L1 REGULARIZED STAP ALGORITHM WITH A GENERALIZED SIDELOBE CANCELER ARCHITECTURE FOR AIRBORNE RADAR

Zhaocheng Yang, Rodrigo C. de Lamare and Xiang Li

Communications Research Group

Department of Electronics

University of York, UK

rcdl500@york.ac.uk

Electronics Science and Engineering School,

National University of Defense Technology, China

yangzhaocheng@gmail.com and lixiang01@vip.sina.com

THE UNIVERSITY *of York*

Outline

- Motivation
- Prior Work
- Contributions
- Signal Model
- Proposed L1 Regularized Algorithm
- Simulations
- Conclusions

Motivation

- Full-rank STAP:
  - Existing problems: large-sample support and expensive computation.
  - Faster convergence, lower computational complexity and improved robustness against non-homogeneous interference.
  - Main idea: to devise a new STAP algorithm that exploits the sparsity of the receive data and the filter weights.
- How does it work?
  - Imposing a sparse regularization (L1-norm type) to the minimum mean-square error (MMSE) criterion.
  - The goal is to find an appropriate solution for this kind of mixed L1-norm and L2-norm optimization problem.

Prior Work

- Compressive sensing type STAP:
  - Global matched filter to STAP data, Maria and Fuchs. (2006s).
  - CS-STAP, Sun, Zhang, etc. (2009s).
  - Selesnick, Pillai, etc. (2010s).
  - Parker and Potter. (2010s)
- All these works focus on the recovery of the clutter power in angle-Doppler plane.

THE UNIVERSITY of York

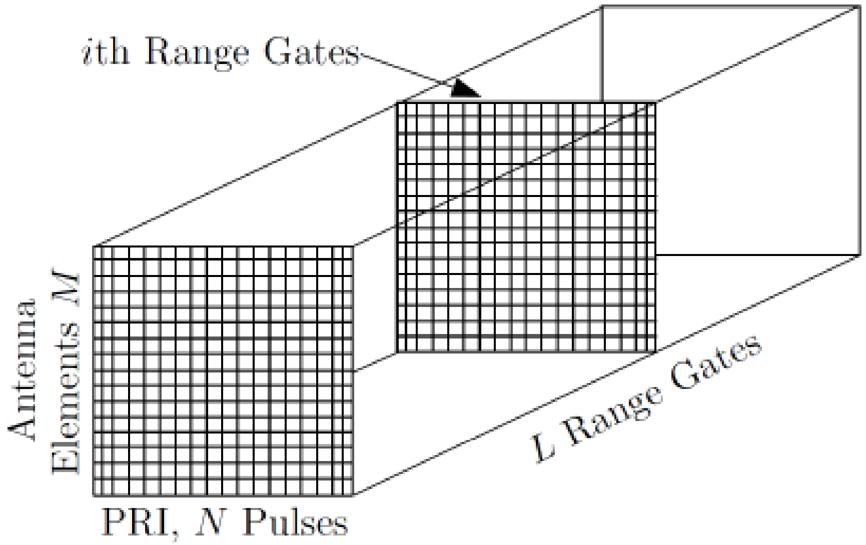
Contributions

- Modify the MSE cost function (imposing a sparse regularization to the MSE criterion) .
- Propose a L1-based online coordinate descent (OCD) adaptive algorithm to compute the weights.
- Do not need matrix inversion (nearly the same computation complexity as RLS algorithm).
- Show faster convergence and better performance than conventional RLS algorithm.

Signal Model and Problem Statement

- Detection problem:

$$H_0 : \quad \mathbf{r} = \mathbf{r}_u$$
$$H_1 : \quad \mathbf{r} = \alpha_t \mathbf{r}_t + \mathbf{r}_u.$$



where

H_0 denotes target absence,

H_1 denotes target present,

$\mathbf{r}_u$ is a MN x 1 the undesired interference vector,

$\mathbf{r}_t$ is a MN x 1 the normalized target space-time steering vector,

α_t is a complex gain of the target,

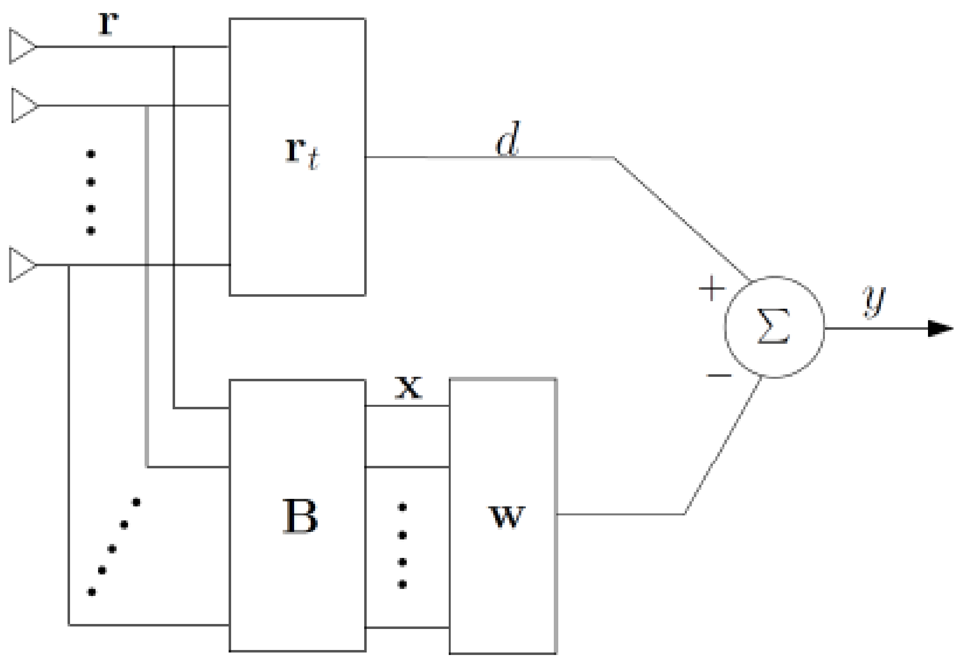
M is the number of antenna elements,

N is the number of pulses in one CPI

GSC-STAP for Airborne Radar

- Blocking matrix $\boldsymbol{B}$:

$$\boldsymbol{B}\boldsymbol{r}_t = \mathbf{0},$$



- STAP filter:

$$y = d - \boldsymbol{w}^H \boldsymbol{x} = \boldsymbol{r}_t^H \boldsymbol{r} - \boldsymbol{w}^H \boldsymbol{B}\boldsymbol{r}.$$

- Main problem: design of $\boldsymbol{w}$

THE UNIVERSITY *of York*

Optimal Linear MMSE Design for GSC-STAP

- Optimization problem:

$$\boldsymbol{w} = \arg\min_{\boldsymbol{w}} E\Big[||d - \boldsymbol{w}^H \boldsymbol{x}||^2\Big],$$

- Optimal GSC-STAP filter weights:

$$\boldsymbol{w}_{\text{MMSE}} = \boldsymbol{R}_x^{-1} \boldsymbol{r}_{xd}$$

where

$$\boldsymbol{r}_{xd} = E\Big[\boldsymbol{x} d^*\Big] \qquad \boldsymbol{R}_x = E\Big[\boldsymbol{x}\boldsymbol{x}^H\Big]$$

- Associated output SINR:

$$\text{SINR} = \frac{|\alpha_t|^2}{\boldsymbol{r}_t^H \boldsymbol{R} \boldsymbol{r}_t - \boldsymbol{r}_{xd}^H \boldsymbol{R}_x^{-1} \boldsymbol{r}_{xd}}$$

Proposed L1 Regularized STAP

- Modified MMSE cost function:

$$\mathcal{J}(\boldsymbol{w}) = \min_{\boldsymbol{w}} E\left[||d - \boldsymbol{w}^H \boldsymbol{x}||^2\right] + 2\lambda||\boldsymbol{w}||_1$$

where λ is a positive scalar.

- Computing the gradient terms with respect to

$$\frac{\partial \mathcal{J}(\boldsymbol{w})}{\partial \boldsymbol{w}^*(j)}) = -\boldsymbol{r}_{xd}(j) + \mathbf{R}_x(j,j)\boldsymbol{w}(j) + \sum_{i=1, i\neq j}^{NM-1} \boldsymbol{R}_x(j,i)\boldsymbol{w}(i) + \lambda \text{sign}(\boldsymbol{w}(\text{j}))$$

where

$$\text{sign}(\text{x}) = \begin{cases} x/|x| & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$$

How to solve ?

THE UNIVERSITY of York

Proposed L1 Regularized STAP

- Optimal weights by shrinkage method:

$$\boldsymbol{w}_{opt}(j) = \frac{S(z(j), \lambda)}{\boldsymbol{R}_x(j,j)}$$

where

$$z(j) = \boldsymbol{r}_{xd}(j) - \sum_{i=1, i\neq j}^{NM-1} \boldsymbol{R}_x(j,i)\boldsymbol{w}(i)$$

and $S(z(j), \lambda)$ is the soft-thresholding operator, given by

$$S(z(j), \lambda) = \mathrm{sign}(z)(|z| - \lambda)_+ = \mathrm{sign}(z)\max(|z| - \lambda, 0)$$

THE UNIVERSITY *of York*

L1-based OCD Adaptive Algorithm

- The noise-subspace data covariance matrix and the cross-correlation vector (i is sample index):

$$\boldsymbol{R}_x^i = \beta \boldsymbol{R}_x^{i-1} + \boldsymbol{x}_i \boldsymbol{x}_i^H \qquad \boldsymbol{r}_{xd}^i = \beta \boldsymbol{r}_{xd}^{i-1} + \boldsymbol{x}_i d_i^*$$

- The updated filter weights:

$$\widehat{w}_i(p) = \frac{\text{sign}(z_i(p))(|z_i(p)| - \lambda)_+}{\boldsymbol{R}_x(p,p)}$$

where

$$z_i(p) = \boldsymbol{r}_{xd}^i(p) - \sum_{q=1, q\neq p}^{NM-1} \boldsymbol{R}_x^i(p,q) \boldsymbol{w}_{i-1}^p(q)$$

$$\begin{aligned} \boldsymbol{w}_{i-1}^p = & [w_i(1), \cdots, w_i(p-1), \widehat{w}_i(p), \\ & w_{i-1}(p+1), \cdots, w_{i-1}(NM-1)]^T \end{aligned}$$

Complexity Analysis

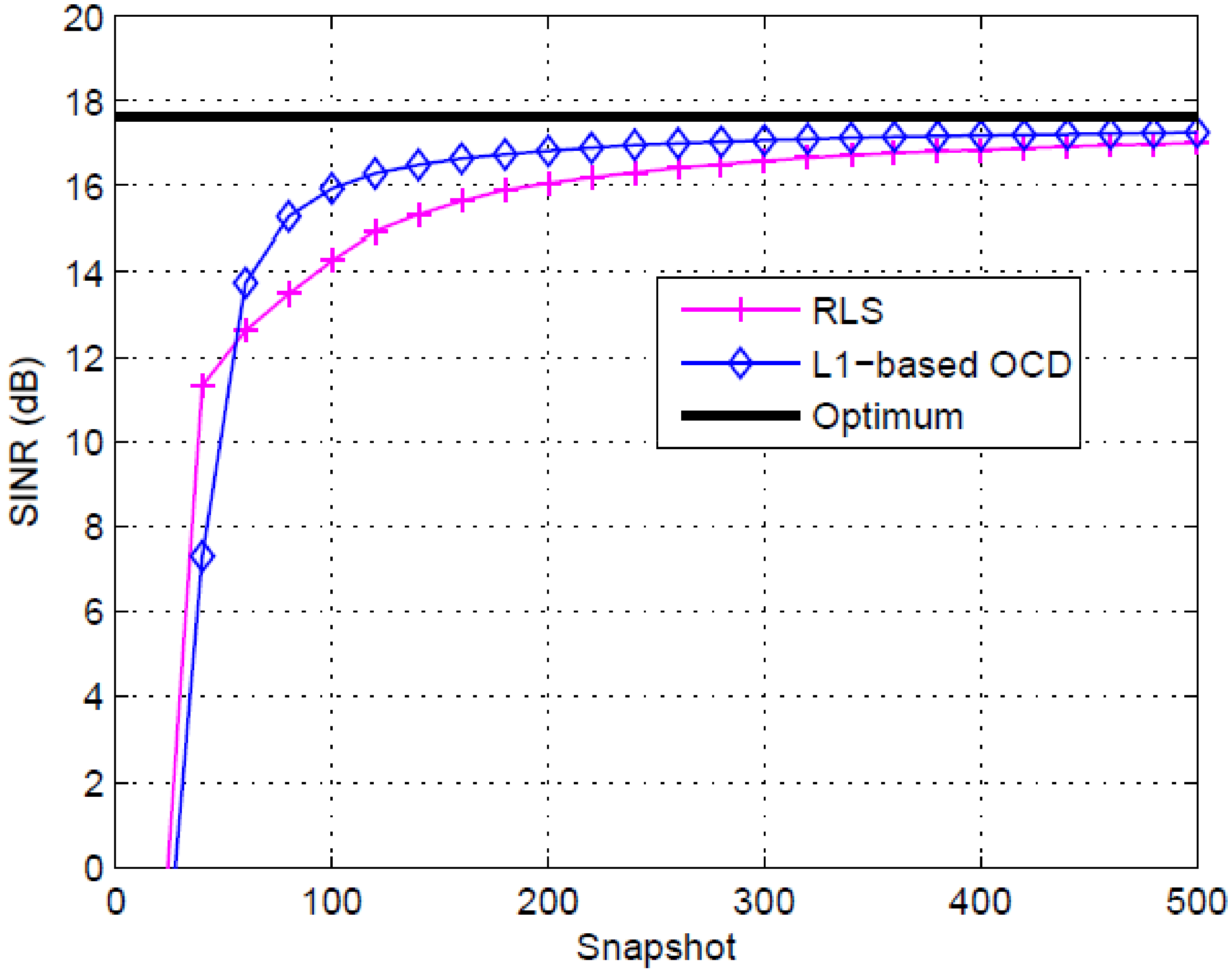
Computational Complexity for Per Snapshot

Algorithm	RLS	l_1-based OCD
Additions	$3(NM)^2 - 2NM - 2$	$3(NM)^2 - 2NM - 1$
Multiplications	$4(NM)^2 - 3NM$	$4(NM)^2 - 3NM$
Divisions	2	$2NM - 2$
Absolute	0	$NM - 1$

Simulation: Scenario and Parameters

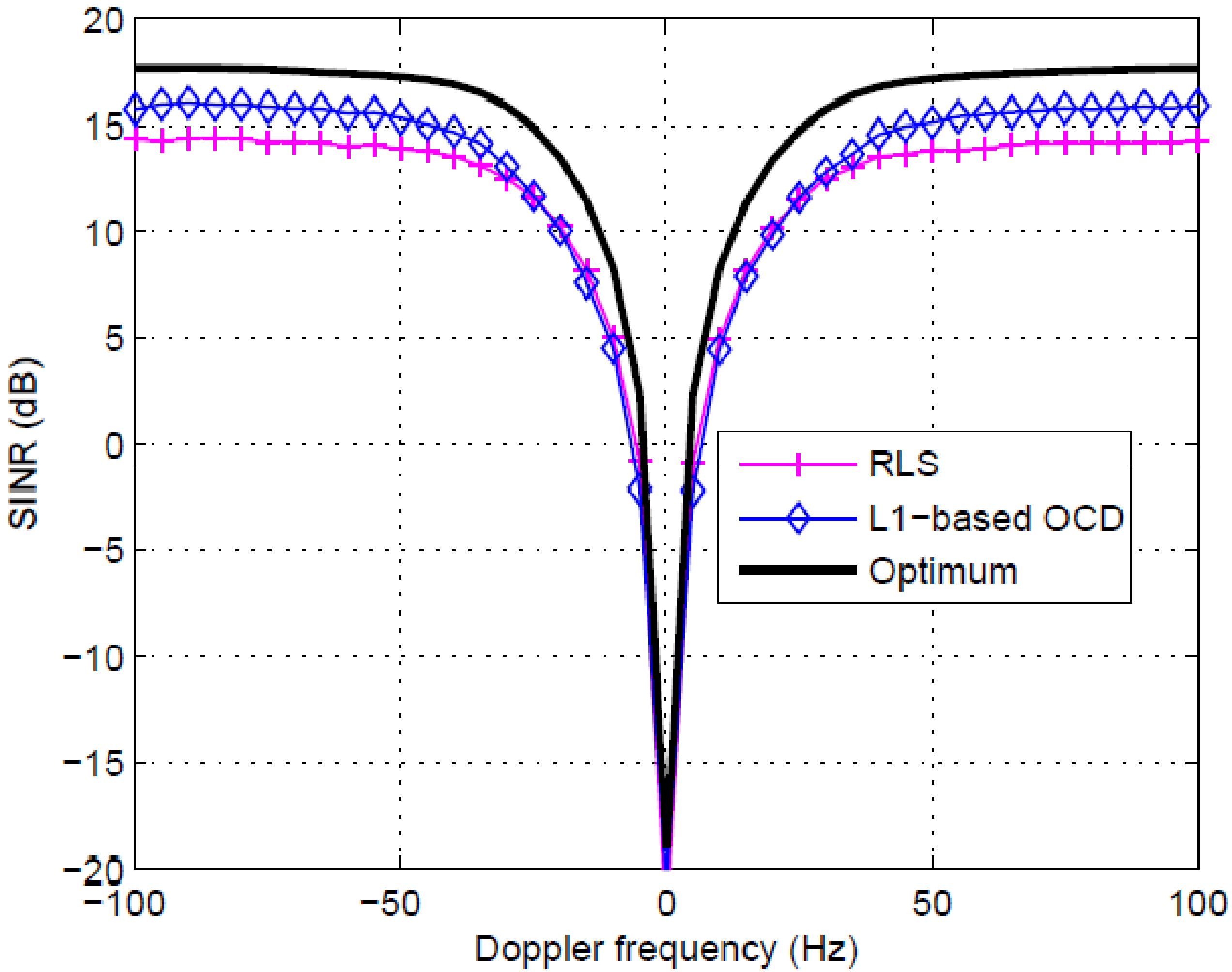
Parameter	Value
Antenna array	sideway-looking array
Antenna array spacing	$\lambda/2$
Carrier frequency	450MHz
Transmit pattern	Uniform
Mainbeam azimuth	0°
PRF	300Hz
Platform velocity	50m/s
Platform height	9000 m
Thermal noise power	10^{-2}
Clutter-to-noise-ratio (CNR)	30dB
Jammer-to-noise-ratio (JNR)	30dB
Jammer azimuth	60° and -45°
Signal-to-noise-ratio (SNR)	0dB
Target Doppler	100Hz
Target azimuth	0°
Antenna elements number	8
Pulse number in one CPI	8

Simulations: SINR vs Snapshots



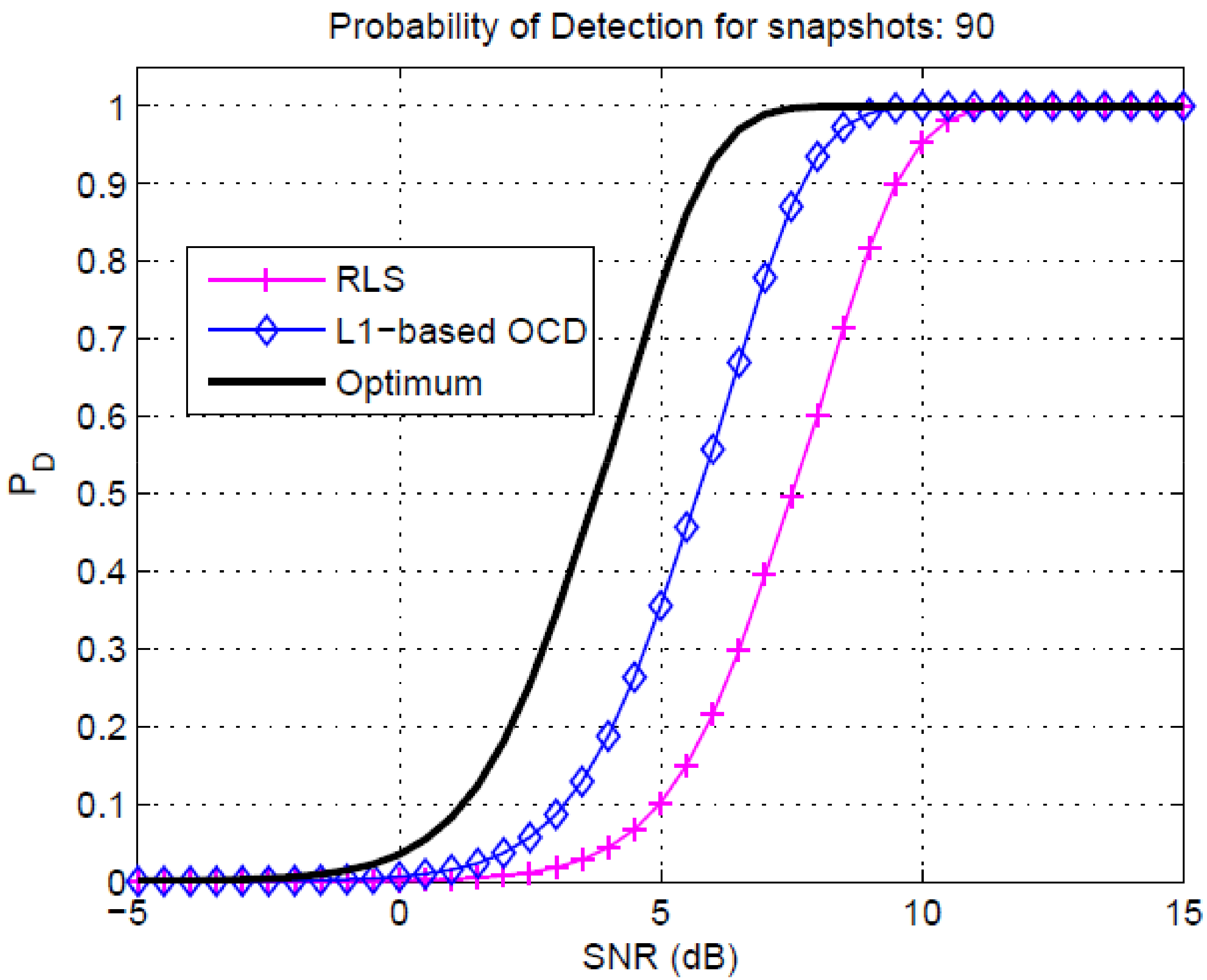
THE UNIVERSITY of York

Simulations: SINR vs Doppler Frequency



THE UNIVERSITY of York

Simulations: Probability of Detection



THE UNIVERSITY of York

Conclusions

- A new L1 regularized STAP algorithm is proposed with GSC architecture for airborne radar.
- A L1-based OCD adaptive algorithm is developed to compute the filter weights.
- Nearly the same computational complexity as RLS algorithm, without the need for a matrix inversion.
- The proposed algorithm outperforms the conventional RLS STAP algorithm.

References

[1] W.L. Melvin, "A stap overview," IEEE A\&E Sys. Mag., vol.19, no.1, pp.19-35, 2004.

[2] J. S. Goldstein, I. S. Reed and P. A. Zulch, "Multistage Partially Adaptive STAP CFAR Detection Algorithm," IEEE Trans. Aes. Ele. Sys., vol.35, no.2, pp.645-661, 1999.

[3] R. Fa, R.C. de Lamare and L. Wang, "Reduced-rank STAP schemes for airborne radar based on swithced joint interpolation, decimation and filtering algorithm," IEEE Trans. Sig. Proc., vol.58, no.8, pp.4182-4194, 2010.

[4] R. C. de Lamare and R. Sampaio-Neto, "Adaptive reduced-rank processing based on joint and iterative interpolation, decimation, and filtering", IEEE Trans. Sig. Proc., vol. 57, No. 7, July 2009, pp. 2503 -2514.

[5] M. Zibulevsky and M. Elad, "L1-L2 optimization in signal and image processing," IEEE Sig. Proc. Mag., vol. 27, no. 3, pp. 76-88, May 2010.

[6] D. Angelosante, J.A. Bazerque and G.B. Giannakis, "Online adaptive estimation of sparse signals: where RLS meets the L_1-norm," IEEE Trans. Sig. Proc., vol. 58, no. 7, pp. 3436-3446, 2010.

[7] J. Friedman, T. Hastie and R. Tibshirani, "Regularization paths for generalized linear models via coordinate descent," Journal of Statistical Software, vol. 33, no. 1, pp. 1-22, 2010.

THE UNIVERSITY of York