L1 REGULARIZED STAP ALGORITHM WITH A GENERALIZED SIDELOBE CANCELER ARCHITECTURE FOR AIRBORNE RADAR

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Outline

- Motivation
- Prior Work
- Contributions
- Signal Model
- Proposed L1 Regularized Algorithm
- Simulations
- Conclusions
Motivation

- Full-rank STAP:
  - Existing problems: large-sample support and expensive computation.
  - Faster convergence, lower computational complexity and improved robustness against non-homogeneous interference.
  - Main idea: to devise a new STAP algorithm that exploits the sparsity of the receive data and the filter weights.

- How does it work?
  - Imposing a sparse regularization (L1-norm type) to the minimum mean-square error (MMSE) criterion.
  - The goal is to find an appropriate solution for this kind of mixed L1-norm and L2-norm optimization problem.
Prior Work

- Compressive sensing type STAP:
  - Global matched filter to STAP data, Maria and Fuchs. (2006s).
  - CS-STAP, Sun, Zhang, etc. (2009s).
  - Selesnick, Pillai, etc. (2010s).
  - Parker and Potter. (2010s)

- All these works focus on the recovery of the clutter power in angle-Doppler plane.
Contributions

- Modify the MSE cost function (imposing a sparse regularization to the MSE criterion).
- Propose a L1-based online coordinate descent (OCD) adaptive algorithm to compute the weights.
- Do not need matrix inversion (nearly the same computation complexity as RLS algorithm).
- Show faster convergence and better performance than conventional RLS algorithm.
Signal Model and Problem Statement

- Detection problem:
  \[ H_0 : \quad \mathbf{r} = \mathbf{r}_u \]
  \[ H_1 : \quad \mathbf{r} = \alpha_t \mathbf{r}_t + \mathbf{r}_u. \]

where

- \( H_0 \) denotes target absence,
- \( H_1 \) denotes target present,
- \( \mathbf{r}_u \) is a \( MN \times 1 \) the undesired interference vector,
- \( \mathbf{r}_t \) is a \( MN \times 1 \) the normalized target space-time steering vector,
- \( \alpha_t \) is a complex gain of the target,
- \( M \) is the number of antenna elements,
- \( N \) is the number of pulses in one CPI
GSC-STAP for Airborne Radar

- **Blocking matrix** $B$:

  $$Br_t = 0,$$

- **STAP filter**:

  $$y = d - \mathbf{w}^H \mathbf{x} = r_t^H \mathbf{r} - \mathbf{w}^H Br.$$

- **Main problem**: design of $\mathbf{w}$
Optimal Linear MMSE Design for GSC-STAP

- Optimization problem:

\[ w = \arg \min_{w} E[\|d - w^H x\|^2], \]

- Optimal GSC-STAP filter weights:

\[ w_{\text{MMSE}} = R_{xx}^{-1} r_{xd} \]

where

\[ r_{xd} = E[xd^*] \quad R_{xx} = E[xx^H] \]

- Associated output SINR:

\[ \text{SINR} = \frac{|\alpha_t|^2}{r_t^H R r_t - r_{xd}^H R_{xx}^{-1} r_{xd}} \]
Proposed L1 Regularized STAP

- **Modified MMSE cost function:**

  \[ J(w) = \min_w E[|d - w^H x|^2] + 2\lambda \|w\|_1 \]

  where \( \lambda \) is a positive scalar.

- **Computing the gradient terms with respect to**

  \[ \frac{\partial J(w)}{\partial w^*(j)} = -r_{xd}(j) + R_x(j, j)w(j) + \sum_{i=1, i \neq j}^{NM-1} R_x(j, i)w(i) + \lambda \text{sign}(w(j)) \]

  where

  \[ \text{sign}(x) = \begin{cases} 
  x/|x| & \text{for } x \neq 0 \\
  0 & \text{for } x = 0 
  \end{cases} \]

**How to solve?**
Proposed L1 Regularized STAP

Optimal weights by shrinkage method:

\[ w_{opt}(j) = \frac{S(z(j), \lambda)}{R_x(j, j)} \]

where

\[ z(j) = r_{xd}(j) - \sum_{i=1,i\neq j}^{NM-1} R_x(j, i)w(i) \]

and \( S(z(j), \lambda) \) is the soft-thresholding operator, given by

\[ S(z(j), \lambda) = \text{sign}(z)(|z| - \lambda)_+ = \text{sign}(z) \max(|z| - \lambda, 0) \]
L1-based OCD Adaptive Algorithm

The noise-subspace data covariance matrix and the cross-correlation vector (i is sample index):

\[ R_x^i = \beta R_x^{i-1} + x_i x_i^H \quad r_{xd}^i = \beta r_{xd}^{i-1} + x_i d_i^* \]

The updated filter weights:

\[ \hat{w}_i(p) = \frac{\text{sign}(z_i(p))(|z_i(p)| - \lambda)}{R_x(p, p)} \]

where

\[ z_i(p) = r_{xd}^i(p) - \sum_{q=1, q \neq p}^{NM-1} R_x^i(p, q)w_{i-1}^p(q) \]

\[ w_{i-1}^p = [w_i(1), \cdots, w_i(p-1), \hat{w}_i(p), w_{i-1}(p+1), \cdots, w_{i-1}(NM-1)]^T \]
## Complexity Analysis

### Computational Complexity for Per Snapshot

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>RLS</th>
<th>$l_1$-based OCD</th>
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</thead>
<tbody>
<tr>
<td>Additions</td>
<td>$3(NM)^2 - 2NM - 2$</td>
<td>$3(NM)^2 - 2NM - 1$</td>
</tr>
<tr>
<td>Multiplications</td>
<td>$4(NM)^2 - 3NM$</td>
<td>$4(NM)^2 - 3NM$</td>
</tr>
<tr>
<td>Divisions</td>
<td>$2$</td>
<td>$2NM - 2$</td>
</tr>
<tr>
<td>Absolute</td>
<td>$0$</td>
<td>$NM - 1$</td>
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</tbody>
</table>
Simulation: Scenario and Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna array</td>
<td>sideway-looking array</td>
</tr>
<tr>
<td>Antenna array spacing</td>
<td>$\lambda/2$</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>450MHz</td>
</tr>
<tr>
<td>Transmit pattern</td>
<td>Uniform</td>
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<tr>
<td>Mainbeam azimuth</td>
<td>0°</td>
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<tr>
<td>PRF</td>
<td>300Hz</td>
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<tr>
<td>Platform velocity</td>
<td>50m/s</td>
</tr>
<tr>
<td>Platform height</td>
<td>9000 m</td>
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<tr>
<td>Thermal noise power</td>
<td>$10^{-2}$</td>
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<tr>
<td>Clutter-to-noise-ratio (CNR)</td>
<td>30dB</td>
</tr>
<tr>
<td>Jammer-to-noise-ratio (JNR)</td>
<td>30dB</td>
</tr>
<tr>
<td>Jammer azimuth</td>
<td>60° and $-45°$</td>
</tr>
<tr>
<td>Signal-to-noise-ratio (SNR)</td>
<td>0dB</td>
</tr>
<tr>
<td>Target Doppler</td>
<td>100Hz</td>
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<tr>
<td>Target azimuth</td>
<td>0°</td>
</tr>
<tr>
<td>Antenna elements number</td>
<td>8</td>
</tr>
<tr>
<td>Pulse number in one CPI</td>
<td>8</td>
</tr>
</tbody>
</table>
Simulations: SINR vs Snapshots

- **RLS**
- **L1-based OCD**
- **Optimum**

**SINR (dB)** vs **Snapshot**
Simulations: SINR vs Doppler Frequency
Simulations: Probability of Detection

Probability of Detection for snapshots: 90

- RLS
- L1-based OCD
- Optimum

SNR (dB)

$P_D$
Conclusions

- A new L1 regularized STAP algorithm is proposed with GSC architecture for airborne radar.
- A L1-based OCD adaptive algorithm is developed to compute the filter weights.
- Nearly the same computational complexity as RLS algorithm, without the need for a matrix inversion.
- The proposed algorithm outperforms the conventional RLS STAP algorithm.
References


