

# Low-Complexity Lattice Reduction-Aided Channel Inversion Methods for Large Multi-User MIMO Systems

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**Abstract**—Low-complexity precoding algorithms are proposed in this work to reduce the computational complexity and improve the performance of regularized block diagonalization (RBD) based precoding schemes for large multi-user MIMO (MU-MIMO) systems. The proposed algorithms are based on a channel inversion technique, QR decompositions, and lattice reductions to decouple the MU-MIMO channel into equivalent SU-MIMO channels. Simulation results show that the proposed precoding algorithms can achieve almost the same sum-rate performance as RBD precoding, substantial bit error rate (BER) performance gains, and a simplified receiver structure, while requiring a lower complexity.

## I. INTRODUCTION

Block diagonalization (BD) based precoding techniques [1], [2] are well-known precoding strategies for multi-user multiple-input multiple-output (MU-MIMO) systems. By implementing two SVD operations, BD precoding can eliminate the multi-user interference (MUI). Since BD precoding focuses on canceling the MUI, it suffers a performance loss at low signal to noise ratios (SNRs) when the noise is the dominant factor. By relaxing the zero MUI constraint, the regularized block diagonalization (RBD) precoding scheme has been proposed in [3]. Instead of achieving strictly independent parallel channels between the users as BD precoding, RBD precoding allows a small level of interference between the users. Although a better performance is obtained by the RBD precoding, it still needs two SVD operations as BD precoding. As revealed in this paper, the computational complexity of the RBD precoding algorithm depends on the number of users and the dimensions of each user's channel matrix which could result in a considerable computational cost for large MIMO systems. The high cost of the two SVD operations required by the RBD precoding suggests that precoding algorithms with lower complexity should be investigated for use in very large MIMO systems.

In order to reduce the computational complexity, a generalized MMSE channel inversion (GMI) precoding algorithm has been proposed in [4] to implement the RBD precoding scheme. The first SVD operation of the RBD precoding is implemented by a matrix inversion method in GMI precoding. In [5], the first SVD operation of the RBD precoding is replaced with a less complex QR decomposition, and we term it as QR/SVD RBD precoding. For the second SVD operation, however, both the GMI and the QR/SVD RBD precoding algorithms require the same number of operations as the original RBD precoding

scheme. If the second SVD operation is implemented at the transmit side, then the corresponding unitary decoding matrix needs to be known by each distributed receiver, which requires an extra control overhead [6]. In this work, we develop a simplified GMI (S-GMI) method to obtain the first precoding filters. In order to reduce the complexity further and to obtain a better BER performance, we transform the equivalent SU-MIMO channels into the lattice space after the first precoding process by utilizing the lattice reduction (LR) technique [7] whose complexity is mainly due to a QR decomposition. Then, a linear precoding algorithm is employed instead of the second SVD operation to parallelize each user's streams.

The essential premise of using transmit processing techniques is the knowledge of the channel state information (CSI) at the transmitter [1] - [6]. In time-division duplexing (TDD) systems, CSI can be obtained at the BS by exploiting reciprocity between the forward and reverse links. In frequency-division duplexing (FDD) systems, reciprocity is usually not available, but the BS can obtain knowledge of the downlink user channels by allowing the users to send a small number of feedback bits on the uplink [8], [9]. We assume that full CSI is available at the transmit side since limited feedback techniques are not the main focus of this work. In this context, it is worth noting that the two SVD operations and the decoding matrix at each receiver are no longer required. The computational complexity is reduced and the receiver structure can be simplified. A significant amount of power can be saved which is very important considering the mobility of distributed users. For convenience, the proposed precoding algorithm is abbreviated as LR-S-GMI. According to the specific precoding constraint, the proposed LR-S-GMI precoding algorithms are categorized as LR-S-GMI-ZF and LR-S-GMI-MMSE precoding, respectively. We compare the proposed LR-S-GMI technique to the precoding algorithms reported in the literature including the BD, RBD, QR/SVD RBD and GMI precoding algorithms.

This paper is organized as follows. The system model is given in Section II. A brief review of the RBD precoding algorithms is presented in Section III. The proposed LR-S-GMI precoding algorithms are described in detail in Section IV. Simulation results and conclusions are presented in Section V and Section VI, respectively.

## II. SYSTEM MODEL

We consider an uncoded MU-MIMO downlink channel, with  $N_T$  transmit antennas at the base station (BS) and  $N_i$  receive antennas at the  $i$ th user equipment (UE). With  $K$  users in the system, the total number of receive antennas is  $N_R = \sum_{i=1}^K N_i$ . A block diagram of such a system is illustrated in Fig. 1. From the system model, the combined

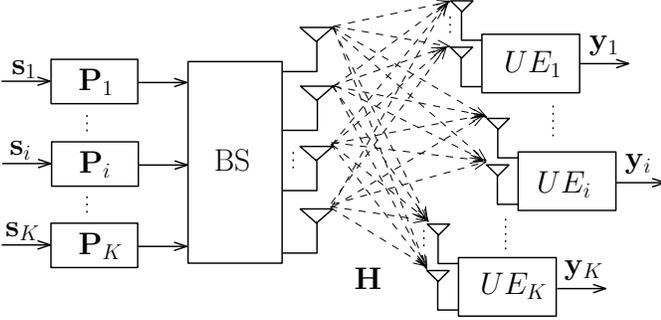


Fig. 1. MU-MIMO System Model

channel matrix  $\mathbf{H}$  and the combined precoding matrix  $\mathbf{P}$  of all users are given by

$$\mathbf{H} = [\mathbf{H}_1^T \ \mathbf{H}_2^T \ \dots \ \mathbf{H}_K^T]^T \in \mathbb{C}^{N_R \times N_T}, \quad (1)$$

$$\mathbf{P} = [\mathbf{P}_1 \ \mathbf{P}_2 \ \dots \ \mathbf{P}_K] \in \mathbb{C}^{N_T \times N_R}, \quad (2)$$

where  $\mathbf{H}_i \in \mathbb{C}^{N_i \times N_T}$  is the  $i$ th user's channel matrix. The quantity  $\mathbf{P}_i \in \mathbb{C}^{N_T \times N_i}$  is the  $i$ th user's precoding matrix. We assume a flat fading MIMO channel and the received signal  $\mathbf{y}_i \in \mathbb{C}^{N_i}$  at the  $i$ th user is given by

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{H}_i \sum_{j=1, j \neq i}^K \mathbf{x}_j + \mathbf{n}_i, \quad (3)$$

where the quantity  $\mathbf{x}_i \in \mathbb{C}^{N_i}$  is the  $i$ th user's transmit signal, and  $\mathbf{n}_i \in \mathbb{C}^{N_i}$  is the  $i$ th user's Gaussian noise with independent and identically distributed (i.i.d.) entries of zero mean and variance  $\sigma_n^2$ . Assuming that the average transmit power for user  $i$  is  $E_{s_i}$ , we construct a normalized signal  $\mathbf{x}_i$  such that

$$\mathbf{x}_i = \frac{\mathbf{s}_i}{\sqrt{\gamma_i}}, \quad (4)$$

where  $\mathbf{s}_i = \mathbf{P}_i \mathbf{d}_i$  with  $\mathbf{d}_i$  being the data vector,  $\gamma_i = \|\mathbf{s}_i\|_2^2 / E_{s_i}$ . With this normalization, the transmit signal  $\mathbf{x}_i$  obeys  $E\|\mathbf{x}_i\|_2^2 = E_{s_i}$  [10].

The received signal  $\mathbf{y}_i$  is weighted by the scalar  $\sqrt{\gamma_i}$  to form the estimate

$$\hat{\mathbf{d}}_i = \sqrt{\gamma_i} \mathbf{y}_i, \quad (5)$$

where the physical meaning of the scalar  $\sqrt{\gamma_i}$  is to make sure that the average transmit power  $E_{s_i}$  is still the same after the precoding process. Note that it is necessary to cancel  $\sqrt{\gamma_i}$  out at the receiver to get the correct amplitude of the desired signal part.

## III. REVIEW OF RBD PRECODING ALGORITHM

The design of the RBD precoding algorithm is performed in two steps [3]. The first precoding filter is used to balance the MUI with noise, then approximate parallel SU-MIMO channels are obtained. The second precoding filter is implemented to parallelize each user's streams. Correspondingly, the precoding matrix  $\mathbf{P}$  can be rewritten as

$$\mathbf{P} = \mathbf{P}^a \mathbf{P}^b, \quad (6)$$

where  $\mathbf{P}^a = [\mathbf{P}_1^a \ \mathbf{P}_2^a \ \dots \ \mathbf{P}_K^a]$  and  $\mathbf{P}^b = \text{diag}\{\mathbf{P}_1^b, \mathbf{P}_2^b, \dots, \mathbf{P}_K^b\}$ . We exclude the  $i$ th user's channel matrix and define  $\bar{\mathbf{H}}_i$  as

$$\bar{\mathbf{H}}_i = [\mathbf{H}_1^T \ \dots \ \mathbf{H}_{i-1}^T \ \mathbf{H}_{i+1}^T \ \dots \ \mathbf{H}_K^T]^T \in \mathbb{C}^{\bar{N}_i \times N_T}, \quad (7)$$

where  $\bar{N}_i = N_R - N_i$ . Thus, the interference generated to the other users is determined by  $\bar{\mathbf{H}}_i \mathbf{P}_i^a$ . In order to balance the MUI and the noise term, an RBD constraint is developed in [3] and given by

$$\mathbf{P}_i^a = \min_{\mathbf{P}_i^a} E\left\{ \sum_{i=1}^K \|\bar{\mathbf{H}}_i \mathbf{P}_i^a\|^2 + \gamma \|\mathbf{n}\|_F^2 \right\} \quad \text{s.t. } E\|\mathbf{x}\|_F^2 = E_s. \quad (8)$$

Assuming that the rank of  $\bar{\mathbf{H}}_i$  is  $\bar{L}_i$ , define the SVD of  $\bar{\mathbf{H}}_i$

$$\bar{\mathbf{H}}_i = \bar{\mathbf{U}}_i \bar{\Sigma}_i \bar{\mathbf{V}}_i^H = \bar{\mathbf{U}}_i \bar{\Sigma}_i [\bar{\mathbf{V}}_i^{(1)} \ \bar{\mathbf{V}}_i^{(0)}]^H, \quad (9)$$

where  $\bar{\mathbf{U}}_i \in \mathbb{C}^{\bar{N}_i \times \bar{N}_i}$  and  $\bar{\mathbf{V}}_i \in \mathbb{C}^{N_T \times N_T}$  are unitary matrices. The diagonal matrix  $\bar{\Sigma}_i \in \mathbb{C}^{\bar{N}_i \times N_T}$  contains the singular values of the matrix  $\bar{\mathbf{H}}_i$ . As shown in [3], the solution for the RBD constraint can be obtained as

$$\mathbf{P}_i^{a(\text{RBD})} = \bar{\mathbf{V}}_i (\bar{\Sigma}_i^T \bar{\Sigma}_i + \alpha \mathbf{I}_{N_T})^{-1/2}, \quad (10)$$

where  $\alpha = \frac{N_R \sigma_n^2}{E_s}$  is the regularization parameter.

After the first RBD precoding process, the MU-MIMO channel is decoupled into a set of  $K$  approximately parallel SU-MIMO channels. Due to the regularization process, there are small residual interferences between these channels, and these interferences tend to zero at high SNRs. Thus, the effective channel matrix for the  $i$ th user can be expressed as

$$\mathbf{H}_{\text{eff}_i} = \mathbf{H}_i \mathbf{P}_i^a. \quad (11)$$

Define  $L_{\text{eff}} = \text{rank}(\mathbf{H}_{\text{eff}_i})$  and consider the second SVD operation on the effective channel matrix

$$\mathbf{H}_{\text{eff}_i} = \mathbf{U}_i \Sigma_i \mathbf{V}_i^H, \quad (12)$$

using the unitary matrices  $\mathbf{U}_i \in \mathbb{C}^{L_{\text{eff}} \times L_{\text{eff}}}$  and  $\mathbf{V}_i \in \mathbb{C}^{N_T \times N_T}$ . The second precoding filters for RBD precoding can be obtained as

$$\mathbf{P}_i^{b(\text{RBD})} = \mathbf{V}_i \mathbf{\Lambda}^{(\text{RBD})}, \quad (13)$$

where  $\mathbf{\Lambda}$  is the power loading matrix that depends on the optimization criterion. An example power loading is the water filling (WF) [11]. The  $i$ th user's decoding matrix is obtained as

$$\mathbf{G}_i = \mathbf{U}_i^H, \quad (14)$$

which needs to be known by each user's receiver.

Note that for the conventional RBD precoding algorithm, there is a **dimensionality** constraint to be satisfied

$$N_T > \max\{\text{rank}(\overline{\mathbf{H}}_1), \text{rank}(\overline{\mathbf{H}}_2), \dots, \text{rank}(\overline{\mathbf{H}}_K)\}. \quad (15)$$

Then, we can get the matrix dimension relationship as  $L_{\text{eff}} \leq N_i < \overline{N}_i < N_R \leq N_T$ . Note that the first SVD operation in (9) needs to be implemented  $K$  times on  $\overline{\mathbf{H}}_i$  with dimension  $\overline{N}_i \times N_T$  and the second SVD operation in (12) needs to be implemented  $K$  times on  $\mathbf{H}_{\text{eff}_i}$  with dimension  $L_{\text{eff}} \times N_T$ . From the above analysis, most of the computational complexity of the RBD precoding algorithm comes from the two SVD operations which make the computational complexity of the RBD precoding algorithm **increase with** the number of users  $K$  and the system dimensions. In order to reduce the computational complexity of the RBD precoding algorithm, low complexity precoding algorithms for MU-MIMO systems are proposed in what follows.

#### IV. PROPOSED LOW COMPLEXITY LR-S-GMI PRECODING ALGORITHMS

In this section, we describe the proposed low-complexity LR-S-GMI precoding algorithms based on a strategy that employs a channel inversion method [4], QR decompositions, and lattice reductions. Similar to the RBD precoding algorithm, the design of the proposed LR-S-GMI precoding algorithms is computed in two steps.

First, we obtain  $\mathbf{P}_i^a$  in the conventional RBD precoding algorithm for the LR-S-GMI precoding algorithms by using one channel inversion and  $K$  QR decompositions. By applying the MMSE channel inversion, we have

$$\mathbf{H}_{\text{mse}}^\dagger = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I})^{-1} \mathbf{H}^H \quad (16)$$

$$= [\mathbf{H}_{1,\text{mse}}, \mathbf{H}_{2,\text{mse}}, \dots, \mathbf{H}_{K,\text{mse}}].$$

Considering a high SNR case, it can be shown that  $\mathbf{H} \mathbf{H}_{\text{mse}}^\dagger \approx \mathbf{I}_{N_T}$  [12]. This means that the off-diagonal block matrices of  $\mathbf{H} \mathbf{H}_{\text{mse}}^\dagger$  converge to zero as the SNR increases. Then, we obtain a condition which shows that  $\mathbf{H}_{i,\text{mse}}$  is in the null space of  $\overline{\mathbf{H}}_i$

$$\overline{\mathbf{H}}_i \mathbf{H}_{i,\text{mse}} \approx \mathbf{0}. \quad (17)$$

Considering the QR decomposition of  $\mathbf{H}_{i,\text{mse}} = \mathbf{Q}_{i,\text{mse}} \mathbf{R}_{i,\text{mse}}$ , we have

$$\overline{\mathbf{H}}_i \mathbf{H}_{i,\text{mse}} = \overline{\mathbf{H}}_i \mathbf{Q}_{i,\text{mse}} \mathbf{R}_{i,\text{mse}} \approx \mathbf{0} \text{ for } i = 1, \dots, K, \quad (18)$$

where  $\mathbf{R}_{i,\text{mse}} \in \mathbb{C}^{N_i \times N_i}$  is an upper triangular matrix and  $\mathbf{Q}_{i,\text{mse}} \in \mathbb{C}^{N_T \times N_i}$  is an orthogonal matrix. Since  $\mathbf{R}_{i,\text{mse}}$  is invertible, we have

$$\overline{\mathbf{H}}_i \mathbf{Q}_{i,\text{mse}} \approx \mathbf{0}. \quad (19)$$

Thus,  $\mathbf{Q}_{i,\text{mse}}$  satisfies the RBD constraint (8) to balance the MUI and the noise.

We have simplified the design of  $\mathbf{P}_i^a$  for the RBD precoding here as compared to [4] where a residual interference suppression filter  $\mathbf{T}_i$  is applied after  $\mathbf{P}_i^a$ . The filter  $\mathbf{T}_i$  increases the complexity and cannot completely cancel the MUI. Therefore,

we omit the residual interference suppression part since it is not necessary for the RBD precoding. We term the simplified GMI as S-GMI precoding in this work. Then, the first precoding matrix can be obtained as

$$\mathbf{P}_i^a = \mathbf{Q}_{i,\text{mse}}, \quad (20)$$

and the first combined precoding matrix is

$$\mathbf{P}^a = [\mathbf{P}_1^a, \mathbf{P}_2^a, \dots, \mathbf{P}_K^a]. \quad (21)$$

Next, we employ the LR-aided linear precoding algorithm instead of the second SVD operation to obtain  $\mathbf{P}_i^b$  and **parallelize** each user's streams. The aim of the LR transformation is to find a new basis  $\tilde{\mathbf{H}}$  which is nearly orthogonal compared to the original matrix  $\mathbf{H}$  for a given lattice  $L(\mathbf{H})$ . The most commonly used LR algorithm has been first proposed by Lenstra, Lenstra and L. Lovász (LLL) in [13] with polynomial time complexity. In order to reduce the computational complexity, a complex LLL (CLLL) algorithm was proposed in [7], which reduces the overall complexity of the LLL algorithm **by nearly half** without sacrificing any performance. We employ the CLLL algorithm to implement the LR transformation in this work.

After the first precoding, we transform the MU-MIMO channel into approximate parallel SU-MIMO channels and the effective channel matrix for the  $i$ th user is

$$\mathbf{H}_{\text{eff}_i} = \mathbf{H}_i \mathbf{P}_i^a. \quad (22)$$

We perform the LR transformation on  $\mathbf{H}_{\text{eff}_i}^T$  in the precoding scenario [14], that is

$$\tilde{\mathbf{H}}_{\text{eff}_i} = \mathbf{T}_i \mathbf{H}_{\text{eff}_i} \text{ and } \mathbf{H}_{\text{eff}_i} = \mathbf{T}_i^{-1} \tilde{\mathbf{H}}_{\text{eff}_i}, \quad (23)$$

where  $\mathbf{T}_i$  is a unimodular matrix ( $\det|\mathbf{T}_i| = 1$ ) and all elements of  $\mathbf{T}_i$  are complex integers, i.e.  $t_{l,k} \in \mathbb{Z} + j\mathbb{Z}$ .

Following the LR transformation, we employ the linear precoding constraint to get the second precoding matrix instead of the second SVD operation in (12). The ZF precoding constraint is implemented for user  $i$  as

$$\tilde{\mathbf{P}}_{\text{ZF}_i}^b = \tilde{\mathbf{H}}_{\text{eff}_i}^H (\tilde{\mathbf{H}}_{\text{eff}_i} \tilde{\mathbf{H}}_{\text{eff}_i}^H)^{-1}. \quad (24)$$

It is well-known that the performance of MMSE precoding is always better than that of ZF precoding. We can get the second precoding filter by employing an MMSE precoding constraint. The MMSE precoding is actually equivalent to ZF precoding with respect to an extended system model [15], [16]. The extended channel matrix  $\underline{\mathbf{H}}$  for the MMSE precoding scheme is defined as

$$\underline{\mathbf{H}} = [\mathbf{H}, \sqrt{\alpha} \mathbf{I}_{N_R}]. \quad (25)$$

By introducing the regularization factor  $\alpha$ , a trade-off between the level of MUI and noise is introduced [12]. Then, the MMSE precoding filter is obtained as

$$\mathbf{P}_{\text{MMSE}} = \mathbf{A} \underline{\mathbf{H}}^H (\underline{\mathbf{H}} \underline{\mathbf{H}}^H)^{-1}, \quad (26)$$

where  $\mathbf{A} = [\mathbf{I}_{N_T}, \mathbf{0}_{N_T, N_R}]$ , and the multiplication of  $\mathbf{A}$  will not result in transmit power amplification since  $\mathbf{A} \mathbf{A}^H = \mathbf{I}_{N_T}$ . From the mathematical expression in (26), the rows of  $\underline{\mathbf{H}}$

determine the effective transmit power amplification of the MMSE precoding. Correspondingly, the LR transformation for MMSE precoding should be applied to the transpose of the extended channel matrix  $\underline{\mathbf{H}}_{\text{eff}_i}^T = [\mathbf{H}_{\text{eff}_i}, \sqrt{\alpha}\mathbf{I}_{N_i}]^T$  for the MMSE precoding, and the LR transformed channel matrix  $\tilde{\underline{\mathbf{H}}}_{\text{eff}_i}$  is obtained as

$$\tilde{\underline{\mathbf{H}}}_{\text{eff}_i} = \underline{\mathbf{T}}_i \underline{\mathbf{H}}_{\text{eff}_i}. \quad (27)$$

Then, the LR-aided MMSE precoding filter is given by

$$\tilde{\mathbf{P}}_{\text{MMSE}_i}^b = \mathbf{A}_i \tilde{\underline{\mathbf{H}}}_{\text{eff}_i}^H (\tilde{\underline{\mathbf{H}}}_{\text{eff}_i} \tilde{\underline{\mathbf{H}}}_{\text{eff}_i}^H)^{-1}. \quad (28)$$

Finally, the second precoding matrix  $\tilde{\mathbf{P}}^b$  for all users is

$$\tilde{\mathbf{P}}^b = \text{diag}\{\tilde{\mathbf{P}}_1^b, \tilde{\mathbf{P}}_2^b, \dots, \tilde{\mathbf{P}}_K^b\}. \quad (29)$$

The resulting precoding matrix is  $\tilde{\mathbf{P}} = \mathbf{P}^a \tilde{\mathbf{P}}^b$ . Since the lattice reduced precoding matrix  $\tilde{\mathbf{P}}^b$  has near orthogonal columns, the required transmit power will be reduced compared to the BD or RBD precoding algorithms. Thus, a better BER performance than the RBD precoding algorithm can be achieved by the proposed LR-S-GMI precoding algorithms.

The received signal is finally obtained as

$$\mathbf{y} = \mathbf{H} \tilde{\mathbf{P}} \mathbf{d} + \sqrt{\gamma} \mathbf{n}. \quad (30)$$

The main processing work left for the receiver is to quantize the received signal  $\mathbf{y}$  to the nearest data vector and the decoding matrix  $\mathbf{G}$  in (14) is not needed anymore.

The proposed precoding algorithms are called LR-S-GMI-ZF and LR-S-GMI-MMSE depending on the choice of the second precoding filter as given in (24) and (28), respectively. We will focus on the LR-S-GMI-MMSE since a better performance is achieved. The implementing steps of the LR-S-GMI-MMSE precoding algorithm are summarized in Table 1. By replacing the steps 8 and 9 in Table I with the formulation in (24), the LR-S-GMI-ZF precoding algorithm can be obtained.

## V. SIMULATION RESULTS

A system with  $N_T = 8$  transmit antennas and  $K = 4$  users each equipped with  $N_i = 2$  receive antennas is considered; this scenario is denoted as [the](#)  $(2, 2, 2, 2) \times 8$  case. The vector  $\mathbf{d}_i$  of the  $i$ th user represents data transmitted with QPSK modulation.

The channel matrix  $\mathbf{H}_i$  of the  $i$ th user is modeled as a complex Gaussian channel matrix with zero mean and unit variance. We assume an uncorrelated block fading channel. We also assume that the channel estimation is perfect at the receive side and the feedback channel is error free. The number of simulation trials is  $10^6$  and the packet length is  $10^2$  symbols. The  $E_b/N_0$  is defined as  $E_b/N_0 = \frac{N_R E_s}{N_T M N_0}$  with  $M$  being the number of transmitted information bits per channel symbol.

Fig. 2. shows the BER performance of the proposed and existing precoding algorithms. The QR/SVD RBD and GMI precoding algorithms achieve almost the same BER performance as the conventional RBD precoding. It is clear that the S-GMI precoding has a better BER performance compared to BD, RBD, QR/SVD RBD and GMI precoding algorithms.

TABLE I  
THE PROPOSED LR-S-GMI-MMSE PRECODING ALGORITHM

Steps	Operations
	<b>Applying the MMSE Channel Inversion</b>
(1)	$\mathbf{H}_{\text{mse}}^\dagger = (\mathbf{H}^H \mathbf{H} + \alpha \mathbf{I})^{-1} \mathbf{H}^H$
(2)	for $i = 1 : K$
(3)	$[\mathbf{Q}_{i,\text{mse}}^\dagger \mathbf{R}_{i,\text{mse}}^\dagger] = \text{QR}(\mathbf{H}_{i,\text{mse}}^\dagger, 0)$
(4)	$\mathbf{P}_i^a = \mathbf{Q}_{i,\text{mse}}^\dagger$
(5)	$\mathbf{H}_{\text{eff}_i} = \mathbf{H}_i \mathbf{P}_i^a$
(6)	$\underline{\mathbf{H}}_{\text{eff}_i} = [\mathbf{H}_{\text{eff}_i} \sqrt{\alpha} \mathbf{I}_{N_i}]$
(7)	$[\underline{\mathbf{T}}_i^T \underline{\mathbf{H}}_{\text{eff}_i}^T] = \text{CLLL}(\underline{\mathbf{H}}_{\text{eff}_i}^T)$
(8)	$\mathbf{A}_i = [\mathbf{I}_i \mathbf{0}_i]$
(9)	$\tilde{\mathbf{P}}_{\text{MMSE}_i}^b = \mathbf{A}_i \tilde{\underline{\mathbf{H}}}_{\text{eff}_i}^H (\tilde{\underline{\mathbf{H}}}_{\text{eff}_i} \tilde{\underline{\mathbf{H}}}_{\text{eff}_i}^H)^{-1}$
(10)	end
	<b>Compute the overall precoding matrix</b>
(11)	$\mathbf{P}^a = [\mathbf{P}_1^a, \mathbf{P}_2^a, \dots, \mathbf{P}_K^a]$
(12)	$\tilde{\mathbf{P}}^b = \text{diag}\{\tilde{\mathbf{P}}_1^b, \tilde{\mathbf{P}}_2^b, \dots, \tilde{\mathbf{P}}_K^b\}$
(13)	$\tilde{\mathbf{P}} = \mathbf{P}^a \tilde{\mathbf{P}}^b$
	<b>Calculate the scaling factor <math>\gamma</math></b>
(14)	$\gamma = (\ \tilde{\mathbf{P}} \mathbf{d}\ _F^2 / E_s)$
	<b>Get the received signal</b>
(15)	$\mathbf{y} = \mathbf{H} \tilde{\mathbf{P}} \mathbf{d} + \sqrt{\gamma} \mathbf{n}$
	<b>Transform back from lattice space</b>
(16)	$\hat{\mathbf{d}} = \underline{\mathbf{T}}[\mathbf{y}]$

The proposed LR-S-GMI-MMSE precoding algorithm shows the best BER performance. At the BER of  $10^{-2}$ , the LR-S-GMI-MMSE [precoding](#) has a gain of more than 5.5 dB compared to the RBD precoding. It is worth noting that the BER performance of the RBD precoding is outperformed by the proposed LR-S-GMI-MMSE precoding in the whole  $E_b/N_0$  range and the BER gains become more significant with the increase of  $E_b/N_0$ . [The reason why the proposed LR-S-GMI-MMSE precoding algorithm provides a better BER performance than the exiting algorithm is because it provides a better channel quality as measured by the condition number of the effective channel.](#)

Fig. 3. illustrates the sum-rate of the above precoding algorithms. The information rate is calculated using [17]:

$$C = \log(\det(\mathbf{I} + \sigma_n^{-2} \mathbf{H} \mathbf{P} \mathbf{P}^H \mathbf{H}^H)) \quad (\text{bits/Hz}). \quad (31)$$

BD precoding with WF power loading shows a better sum-rate performance than BD precoding without power loading. However, as shown in Fig. 2., the BER performance is degraded by applying this WF scheme. Similar to the BER figure, the RBD, QR/SVD RBD and GMI precoding algorithms show [a comparable](#) sum-rate performance. The S-GMI precoding also achieves the sum-rate performance of the RBD precoding. The proposed LR-S-GMI-MMSE precoding algorithm shows almost the same sum-rate performance as the RBD precoding at low  $E_b/N_0$ s. At high  $E_b/N_0$ s, however, the sum-rate performance of LR-S-GMI-MMSE precoding is slightly inferior to that of the RBD precoding and [approaches the performance of BD precoding.](#)

The required floating point operations (FLOPs) for the conventional BD, RBD and QR/SVD RBD precoding algorithms [are](#) given in [18], [19]. The reduction in the number of FLOPs

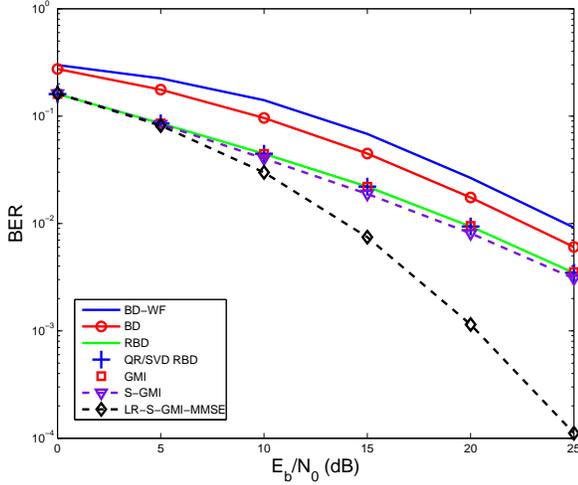


Fig. 2. BER performance,  $(2, 2, 2, 2) \times 8$  MU-MIMO

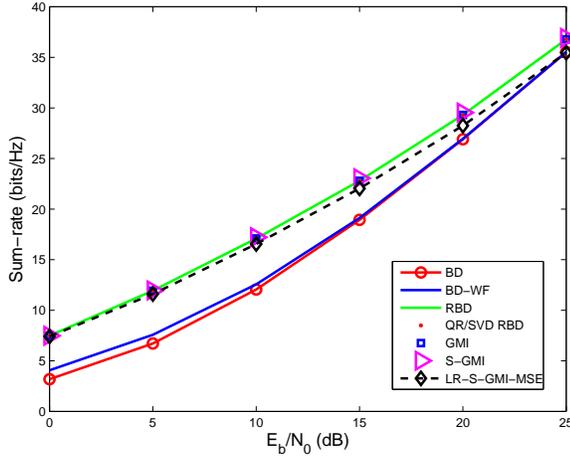


Fig. 3. Sum-rate performance,  $(2, 2, 2, 2) \times 8$  MU-MIMO

obtained by the proposed LR-S-GMI-MMSE is 73.6%, 69.5% and 59.1% as compared to the RBD, BD and QR/SVD RBD precoding algorithms, respectively.

## VI. CONCLUSION

In this paper, low-complexity precoding algorithms based on a channel inversion technique, QR decompositions, and lattice reductions have been proposed for MU-MIMO systems. The complexity of the precoding process is reduced and a considerable BER gain is achieved by the proposed LR-S-GMI precoding algorithms at a cost of a slight sum-rate loss at high SNRs. Since the proposed LR-S-GMI precoding algorithms are only implemented at the transmit side, the decoding matrix is not needed anymore at the receive side compared to the RBD precoding algorithm. Then, the structure of the receiver can be simplified, which is an additional benefit of the proposed LR-S-GMI precoding algorithms.

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