GENERALIZED LOW-RANK DECOMPOSITIONS WITH SWITCHING AND ADAPTIVE ALGORITHMS FOR SPACE-TIME ADAPTIVE PROCESSING

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Motivation

- **Low-rank signal processing:**
  - Key for dealing with high-dimensional data, low-sample support and large problems.
  - Faster convergence, enhanced tracking and improved robustness against interference.
  - Main idea: to devise a decomposition that performs dimensionality reduction so that the data can be represented by a reduced number of effective features.

- **How does it work?**
  - Two stage processing: a transformation matrix that performs dimensionality reduction and a low-rank filter.
  - The goal is to find an appropriate trade-off between the compression ratio and the reconstructed error.
Prior Work

- **Principal component analysis (PCA):**
  - Pearson, Hotelling, and others (early 1900s).
  - Eigen-decomposition or subspace tracking is required.

- **Krylov subspace techniques:**
  - Conjugate gradient algorithm by Hestenes and Stiefel (1952).
  - The multistage Wiener filter (MSWF) by Goldstein, Reed and Scharf (1998).
  - The auxiliary vector filtering (AVF) algorithm by Pados and Batallama (1997).

- **Joint and iterative optimisation (JIO):**
  - Iterative approach by Hua, Nikpour and Stoica (2001).
  - The optimization algorithm dictates the performance and the complexity.

- **Joint interpolation, decimation and filtering (JIDF):**
  - By de Lamare and Sampaio-Neto (2007)
  - Use of switching with simple structures, very fast and powerful.
Contributions

- A scheme is devised to compute low-rank signal decompositions with switching techniques and adaptive algorithms, without eigen-decompositions.
- The generalized low-rank decomposition with switching (GLRDS) scheme computes the subspace and the low-rank filter that best match the problem.
- GLRDS imposes constraints on the decomposition and performs iterations between the computed subspace and the low-rank filter.
- An alternating optimization strategy with switching and iterations based on RLS algorithms is presented to compute the parameters.
- An application to multichannel space-time interference suppression in DS-CDMA systems is considered.
- Simulations show that the GLRDS scheme and algorithms obtain significant gains in performance over existing schemes.
Signal Model and Problem Statement

- Linear signal model:
  \[ r[i] = Hs[i] + n[i], \quad i = 1, 2, \ldots, P \]

  where

  \( H \) is an \( M \times M \) matrix that describes the mixing,

  \( s[i] \) is an \( M \times 1 \) vector with the signal,

  \( n[i] \) is an \( M \times 1 \) the noise vector,

  \([i]\) denotes the time instant and \( P \) is the data record.

- The signal processing scheme observes \( r[i] \) and performs linear filtering.
Low-Rank Signal Processing and Problem Statement

- Dimensionality reduction:

\[
\mathbf{r}_D[i] = \mathbf{S}_D^H \mathbf{r}[i] = \sum_{d=1}^{D} s_d^H \mathbf{r}[i] q_d,
\]

- Low-rank filtering:

\[
\hat{\mathbf{x}}[i] = \mathbf{W}_D^H \mathbf{S}_D^H \mathbf{r}[i] = \mathbf{W}_D^H \sum_{d=1}^{D} s_d^H \mathbf{r}[i] q_d = \sum_{k=1}^{K} \mathbf{w}_{D,k}^H \left( \sum_{d=1}^{D} s_d^H \mathbf{r}[i] q_d \right) q_k.
\]

- Main problem: design of \( \mathbf{S}_D \)
Optimal Linear MMSE Design

- **Optimization problem:**
  \[
  \left[ S_{D,\text{opt}}, W_{D,\text{opt}} \right] = \arg \min_{S_d, W_D} E \left[ \| x[i] - W_D^H S_D^H r[i] \|^2 \right],
  \]

- **Optimal linear MMSE low-rank filter:**
  \[ W_{D,\text{opt}} = \bar{R}^{-1} \bar{P} = (S_D^H R S_D)^{-1} S_D^H P, \]

- **Associated MMSE:**
  \[ \text{MMSE} = \sigma_x^2 - \text{tr} \left[ P^H S_D (S_D^H R S_D)^{-1} S_D^H P \right], \]

- **Optimal dimensionality reduction (when R is known):**
  \[ S_{D,\text{opt}} = \Phi_{1:M,1:D}, \]
  where \[ R = \Phi A \Phi^H \]
Proposed GLRDS Scheme
GLRDS Scheme: Signal Processing Tasks

Parameter Estimates:

\[
\hat{x}_b[i] = W_H^D[i] S_{D,b}^H[i] r[i] = W_H^D[i] \left( \sum_{d=1}^{D} q_d d_{d,b}^H C_{s_{d,b}[i]} \right) r[i] = W_H^D[i] \left( \sum_{d=1}^{D} q_d d_{d,b}^H C_{r}[i] \right) s_{d,b}[i],
\]

where the \( M \times 1 \) vector is \( d_{d,b}[i] = [0 \ldots 0 1 \underbrace{0 \ldots 0}_{\gamma_d \text{ zeros}} (M-\gamma_{j-1}) \text{ zeros}]^T \), and the \( M \times D \) matrices \( C_{r}[i] \) and \( C_{s_{d,b}[i]} \) are Hankel matrices given by

\[
C_r[i] = \begin{bmatrix}
    r_0[i] & r_1[i] & \cdots & r_{I_d-1}[i] \\
    r_1[i] & r_2[i] & \cdots & r_{I_d}[i] \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{M-2}[i] & r_{M-1}[i] & \cdots & 0 \\
    r_{M-1}[i] & 0 & \cdots & 0 \\
\end{bmatrix},
\]

\[
C_{s_{d,b}[i]} = \begin{bmatrix}
    s_{d,b,0}[i] & s_{d,b,1}[i] & \cdots & s_{d,b,I_d-1}[i] \\
    s_{d,b,1}[i] & s_{d,b,2}[i] & \cdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{d,b,M-2}[i] & s_{d,b,M-1}[i] & \cdots & 0 \\
    s_{d,b,M-1}[i] & 0 & \cdots & 0 \\
\end{bmatrix}.
\]
GLRDS Scheme: LS Design

- **Optimization problem:**
  
  \[
  [s_{d,b}^{\text{opt}}, W_D^{\text{opt}}] = \arg \min_{s_{d,b}[i], W_D[i]} \sum_{l=1}^{i} \lambda^{i-l} \| x[l] - \hat{x}_b[l] \|^2, \quad d = 1, \ldots, D, \quad b = 1, \ldots, B.
  \]

- **Decomposition parameters:**
  
  \[
  s_{d,b}[i] = R_{d,b}^{-1}[i] (p_{d,b}[i] - \sum_{j \neq d}^{D} P_{j,b}[i] s_{j,b}[i]), \quad d, j = 1, \ldots, D, \quad b = 1, \ldots, B.
  \]

  where \( R_{d,b}[i] = \sum_{l=1}^{i} \lambda^{i-l} q_d^H W_D[i] W_D^H[i] q_d C_r^T[l] d_{d,b} d_{d,b}^H C_r^*[l] \) is an \( I_d \times I_d \) correlation matrix, \( p_{d,b}[i] = \sum_{l=1}^{i} \lambda^{i-l} x^H[l] W_D^H[i] q_d C_r^T[l] d_{d,b} \) is an \( I_d \times I_d \) correlation matrix and \( P_{j,b}[i] = \sum_{l=1}^{i} \lambda^{i-l} q_j^H W_D[i] W_D^H[i] q_d C_r^T[l] d_{d,b} d_{j,b}^H C_r^*[l] \) and is an \( I_d \times 1 \) cross-correlation vector.
GLRDS Scheme: LS Design (cont.)

- **Switching:**
  
  $$S_{D,b}[i] = S_{D,b_s}[i] \text{ when } b_s = \arg \min_{1 \leq b \leq B} \| \hat{x}[i] - \hat{x}_b[i] \|_2^2,$$

- **Low-rank filter:**
  
  $$W_D[i+1] = R^{-1}[i]P[i],$$

where

$$R[i] = \sum_{l=1}^{i} \lambda^{i-l}S_{d,b}^{H(t)}[l]r[l]r^H[l]S_{d,b}^{(t)}[l]$$

is an $D \times D$ correlation matrix and

$$P^{(t)}[i] = \sum_{l=1}^{i} \lambda^{i-l}S_{d,b}^{H(t)}[l]r[l]x^H[l]$$

is a $D \times K$ cross-correlation vector.
Proposed Recursive Alternating Least Squares (RALS) Algorithm (1/3)

- **Main strategy:**
  - RALS-based algorithms -> complexity from cubic to quadratic in D.
  - Estimate subspace bases.
  - Perform switching.
  - Estimate low-rank filter.
  - Iterate between subspace bases and low-rank filter.

- **Alternating optimisation:**

  ![Diagram](Image)

  Estimate decomposition parameters
  $t = 1, \ldots, T$ Iterations

  Estimate Low-Rank Filter
  Switching

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Proposed RALS Algorithm (2/3)

- Estimate decomposition parameters:

\[
s_{d,b}^{(t)}[i] = P_{d,b}[i] \left( p_{d,b}[i] - \sum_{j \neq d}^{D} P_{j,b}[i] s_{j,b}^{(t)}[i] \right),
\]

where

\[
P_{d,b}[i] = \lambda^{-1} P_{d,b}[i-1] - \lambda^{-1} k_{d,b}[i] C_{r}^{T}[i] d_{d,b} P_{d,b}[i-1],
\]

\[
k_{d,b}[i] = \frac{\lambda^{-1} P_{d,b}[i-1] d_{d,b}^{H} C_{r}^{*}[i]}{\left( \sum_{k=1}^{K} |w_{d,k}[i]|^{2} \right)^{-1} + \lambda^{-1} d_{d,b}^{H} C_{r}^{*}[i] P_{d,b}[i-1] C_{r}^{T}[i] d_{d,b}},
\]

\[
P_{j,b}[i] = \lambda^{-1} P_{j,b}[i-1] + q_{j}^{H} W_{D}[i] W_{D}^{H}[i] q_{d} C_{r}^{T}[i] d_{d,b} a_{j,b}^{H} C_{r}^{*}[i],
\]

\[
p_{d,b}[i] = \lambda p_{d,b}[i-1] + x^{H}[i] W_{D}^{H}[i] q_{d} C_{r}^{T}[i] d_{d,b}
\]

- Switching:

\[
S_{D,b}^{(t)}[i] = S_{D,b_s}^{(t)}[i] \quad \text{when} \quad b_s = \arg \min_{1 \leq b \leq B} \| x[i] - \hat{x}_{b}^{(t)}[i] \|^{2},
\]
Proposed RALS Algorithm (3/3)

- Compute the low-rank input data:
  \[ r_D[i] = S_{D,b_S}^{H^{(t=1)}}[i]r[i] \]

- Estimate the low-rank filter:

  \[ W_D^{(t)}[i + 1] = W_D[i] + k_D[i]e^{H^{(t)}}[i], \]

  where \( e^{(t)}[i] = x[i] - \hat{x}_{bs}^{(t)}[i] \) and

  \[ k_D[i] = \frac{\lambda^{-1}P_D[i - 1]r_D[i]}{1 + \lambda^{-1}r_D^H[i]P_D[i - 1]r_D[i]}, \]

  \[ P_D[i] = \lambda^{-1}P_D[i - 1] - \lambda^{-1}k_D[i]r_D^H[i]P_D[i - 1], \]
Simulations: Scenario and Parameters

- DS-CDMA system with random spreading codes, processing gain N=16, channel with delay spread of L chips, equipped with an antenna array of J sensor elements and linear receivers.

- We consider a space-time interference suppression application. The space-time received signal organised in JM x 1 vector, where M=J(N+L-1):

  \[ r[i] = \sum_{k=1}^{K} A_k x_k[i] p_k[i] + \eta[i] + j[i] + n[i], \]

- We assess the BER of the following algorithms:
  - Proposed GLRDS scheme with RALS algorithm \( \rightarrow \) GLRDS-RALS.
  - Full-rank RLS algorithm \( \rightarrow \) Full-rank-RLS.
  - Low-rank eigendecomposition algorithm with RLS - EIG-RLS.
  - Multistage Wiener filter \( \rightarrow \) MSWF-RLS.
  - Auxiliary vector filtering algorithm \( \rightarrow \) AVF.
  - Joint and iteration optimization (JIO) scheme \( \rightarrow \) JIO-RLS.
  - Joint interpolation and decimation (JIDF) scheme \( \rightarrow \) JIDF-RLS.
  - Linear full-rank MMSE estimator \( \rightarrow \) MMSE

- The time-varying channels are modelled by an FIR filter and the Jakes model, have L=9, 3 effective paths with powers equal to 0, -3 and -6 dB and spacing given by a discrete random variable between 1 and 2 chips.
Simulations: BER $\times$ Rank

N=16, J=3, K=8 users, SNR=15 dB, P = 150 symbols, $f_dT=0.0001$

$\begin{align*}
\text{MSE (dB)} &\quad \text{Rank (D)} \\
10^0 &\quad 16 \\
10^{-1} &\quad 10 \\
10^{-2} &\quad 6 \\
10^{-3} &\quad 2 \\
10^{-4} &\quad 0
\end{align*}$

- GLRDS-RALS($l_d=3$, $B=4$, $T=1$)
- GLRDS-RALS($l_d=3$, $B=4$, $T=3$)
- Full-Rank-RLS
- EIG-RLS
- MSWF-RLS
- AVF
- IIO-RLS
- JIDF-RLS ($N_l=3$, $B=12$)
- MMSE
Simulations: BER X Symbols

$N=16, J=3, K=8$ users, $\text{SNR}=12$ dB, $f_dT=0.0001$, $P=1500$ symbols

- GLRDS-RALS($D=4, I_d=3, B=4, T=1$)
- GLRDS-RALS($D=4, I_d=3, B=4, T=3$)
- Full-Rank-RLS ($M=75$)
- Eig-RLS($D=8$)
- MSWF-RLS ($D=5$)
- AVF($D=6$)
- JIO-RLS($D=5$)
- JIDF-RLS($D=5, N_1=3, B=12$)
- MMSE

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Conclusions

- A GLRDS scheme has been proposed and compared with the state-of-the-art low-rank signal processing techniques for space-time interference suppression.

- Adaptive RALS algorithms have been devised to estimate the parameters of the decomposition, the low-rank filter with the aid of switching techniques.

- The application of the GLRDS scheme with RALS for interference suppression has shown a performance significantly better than existing techniques.

- The proposed techniques require less training than prior art and can converge twice faster than the best available scheme.

- The complexity of the proposed algorithms is about 50% higher than the simplest available scheme, i.e. the JIDF approach.
References


