_{j,l}$, which is given by

$$\boldsymbol{S}_{j,l}\boldsymbol{f}_{j,l} = \mathbf{0}, \quad l = 2, \ldots, L$$
$$\boldsymbol{S}_{j,l} = \phi_l \left[\begin{array}{cc} \mathbf{0}_{j-1} & \mathbf{0}_{j-1,N_T-j+1} \\ \mathbf{0}_{N_T-j+1,j-1} & \boldsymbol{I}_{N_T-j+1} \end{array} \right], \quad j = 1,$$

where the operator $\phi_l[\cdot]$ permutes the columns of the argument matrix such that one can exploit different orderings via SIC.

– These permutations are straightforward to implement and allow the increase of the diversity order of the proposed MB-MMSE-DF detector.

– An alternative approach for shaping $\boldsymbol{S}_{j,l}$ for one of the L branches is to use a PIC approach and design the matrices as follows

$$\boldsymbol{S}_{j,l}\boldsymbol{f}_{j,l} = \mathbf{0}, \quad l$$
$$\boldsymbol{S}_{j,l} = \text{diag}\,(\boldsymbol{\delta}_j), \quad j = 1, \ldots, N_T,$$

Ordering Algorithm

– The proposed ordering algorithm for $l = 1, \ldots, L$ is given by

$$\{o_{1,l}, \ldots, o_{N_T,l}\} = \arg \min_{o_{1,l},\ldots,o_{N_T,l}} \sum_{l=1}^{L} \sum_{j=1}^{N_T} \mathrm{MMSE}(s_j[i],$$

– The ordering for the proposed MB-MMSE-DF detector is based on determining the optimal ordering for the first branch, which employs a SIC-based DFE, and then uses phase shifts for increasing the diversity for the remaining branches.
– The algorithm finds the optimal ordering for each branch. For a single branch detector this corresponds to the optimal ordering of the V-BLAST detector.
– The idea with the multiple branches and their orderings is to attempt to benefit a given data stream or group for each decoding branch.

Multistage Detection

- Main ideas : to combat error propagation by refining the decision vectors with multiple stages, and to equalize the performance over the data streams.
- The MB-MMSE-DF detector with M stages can be described by

  $$z_{j,l}^{(m+1)}(i) = \tilde{\boldsymbol{w}}_{j,l}^{H}\boldsymbol{r}[i] - \tilde{\boldsymbol{f}}_{j,l}^{H}\hat{\boldsymbol{s}}_{j,l}^{o,(m)}[i], \; m = 0, 1, \ldots, M$$

  where $\hat{\boldsymbol{s}}_{j,l}^{o,(m)}[i]$ is the vector of tentative decisions from the preceding iteration that is described by $\hat{s}_{k,j,l}^{o,(1)}[i] = \mathrm{Q}\Big(\boldsymbol{w}_{j,l}^{H}\boldsymbol{r}[i]\Big), \quad k = 1, \ldots, N_T$, and $\hat{s}_{k,j,l}^{o,(m)}[i] = \mathrm{Q}\Big(z_{j,l}^{(m)}[i]\Big), \; m = 2, \ldots, M$,.
- In order to equalize the performance over the data streams population, we consider an M-stage structure with output given by

  $$z_{j,l}^{(m+1)}[i] = [\boldsymbol{T}\boldsymbol{w}_{j,l}]^{H}\boldsymbol{r}[i] - [\boldsymbol{T}\boldsymbol{f}_{j,l}]^{H}]\hat{\boldsymbol{s}}_{j,l}^{0,(m)}[i] \tag{1}$$

  where $z_{j,l}^{(m+1)}[i]$ is the output of jth data stream and $\boldsymbol{T}$ is a square permutation matrix with ones along the reverse diagonal and zeros elsewhere.

Simulation Results

– We assess the bit error rate (BER) performance of the proposed and analyzed MIMO detection schemes.

– Compared schemes : the sphere decoder (SD) [3], the linear [5], the VBLAST [2], the S-DF [6], the lattice-reduction versions of the linear and the VBLAST detectors [**?**], the P-DF [7] and the proposed MB-MMSE-DF detector.

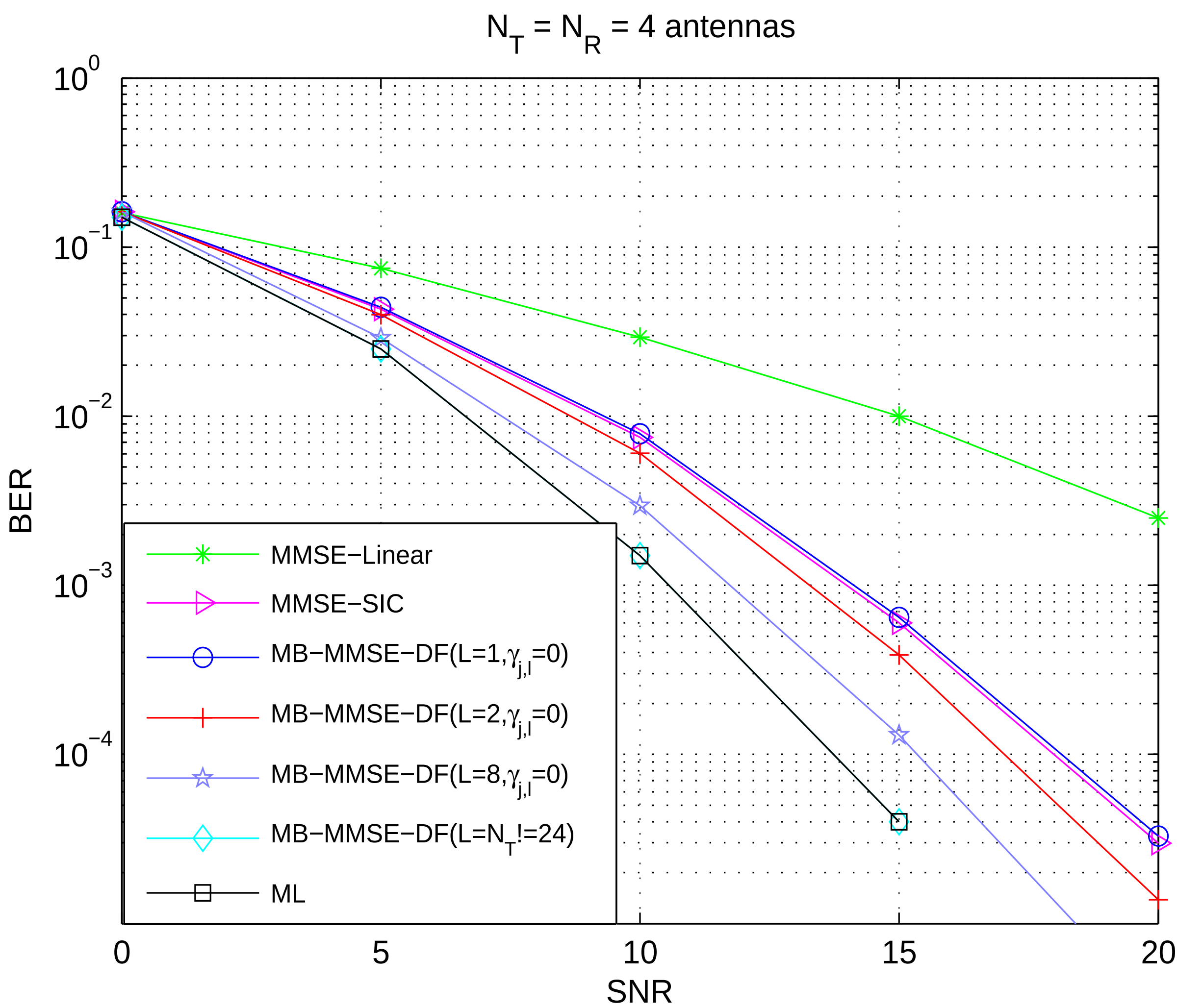
– The channels's coefficients are taken from complex Gaussian random variables with zero mean and unit variance.

– We employ QPSK modulation and use packets with $Q = 200$ symbols.

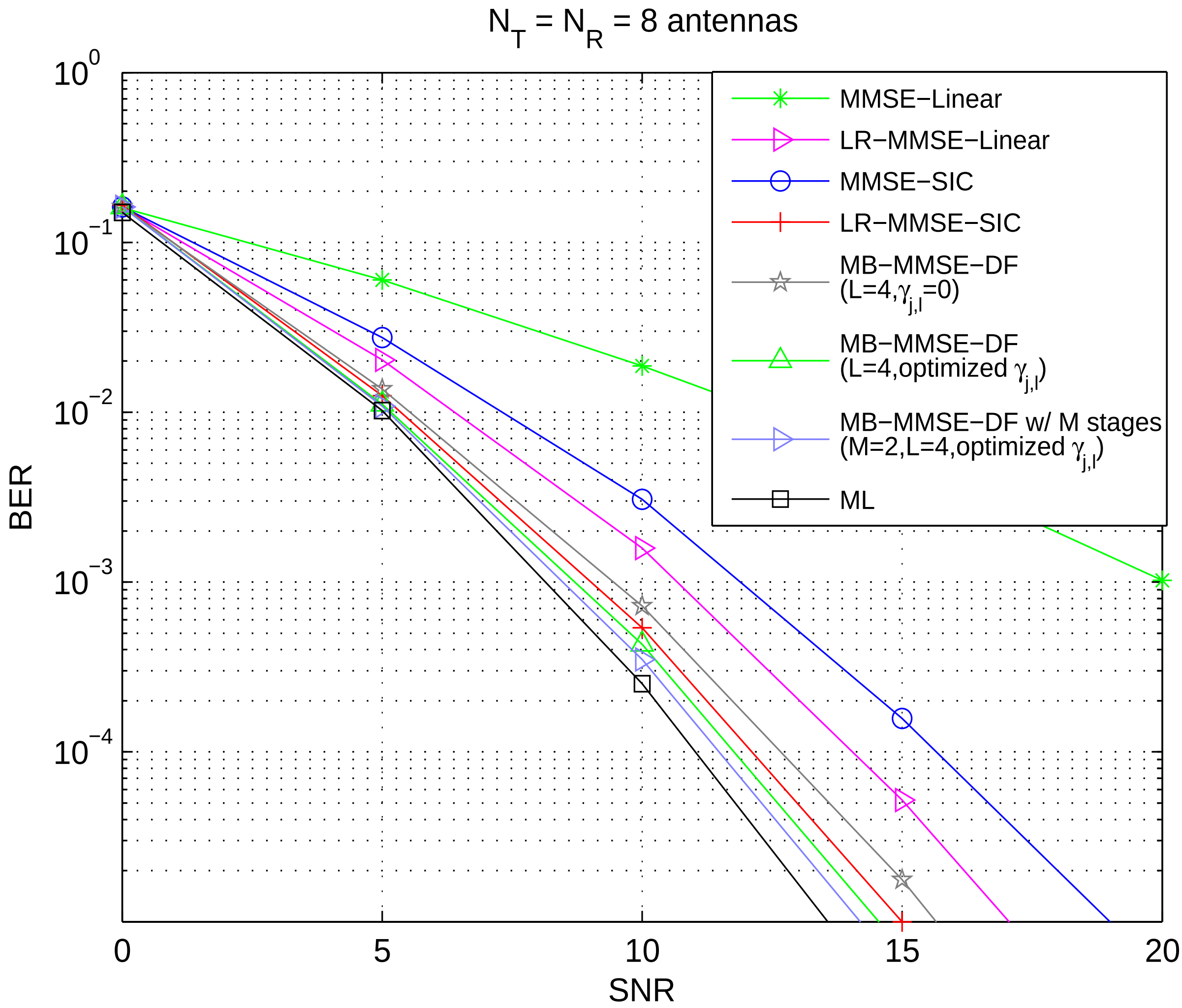
– We average the experiments over 10000 runs .

– The signal-to-noise ratio is defined as $\mathrm{SNR} = 10 \log_{10} \frac{N_T}{\sigma}$ where σ_s^2 is the variance of the symbols and σ^2 is the noise variance.

– BER Performance of the detectors with perfect decisions and channel estimation for multiple branches L .



– BER Performance with perfect decisions and estimates.



Conclusions

- A novel MMSE multi-branch decision feedback detector was proposed.
- The key strategy lies in the use of multiple branches for interference cancellation which allows further exploitation of the diversity at the receiver.
- We have derived MMSE expressions for the filters of the proposed multi-branch decision feedback detectors with shape and magnitude constraints on the feedback filters.
- The proposed multi-branch detector has a performance superior to the linear, VBLAST and existing DF detectors and very close to the ML detector, while it is simpler than the SD detector.
- Future work will investigate the performance with channel estimation, multiuser and multicell environments, iterative detection techniques.

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