Set-Membership Constrained Widely Linear Beamforming Algorithms

Ricardo Zelenovsky and Rodrigo C. de Lamare
Department of Electrical Engineering, University of Brasilia, Brazil
Department of Electronics, University of York,
York, United Kingdom
Emails: zele@unb.br, rcdl500@york.ac.uk

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Introduction

- Modern electronic systems like radar and sonar use antenna arrays and rely on adaptive beamforming techniques, whose design is still a major research problem.
- A great deal of research has been done on linearly constrained minimum variance (LCMV) beamforming that employ the second-order statistics of the data.
- Widely-linear (WL) processing can improve the performance of the LCMV based algorithms when the data are second-order noncircular at the expense of a higher computational cost.
- Set-membership filtering (SMF) techniques can reduce costs by performing data selective updates and can provide extra flexibility in the design.
- We propose the combination of SMF techniques with WL processing for the design of LCMV beamforming and develop LMS and RLS algorithms.

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System Model and Problem Statement

- We consider a sensor array processing system equipped with a ULA with M elements and K narrow-band sources in the far field

- The received vector from the linear array can be modelled as

$$\boldsymbol{x} = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s} + \boldsymbol{n} \in \mathbb{C}^M$$

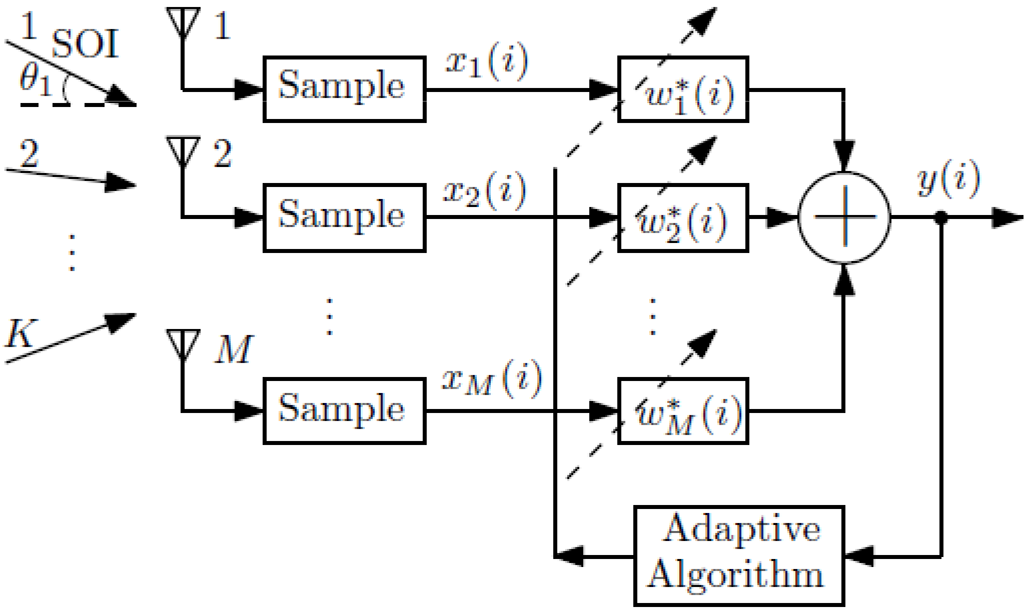
- The problem:
  - Design of LCMV beamformer

$$\text{minimize } E[|\boldsymbol{w}^H\boldsymbol{x}|^2] = \boldsymbol{w}^H\boldsymbol{R}_x\boldsymbol{w}$$
$$\text{subject to } \boldsymbol{w}^H\boldsymbol{a}(\theta_k) = 1$$

  - Batch solution

$$\boldsymbol{w}_{\text{opt}} = \frac{\boldsymbol{R}_x{}^{-1}\boldsymbol{a}(\theta_k)}{\boldsymbol{a}^H(\theta_k)\boldsymbol{R}_x{}^{-1}\boldsymbol{a}(\theta_k)}$$

  - Adaptive algorithms



Set-Membership Widely-Linear Beamforming Techniques

- In the presence of non-circular data, widely-linear processing techniques can improve the performance of adaptive beamforming algorithms.
- This is done by taking into account all the second-order statistics of the received signal ***x***
- A simple way to do that is to use a transformation that augments **x**

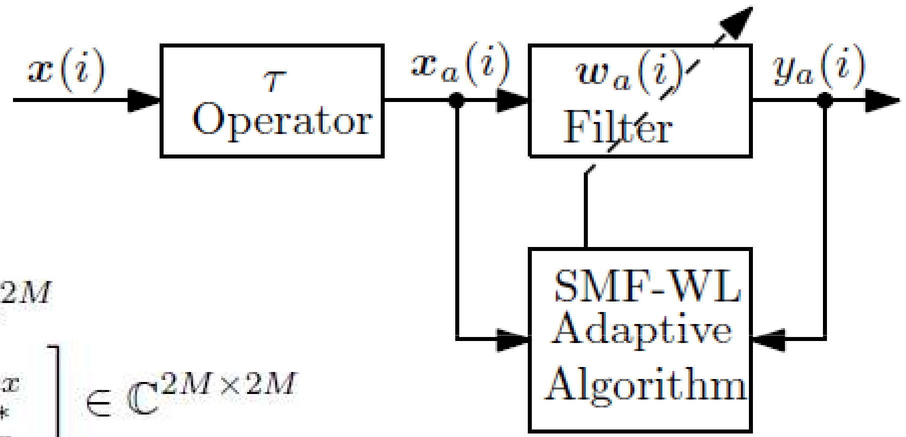
$$x \xrightarrow{T} x_a : x_a = [x^T, x^H]^T \in \mathbb{C}^{2M}$$

- Problem: this doubles the dimension of the data structures and increases the cost.
- We propose a SMF- approach to widely-linear beamforming, which only updates the weights if a bound is satisfied with the following steps:
  - 1) information evaluation and computation of the bound
  - 2) update of the weights if the bound is exceeded.

Proposed SMF-WL Algorithms (1/3)

- Widely-linear LCMV optimization:

$$\text{minimize } E[|y|^2] = E[|\boldsymbol{w}_a^H \mathbf{x}_a|^2] = \boldsymbol{w}_a^H \boldsymbol{R}_{ax} \boldsymbol{w}_a$$
$$\text{subject to } \boldsymbol{w}_a^H \boldsymbol{a}_a(\theta_k) = 1$$



where

$$\boldsymbol{a}_a(\theta_k) = [\boldsymbol{a}^T(\theta_k), \boldsymbol{a}^H(\theta_k)]^T \in \mathbb{C}^{2M}$$
$$\boldsymbol{R}_{ax} = E[\boldsymbol{x}_a(i)\boldsymbol{x}_a^H(i)] = \begin{bmatrix} \boldsymbol{R}_x & \boldsymbol{R}_{cx} \\ \boldsymbol{R}_{cx}^* & \boldsymbol{R}_x^* \end{bmatrix} \in \mathbb{C}^{2M \times 2M}$$

- Solution:

$$\boldsymbol{w}_{\text{a-opt}} = \frac{\boldsymbol{R}_{ax}^{-1}\boldsymbol{a}_a(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{R}_{ax}^{-1}\boldsymbol{a}_a(\theta_k)}$$

- Adaptive algoritnms:
  - LMS
  - RLS

Proposed SMF-WL Algorithms (2/3)

- Consider the Lagrangian associated with the optimisation problem:

$$\mathcal{L}(\boldsymbol{w}_a(i), \lambda_l) = E\big[|\boldsymbol{w}_a^H(i)\boldsymbol{x}_a(i)|^2\big] + 2\Re[\lambda_l(\bar{\boldsymbol{w}}_a^H(i)\boldsymbol{a}_a(\theta_k) - 1]$$

- The SMF-WL-LMS algorithm is given by:

$$\boldsymbol{w}_a(i+1) = \boldsymbol{w}_a(i) - \mu y^*(i)\Big(\boldsymbol{I} - \frac{\boldsymbol{a}_a(\theta_k)\boldsymbol{a}_a^H(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{a}_a(\theta_k)}\Big)\boldsymbol{x}_a(i)$$

- A simple and effective time-varying bound is given by

$$\delta(i) = \beta\delta(i-1) + (1-\beta)\sqrt{\alpha||\boldsymbol{w}_a||^2\hat{\sigma}_n^2}$$

- A step size rule that controls the data selective updates is given by

$$\mu(i+1) = \begin{cases} \dfrac{1 - \frac{\delta(i)}{|y_a(i)|}}{\boldsymbol{x}_a^H(i)\Big(\boldsymbol{I} - \frac{\boldsymbol{a}_a(\theta_k)\boldsymbol{a}_a^H(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{a}_a(\theta_k)}\Big)\boldsymbol{x}_a(i)} & \text{if } |y|^2 > \delta^2 \\ 0 & \text{if } |y|^2 \leq \delta^2 \end{cases}$$

Proposed SMF-WL Algorithms (3/3)

- Consider the Lagrangian associated with the optimisation problem:

$$\mathcal{L}(\boldsymbol{w}_a(i), \mu_l) = \sum_{j=1}^{i} \alpha_l^{i-j} \left| \boldsymbol{w}_a^H(i)\boldsymbol{x}_a(j) \right|^2 + 2\Re[\lambda_l(\boldsymbol{w}_a^H(i)\boldsymbol{a}_a(\theta_k) - 1)]$$

- The SMF-WL-RLS algorithm is given by:

$$\boldsymbol{w}_a(i) = \frac{\hat{\boldsymbol{R}}_{ax}^{-1}(i)\boldsymbol{a}_a(\theta_k)}{\boldsymbol{a}_a^H(\theta_k)\hat{\boldsymbol{R}}_{ax}^{-1}(i)\boldsymbol{a}_a(\theta_k)}$$

$$\boldsymbol{R}_{ax}^{-1}(i) = \alpha^{-1}(i)\boldsymbol{R}_{ax}^{-1}(i-1) - \alpha^{-1}(i)\boldsymbol{G}(i)\boldsymbol{x}_a^H(i)\boldsymbol{R}_{ax}^{-1}(i-1)$$

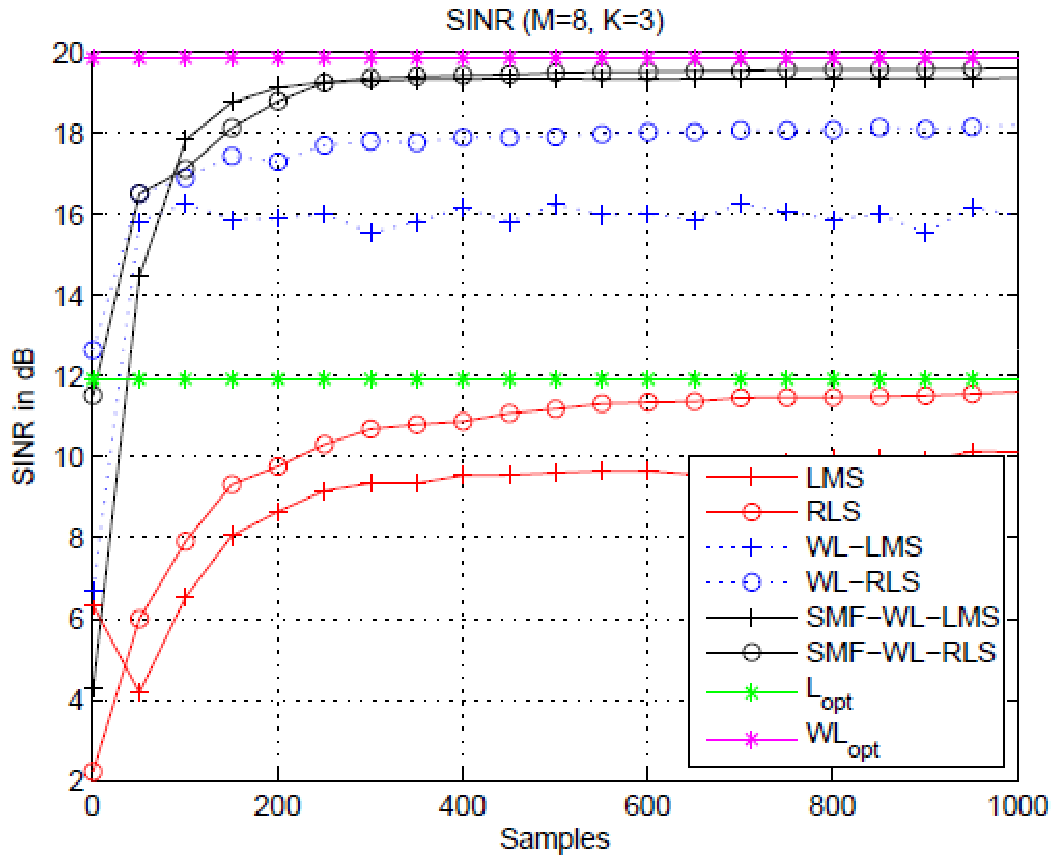
$$\boldsymbol{G}(i) = \frac{\boldsymbol{R}_{ax}^{-1}(i)\boldsymbol{x}_a(i)}{\alpha(i) + \boldsymbol{x}_a^H(i)\boldsymbol{R}_{ax}^{-1}\boldsymbol{x}_a(i)}$$

- The variable forgetting factor rule that controls the data selective updates is:

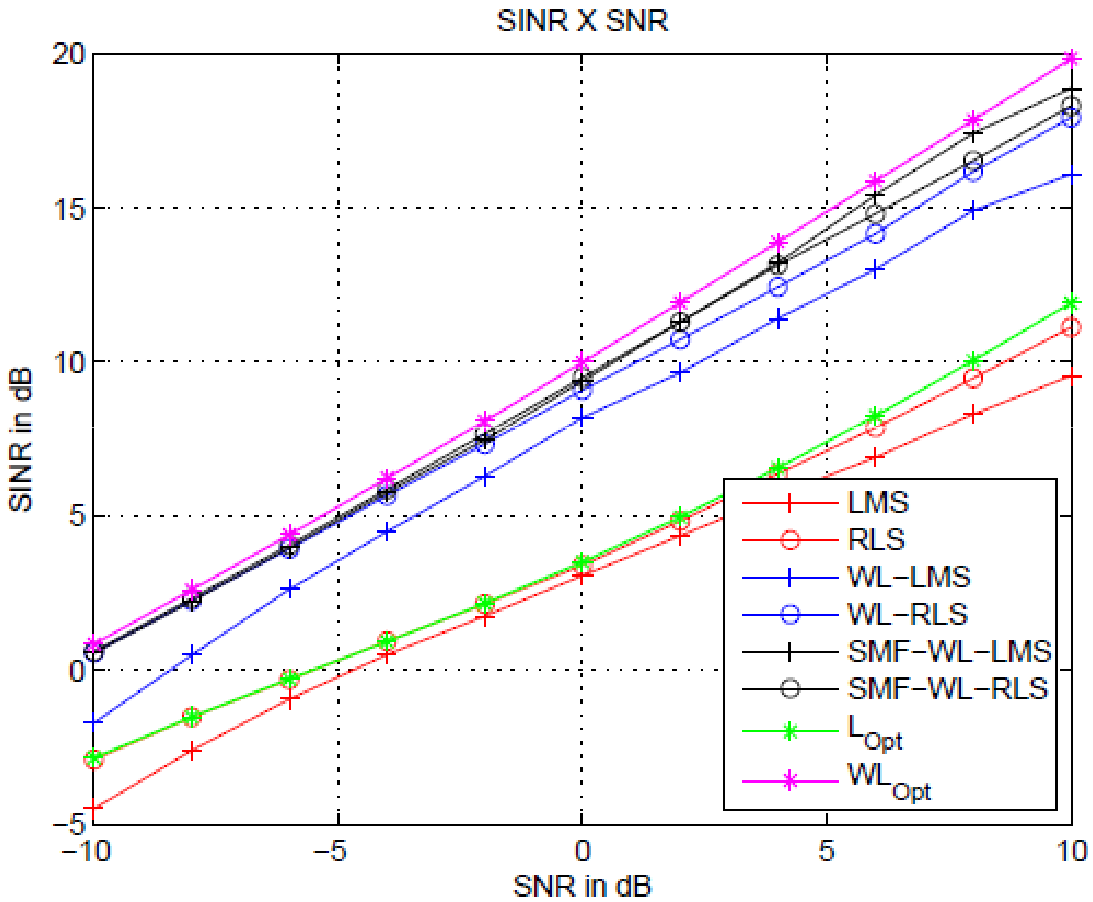
$$\alpha(i) = \begin{cases} \frac{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{R}_{ax}^{-1}(i)[\delta(i)\boldsymbol{a}_a(\theta_k) - \boldsymbol{x}_a(i)]}{\boldsymbol{a}_a^H(\theta_k)\boldsymbol{G}(i)\boldsymbol{x}_a(i)\boldsymbol{R}_{ax}^{-1}(i)[\delta(i)\boldsymbol{a}_a(\theta_k) - \boldsymbol{x}_a(i)]} & \text{if } |y|^2 > \delta^2 \\ 0 & \text{if } |y|^2 \leq \delta^2 \end{cases}$$

Simulations (1/4)

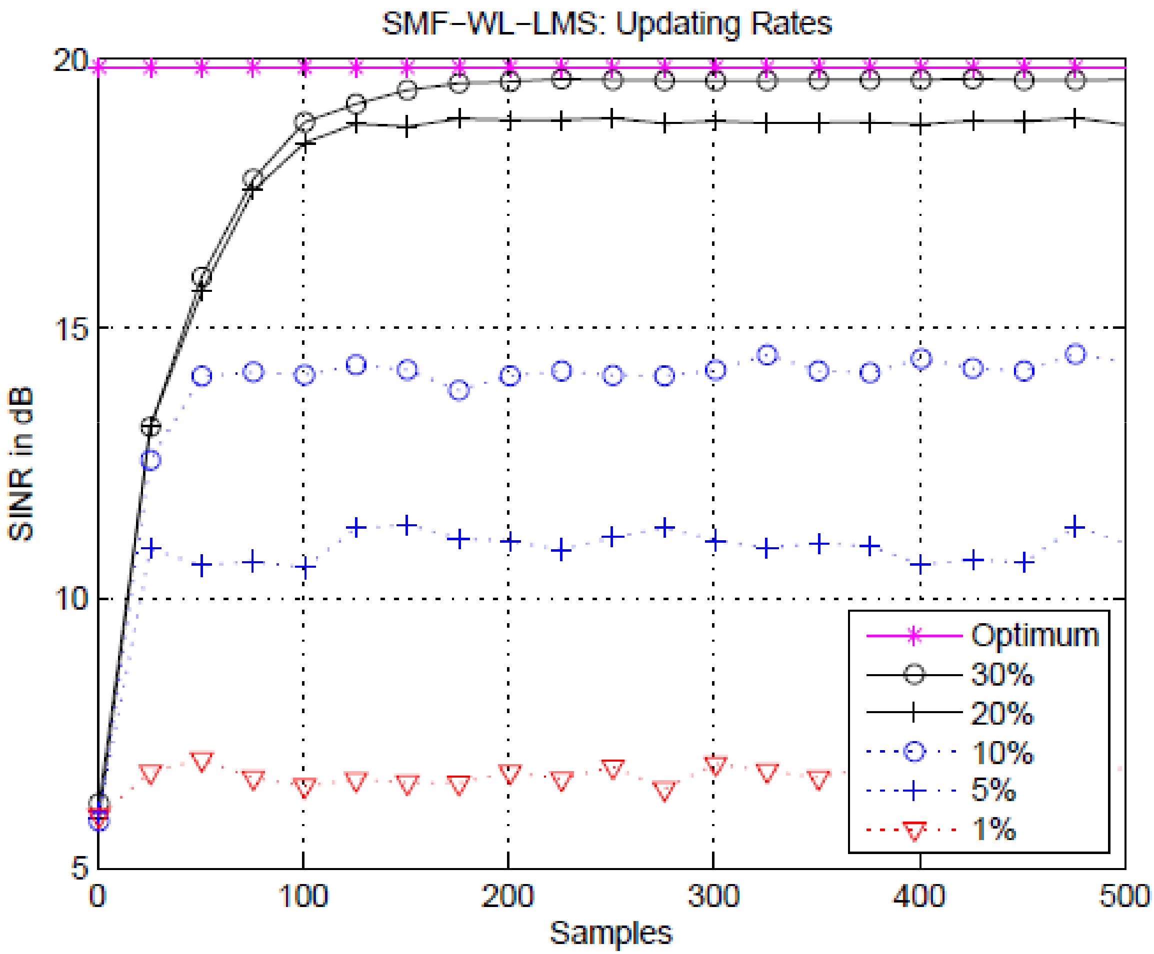
- We consider a ULA with 8 elements.
- The system has 1 desired user and 2 interferers with the same power with DOAs equal to 20, 50, −30 degrees.
- The noise is modelled as AWGN with zero mean and variance σ^2



Simulations (2/4)



Simulations (3/4)



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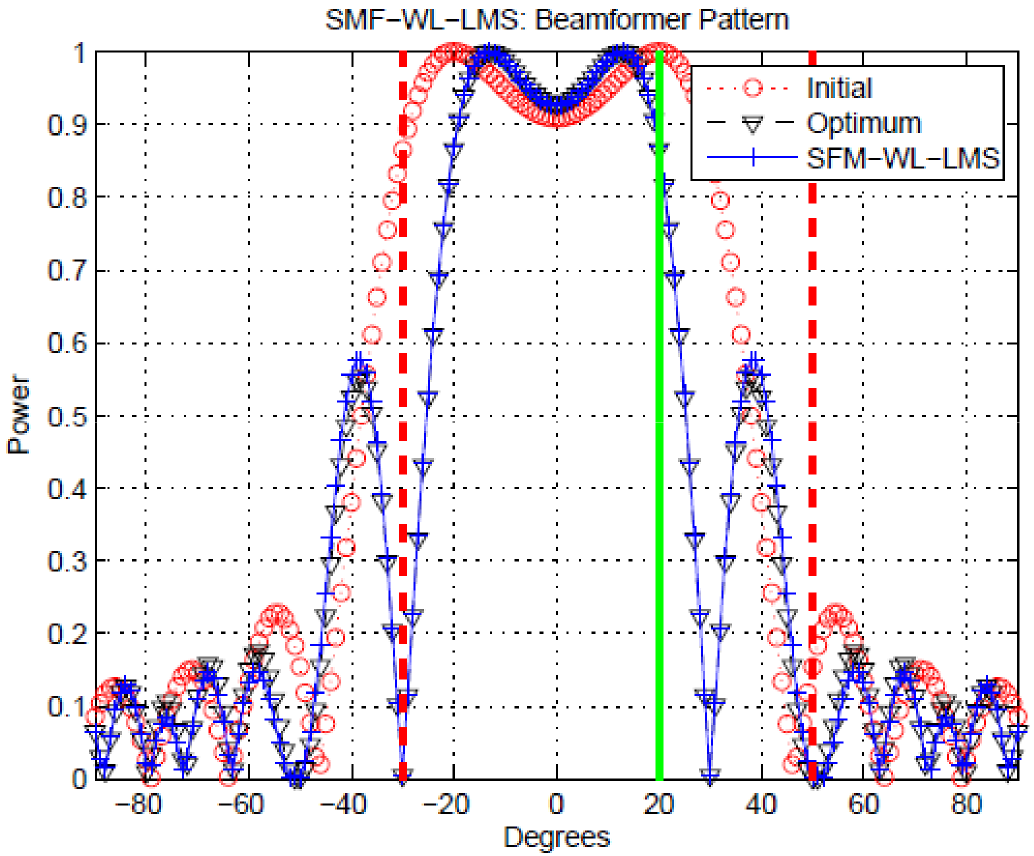
Simulations (4/4)



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Conclusions

- We have developed distributed SMF-WL beamforming algorithms for low-complexity adaptive beamforming applications.
- We have devised both LMS and RLS versions that can be used for various applications in sensor arrays.
- The proposed SMF-WL algorithms can exploit non-circular data for an improved performance and have a reduced computational cost.
- Simulation results have shown that the proposed SMF-WL algorithms perform very close to the optimal solutions.

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