GENERALIZED LOW-RANK DECOMPOSITIONS WITH SWITCHING AND ADAPTIVE ALGORITHMS FOR SPACE-TIME ADAPTIVE PROCESSING

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Outline

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Motivation

- Low-rank signal processing:
  - Key for dealing with high-dimensional data, low-sample support and large problems.
  - Faster convergence, enhanced tracking and improved robustness against interference.
  - Main idea: to devise a decomposition that performs dimensionality reduction so that the data can be represented by a reduced number of effective features.

- How does it work?
  - Two stage processing: a transformation matrix that performs dimensionality reduction and a low-rank filter.
  - The goal is to find an appropriate trade-off between the compression ratio and the reconstructed error.

Prior Work

- Principal component analysis (PCA):
  - Pearson, Hotelling, and others (early 1900s).
  - Eigen-decomposition or subspace tracking is required.
- Krylov subspace techniques:
  - Conjugate gradient algorithm by Hestenes and Stiefel (1952).
  - The multistage Wiener filter (MSWF) by Goldstein, Reed and Scharf (1998).
  - The auxiliary vector filtering (AVF) algorithm by Pados and Batallama (1997).
- Joint and iterative optimisation (JIO):
  - Iterative approach by Hua, Nikpour and Stoica (2001).
  - Alternating approach by de Lamare and Sampaio-Neto (2007).
  - The optimization algorithm dictates the performance and the complexity.
- Joint interpolation, decimation and filtering (JIDF):
  - By de Lamare and Sampaio-Neto (2007)
  - Use of switching with simple structures, very fast and powerful.

Contributions

- A scheme is devised to compute low-rank signal decompositions with switching techniques and adaptive algorithms, without eigen-decompositions.
- The generalized low-rank decomposition with switching (GLRDS) scheme computes the subspace and the low-rank filter that best match the problem.
- GLRDS imposes constraints on the decomposition and performs iterations between the computed subspace and the low-rank filter.
- An alternating optimization strategy with switching and iterations based on RLS algorithms is presented to compute the parameters.
- An application to multichannel space-time interference suppression in DS-CDMA systems is considered.
- Simulations show that the GLRDS scheme and algorithms obtain significant gains in performance over existing schemes.

Signal Model and Problem Statement

- Linear signal model:

$$\boldsymbol{r}[i] = \boldsymbol{H}\boldsymbol{s}[i] + \boldsymbol{n}[i], \quad i = 1, 2, \ldots, P$$

where

$\boldsymbol{H}$ is an M x M matrix that describes the mixing,

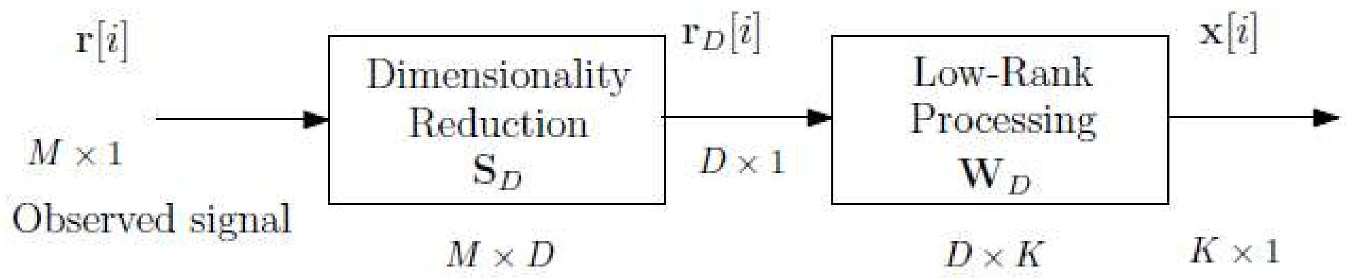
s[i] is an M x 1 vector with the signal,

n[i] is an M x 1 the noise vector,

[i] denotes the time instant and P is the data record.

- The signal processing scheme observes r[i] and performs linear filtering.

Low-Rank Signal Processing and Problem Statement



- Dimensionality reduction:

$$\boldsymbol{r}_D[i] = \boldsymbol{S}_D^H \boldsymbol{r}[i] = \sum_{d=1}^{D} \boldsymbol{s}_d^H \boldsymbol{r}[i] \boldsymbol{q}_d,$$

- Low-rank filtering:

$$\hat{\boldsymbol{x}}[i] = \boldsymbol{W}_D^H \boldsymbol{S}_D^H \boldsymbol{r}[i] = \boldsymbol{W}_D^H \sum_{d=1}^{D} \boldsymbol{s}_d^H \boldsymbol{r}[i] \boldsymbol{q}_d = \sum_{k=1}^{K} \boldsymbol{w}_{D,k}^H \Big(\sum_{d=1}^{D} \boldsymbol{s}_d^H \boldsymbol{r}[i] \boldsymbol{q}_d \Big) \boldsymbol{q}_k.$$

- Main problem: design of $\mathbf{S}_D$

Optimal Linear MMSE Design

- Optimization problem:

$$\left[\boldsymbol{S}_{D,\text{opt}}, \boldsymbol{W}_{D,\text{opt}}\right] = \arg \min_{\boldsymbol{S}_d, \boldsymbol{W}_D} E\left[||\boldsymbol{x}[i] - \boldsymbol{W}_D^H \boldsymbol{S}_D^H \boldsymbol{r}[i]||^2\right],$$

- Optimal linear MMSE low-rank filter:

$$\boldsymbol{W}_{D,\text{opt}} = \bar{\boldsymbol{R}}^{-1} \bar{\boldsymbol{P}} = \left(\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D\right)^{-1} \boldsymbol{S}_D^H \boldsymbol{P},$$

- Associated MMSE:

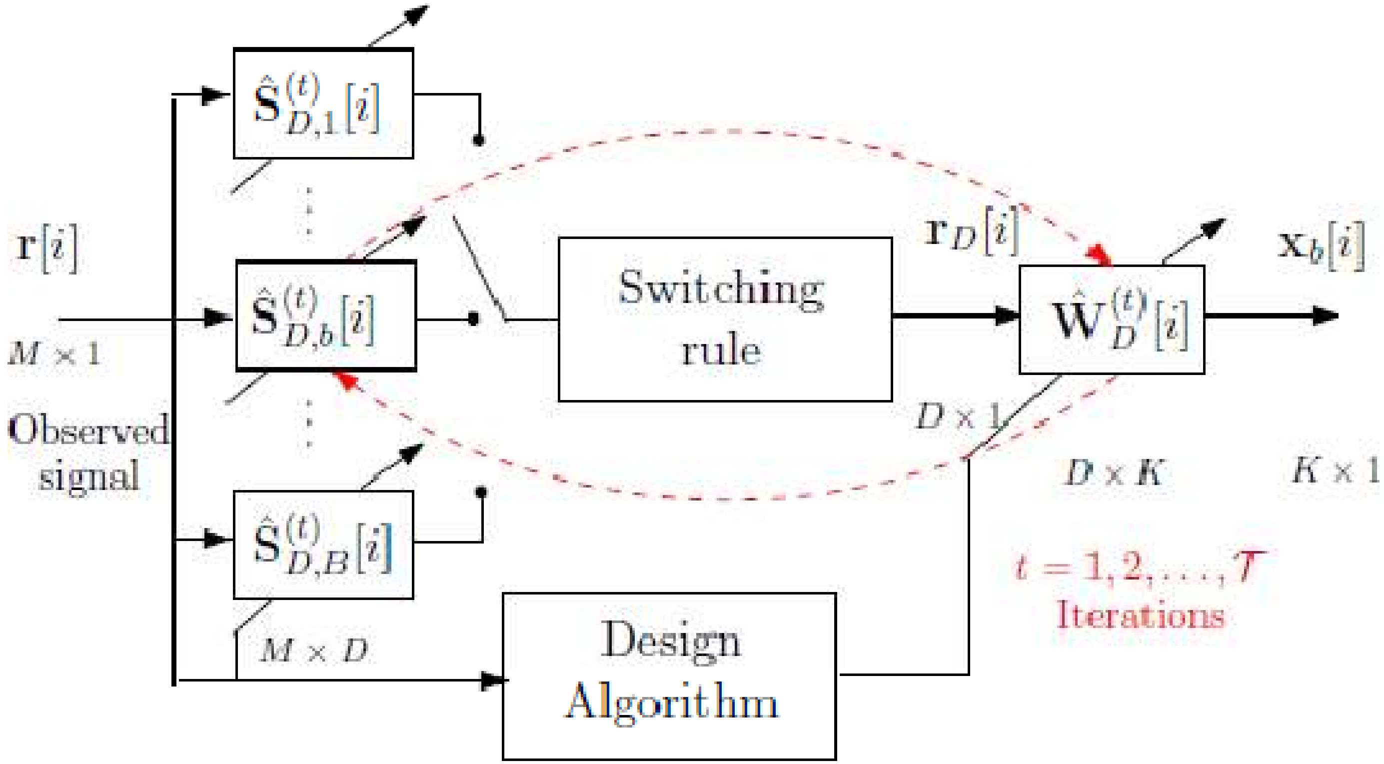
$$\text{MMSE} = \sigma_x^2 - \text{tr}\left[\boldsymbol{P}^H \boldsymbol{S}_D (\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D)^{-1} \boldsymbol{S}_D^H \boldsymbol{P}\right],$$

- Optimal dimensionality reduction (when **R** is known):

$$\boldsymbol{S}_{D,\text{opt}} = \boldsymbol{\Phi}_{1:M,1:D},$$

where $\boldsymbol{R} = \boldsymbol{\Phi} \boldsymbol{\Lambda} \boldsymbol{\Phi}^H$

Proposed GLRDS Scheme



GLRDS Scheme: Signal Processing Tasks

- Parameter Estimates :

$$\hat{\boldsymbol{x}}_b[i] = \boldsymbol{W}_D^H[i]\boldsymbol{S}_{D,b}^H[i]\boldsymbol{r}[i]$$
$$= \boldsymbol{W}_D^H[i]\Big(\sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{d}_{d,b}^H \boldsymbol{C}_{\boldsymbol{s}_{d,b}}[i]\Big)\boldsymbol{r}[i] = \boldsymbol{W}_D^H[i]\Big(\sum_{d=1}^{D} \boldsymbol{q}_d \boldsymbol{d}_{d,b}^H \boldsymbol{C}_{\boldsymbol{r}}[i]\Big)\boldsymbol{s}_{d,b}[i],$$

where the M x 1 vector is $\boldsymbol{d}_{d,b}[i] = [\underbrace{0 \ \ldots \ 0}_{\gamma_d \ zeros} \ \ 1 \ \ \underbrace{0 \ \ldots \ 0}_{(M-\gamma_j-1)\ zeros}]^T,$

and the M X D matrices $\boldsymbol{C}_{\boldsymbol{r}}[i]$ and $\boldsymbol{C}_{\boldsymbol{s}_{d,b}}[i]$ are Hankel matrices given by

$$\boldsymbol{C}_{\boldsymbol{r}}[i] = \begin{bmatrix} r_0^{[i]} & r_1^{[i]} & \ldots & r_{I_d-1}^{[i]} \\ r_1^{[i]} & r_2^{[i]} & \ldots & r_{I_d}^{[i]} \\ \vdots & \vdots & \ddots & \vdots \\ r_{M-2}^{[i]} & r_{M-1}^{[i]} & \ldots & 0 \\ r_{M-1}^{[i]} & 0 & \ldots & 0 \end{bmatrix}, \quad \boldsymbol{C}_{\boldsymbol{s}_{d,b}}[i] = \begin{bmatrix} s_{d,b,0}^{[i]} & s_{d,b,1}^{[i]} & \ldots & s_{d,b,I_d-1}^{[i]} \\ s_{d,b,1}^{[i]} & s_{d,b,2}^{[i]} & \ldots & s_{d,b,I_d}^{[i]} \\ \vdots & \vdots & \ddots & \vdots \\ s_{d,b,M-2}^{[i]} & s_{d,b,M-1}^{[i]} & \ldots & 0 \\ s_{s,b,M-1}^{[i]} & 0 & \ldots & 0 \end{bmatrix}.$$

GLRDS Scheme: LS Design

- Optimization problem:

$$\left[\boldsymbol{s}_{d,b}^{\text{opt}}, \boldsymbol{W}_D^{\text{opt}}\right] = \arg \min_{\boldsymbol{s}_{d,b}[i], \boldsymbol{W}_D[i]} \sum_{l=1}^{i} \lambda^{i-l} ||\boldsymbol{x}[l] - \hat{\boldsymbol{x}}_b[l]||^2 \Big], d = 1, \ \ldots, \ D, \ b = 1, \ \ldots, \ B.$$

- Decomposition parameters:

$$\boldsymbol{s}_{d,b}[i] = \boldsymbol{R}_{d,b}^{-1}[i]\Big(\boldsymbol{p}_{d,b}[i] - \sum_{j \neq d}^{D} \boldsymbol{P}_{j,b}[i]\boldsymbol{s}_{j,b}[i]\Big), d, j = 1, \ \ldots, \ D, \ \ b = 1, \ \ldots, \ B.$$

where $\boldsymbol{R}_{d,b}[i] = \sum_{l=1}^{i} \lambda^{i-l} \boldsymbol{q}_d^H \boldsymbol{W}_D[i] \boldsymbol{W}_D^H[i] \boldsymbol{q}_d \boldsymbol{C}_{\boldsymbol{r}}^T[l] \boldsymbol{d}_{d,b} \boldsymbol{d}_{d,b}^H \boldsymbol{C}_{\boldsymbol{r}}^*[l]$ is an I_d x I_d correlation matrix, $\boldsymbol{p}_{d,b}[i] = \sum_{l=1}^{i} \lambda^{i-l} \boldsymbol{x}^H[l] \boldsymbol{W}_D^H[i] \boldsymbol{q}_d \boldsymbol{C}_{\boldsymbol{r}}^T[l] \boldsymbol{d}_{d,b}$ is an I_d x I_d correlation matrix and $\boldsymbol{P}_{j,b}[i] = \sum_{l=1}^{i} \lambda^{i-l} \boldsymbol{q}_j^H \boldsymbol{W}_D[i] \boldsymbol{W}_D^H[i] \boldsymbol{q}_d \boldsymbol{C}_{\boldsymbol{r}}^T[l] \boldsymbol{d}_{d,b} \boldsymbol{d}_{j,b}^H \boldsymbol{C}_{\boldsymbol{r}}^*[l]$ and is an I_d x 1 cross-correlation vector.

GLRDS Scheme: LS Design (cont.)

- Switching:

$$\boldsymbol{S}_{D,b}[i] = \boldsymbol{S}_{D,b_{\mathsf{s}}}[i] \text{ when } b_{\mathsf{s}} = \arg \min_{1 \le b \le B} || \underbrace{\boldsymbol{x}[i] - \hat{\boldsymbol{x}}_b[i]}_{\boldsymbol{e}_b[i]} ||^2,$$

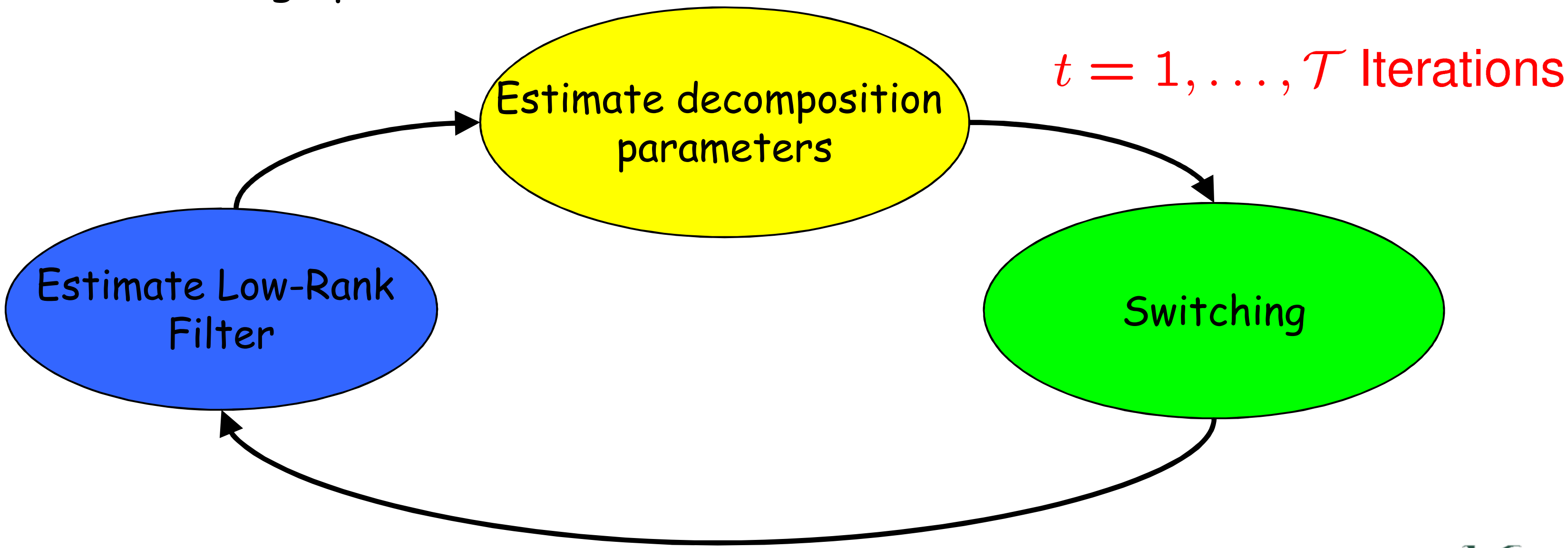
- Low-rank filter:

$$\boldsymbol{W}_D[i+1] = \boldsymbol{R}^{-1}[i]\boldsymbol{P}[i],$$

where $\boldsymbol{R}[i] = \sum_{l=1}^{i} \lambda^{i-l} \boldsymbol{S}_{d,b}^{H(t)}[l]\boldsymbol{r}[l]\boldsymbol{r}^H[l]\boldsymbol{S}_{d,b}^{(t)}[l]$ is an D x D correlation matrix and $\boldsymbol{P}^{(t)}[i] = \sum_{l=1}^{i} \lambda^{i-l} \boldsymbol{S}_{d,b}^{H(t)}[l]\boldsymbol{r}[l]\boldsymbol{x}^H[l]$ is a D x K cross-correlation vector.

Proposed Recursive Alternating Least Squares (RALS) Algorithm (1/3)

- Main strategy:
  - RALS-based algorithms -> complexity from cubic to quadratic in D.
  - Estimate subspace bases.
  - Perform switching.
  - Estimate low-rank filter.
  - Iterate between subspace bases and low-rank filter.
- Alternating optimisation:



Proposed RALS Algorithm (2/3)

- Estimate decomposition parameters:

$$\boldsymbol{s}^{(t)}_{d,b}[i] = \boldsymbol{P}_{d,b}[i]\Big(\boldsymbol{p}_{d,b}[i] - \sum_{j\neq d}^{D} \boldsymbol{P}_{j,b}[i]\boldsymbol{s}^{(t)}_{j,b}[i]\Big),$$

where

$$\boldsymbol{P}_{d,b}[i] = \lambda^{-1}\boldsymbol{P}_{d,b}[i-1] - \lambda^{-1}\boldsymbol{k}_{d,b}[i]\boldsymbol{C}^T_{\boldsymbol{r}}[i]\boldsymbol{d}_{d,b}\boldsymbol{P}_{d,b}[i-1],$$

$$\boldsymbol{k}_{d,b}[i] = \frac{\lambda^{-1}\boldsymbol{P}_{d,b}[i-1]\boldsymbol{d}^H_{d,b}\boldsymbol{C}^*_{\boldsymbol{r}}[i]}{(\sum_{k=1}^{K}|w_{d,k}[i]|^2)^{-1} + \lambda^{-1}\boldsymbol{d}^H_{d,b}\boldsymbol{C}^*_{\boldsymbol{r}}[i]\boldsymbol{P}_{d,b}[i-1]\boldsymbol{C}^T_{\boldsymbol{r}}[i]\boldsymbol{d}_{d,b}},$$

$$\boldsymbol{P}_{j,b}[i] = \lambda^{-1}\boldsymbol{P}_{j,b}[i-1] + \boldsymbol{q}^H_j\boldsymbol{W}_D[i]\boldsymbol{W}^H_D[i]\boldsymbol{q}_d\boldsymbol{C}^T_{\boldsymbol{r}}[i]\boldsymbol{d}_{d,b}\boldsymbol{d}^H_{j,b}\boldsymbol{C}^*_{\boldsymbol{r}}[i],$$

$$\boldsymbol{p}_{d,b}[i] = \lambda\boldsymbol{p}_{d,b}[i-1] + \boldsymbol{x}^H[i]\boldsymbol{W}^H_D[i]\boldsymbol{q}_d\boldsymbol{C}^T_{\boldsymbol{r}}[i]\boldsymbol{d}_{d,b}$$

- Switching:

$$\boldsymbol{S}^{(t)}_{D,b}[i] = \boldsymbol{S}^{(t)}_{D,b_s}[i] \text{ when } b_s = \arg\min_{1\leq b\leq B} ||\boldsymbol{x}[i] - \widehat{\boldsymbol{x}}^{(t)}_b[i]||^2,$$

Proposed RALS Algorithm (3/3)

- Compute the low-rank input data:

$$\boldsymbol{r}_D[i] = \boldsymbol{S}_{D,b_{\mathrm{S}}}^{H(t=1)}[i]\boldsymbol{r}[i]$$

- Estimate the low-rank filter:

$$\boldsymbol{W}_D^{(t)}[i+1] = \boldsymbol{W}_D[i] + \boldsymbol{k}_D[i]\boldsymbol{e}^{H^{(t)}}[i],$$

where $\boldsymbol{e}^{(t)}[i] = \boldsymbol{x}[i] - \hat{\boldsymbol{x}}_{b_{\mathrm{S}}}^{(t)}[i]$ and

$$\boldsymbol{k}_D[i] = \frac{\lambda^{-1}\boldsymbol{P}_D[i-1]\boldsymbol{r}_D[i]}{(1+\lambda^{-1}\boldsymbol{r}_D^H[i]\boldsymbol{P}_D[i-1]\boldsymbol{r}_D[i]},$$

$$\boldsymbol{P}_D[i] = \lambda^{-1}\boldsymbol{P}_D[i-1] - \lambda^{-1}\boldsymbol{k}_D[i]\boldsymbol{r}_D^H[i]\boldsymbol{P}_D[i-1],$$

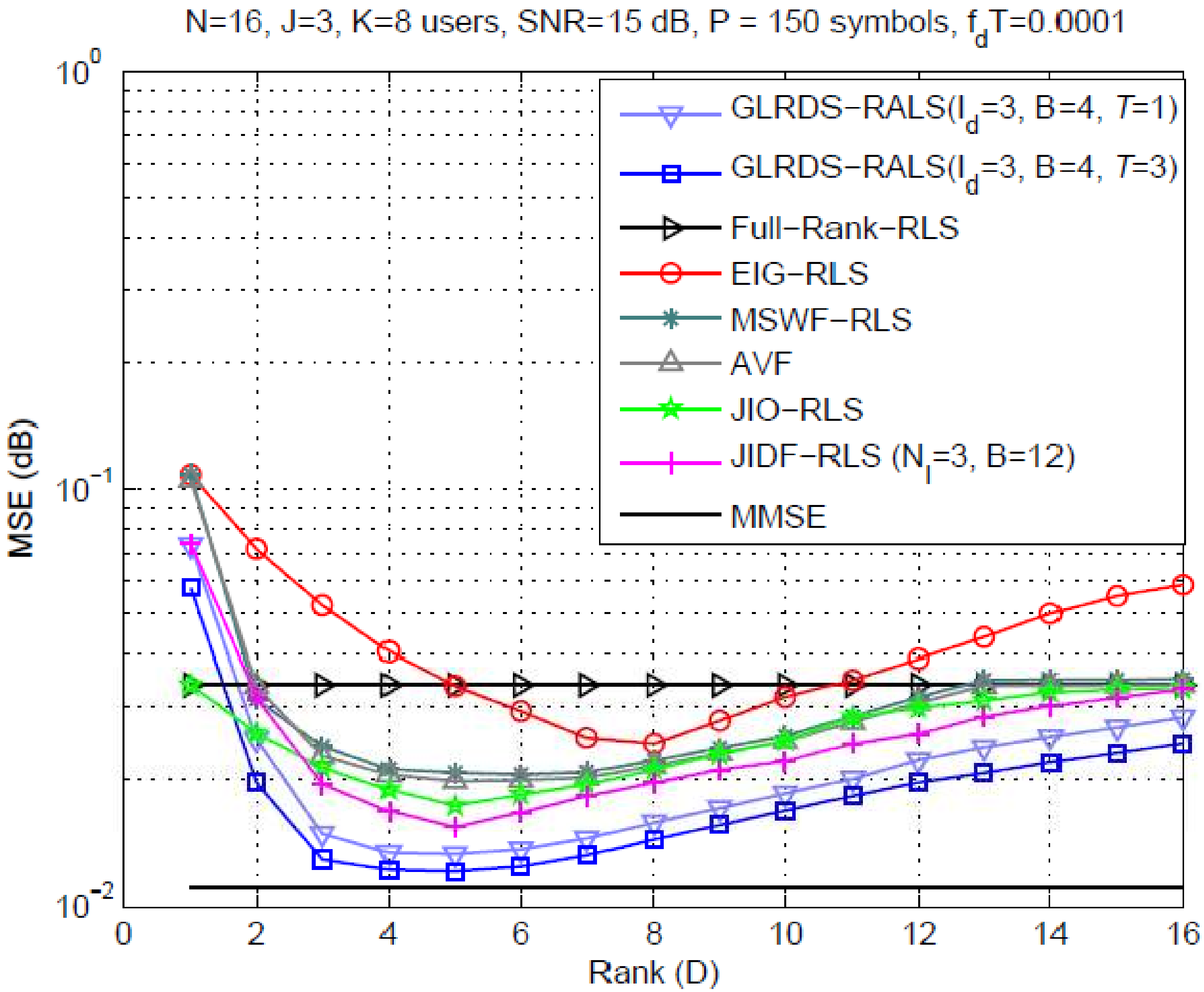
Simulations: Scenario and Parameters

- DS-CDMA system with random spreading codes, processing gain N=16, channel with delay spread of L chips, equipped with an antenna array of J sensor elements and linear receivers.
- We consider a space-time interference suppression application. The space-time received signal organised in JM x 1 vector, where M=J(N+L-1):

$$\mathbf{r}[i] = \sum_{k=1}^{K} A_k x_k[i] \boldsymbol{p}_k[i] + \boldsymbol{\eta}[i] + \boldsymbol{j}[i] + \boldsymbol{n}[i],$$

- We assess the BER of the following algorithms:
  - Proposed GLRDS scheme with RALS algorithm -> GLRDS-RALS.
  - Full-rank RLS algorithm -> Full-rank-RLS.
  - Low-rank eigendecomposition algorithm with RLS – EIG-RLS.
  - Multistage Wiener filter -> MSWF-RLS.
  - Auxiliary vector filtering algorithm -> AVF.
  - Joint and iteration optimization (JIO) scheme -> JIO –RLS.
  - Joint interpolation and decimation (JIDF) scheme -> JIDF-RLS.
  - Linear full-rank MMSE estimator -> MMSE
- The time-varying channels are modelled by an FIR filter and the Jakes model, have L=9, 3 effective paths with powers equal to 0, -3 and -6 dB and spacing given by a discrete random variable between 1 and 2 chips.

Simulations: BER X Rank



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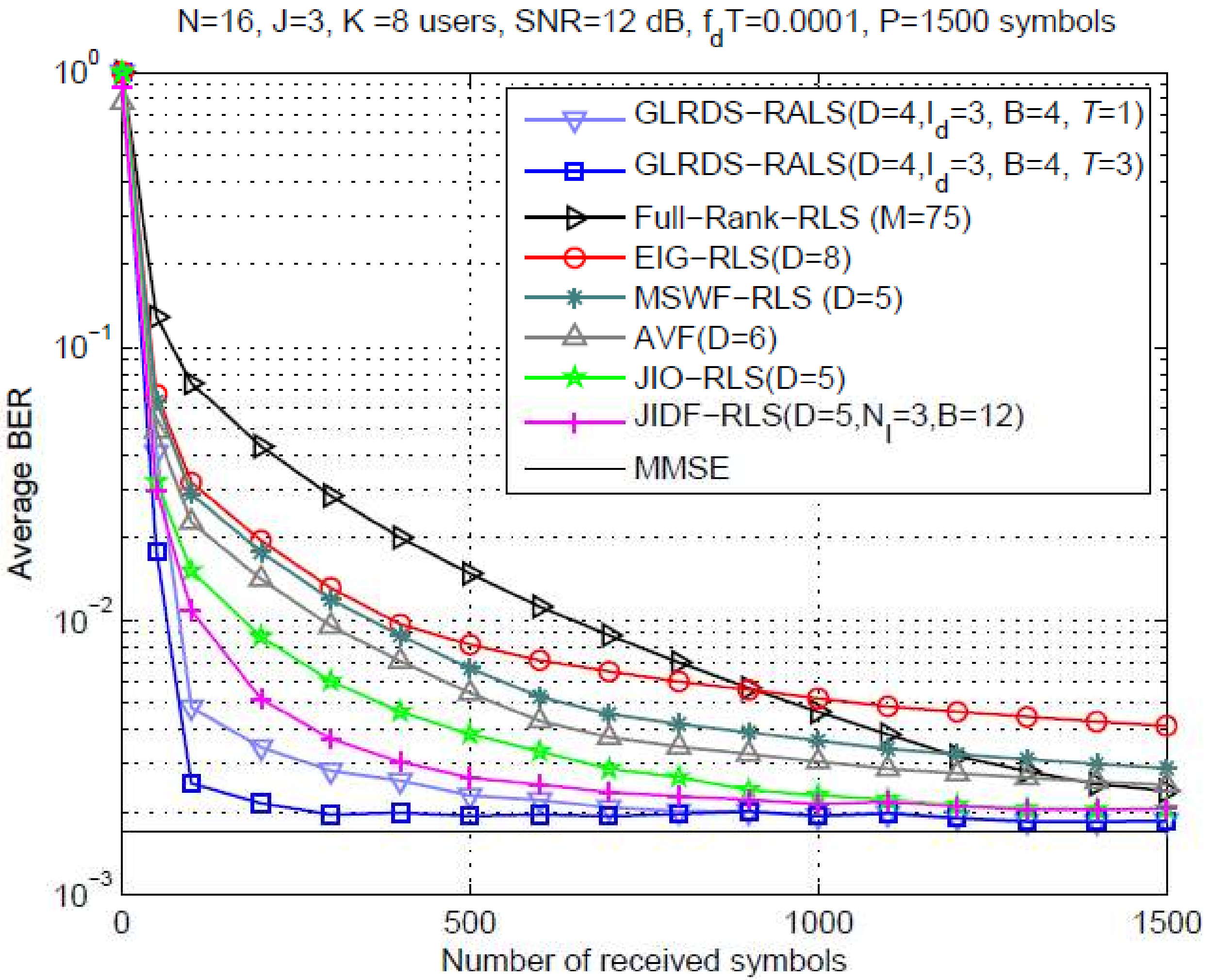
Simulations: BER X Symbols



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Conclusions

- A GLRDS scheme has been proposed and compared with the state-of-the-art low-rank signal processing techniques for space-time interference suppression.
- Adaptive RALS algorithms have been devised to estimate the parameters of the decomposition, the low-rank filter with the aid of switching techniques.
- The application of the GLRDS scheme with RALS for interference suppression has shown a performance significantly better than existing techniques.
- The proposed techniques require less training than prior art and can converge twice faster than the best available scheme.
- The complexity of the proposed algorithms is about 50% higher than the simplest available scheme, i.e. the JIDF approach.

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