Blind Adaptive Constrained Reduced-Rank Parameter Estimation Based on Constant Modulus Design for CDMA Interference Suppression

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***Abstract*—This paper proposes a multistage decomposition for blind adaptive parameter estimation in the Krylov subspace with the code-constrained constant modulus (CCM) design criterion. Based on constrained optimization of the constant modulus cost function and utilizing the Lanczos algorithm and Arnoldi-like iterations, a multistage decomposition is developed for blind parameter estimation. A family of computationally efficient blind adaptive reduced-rank stochastic gradient (SG) and recursive least squares (RLS) type algorithms along with an automatic rank selection procedure are also devised and evaluated against existing methods. An analysis of the convergence properties of the method is carried out and convergence conditions for the reduced-rank adaptive algorithms are established. Simulation results consider the application of the proposed techniques to the suppression of multiaccess and intersymbol interference in DS-CDMA systems.**

***Index Terms*—Blind adaptive constrained algorithms, DS-code-division-multiple-access (CDMA) systems, interference suppression, reduced-rank parameter estimation.**

I. Introduction

LINEARLY constrained blind adaptive estimation algorithms are useful in several areas of communications and signal processing such as beamforming and interference suppression for code-division-multiple-access (CDMA) systems [1]. In these applications, the linear constraints correspond to prior knowledge of certain parameters such as direction of arrival (DoA) of user signals in antenna array processing [2] and the signature sequence of the desired signal in CDMA interference suppression [3], [4]. With respect to the estimation algorithms, a very popular approach is to deploy stochastic gradient (SG) techniques because they represent simple and low complexity solutions that are preferred for implementation although their convergence depends on the eigenvalue spread of the covariance matrix of the received vector. Conversely, recursive least-squares (RLS) type algorithms have fast convergence, are relatively insensitive to variations in the eigenvalue spread of the covariance matrix of the observation data as compared with the convergence of SG algorithms in stationary scenarios but require a significantly higher complexity than SG recursions [1].

Several attempts to provide cost-effective parameter estimators with fast convergence performance have been made in the last few decades through variable step size algorithms [5]–[13], data-reusing [14], [12] averaging methods [15], subband and frequency-domain adaptive filters [16]–[18], and RLS type algorithms with linear complexity such as lattice-based implementations [19], [20], fast RLS algorithms [21]–[24], and QR-decomposition-based RLS techniques [25]–[27]. A challenging problem that remains unsolved by conventional techniques [5]–[27] is that when the number of elements in the filter is very large, the algorithm requires a large number of samples (or data record) to reach its steady-state behavior. In these situations, even RLS algorithms require an amount of data proportional to $2M$ [1], where M is the number of elements of the estimator, in order to converge and this may lead to unacceptable convergence performance. Furthermore, in highly dynamic systems such as those found in wireless communications, estimators with a large number of elements usually fail or provide poor performance in tracking signals embedded in interference and noise.

Reduced-rank filtering is a very powerful technique that has gained considerable attention in the last few years due to its effectiveness in low sample support situations where it can offer improved convergence performance at an affordable complexity [28]–[46]. The advantages of reduced-rank adaptive filters are their faster convergence speed and better tracking performance over existing techniques when dealing with large number of weights. Various reduced-rank methods and systems are based on principal components analysis, in which a computationally expensive singular value decomposition (SVD) to extract the signal subspace is required [30]–[32]. Other recent techniques such as the multistage Wiener filter (MWF) of Goldstein *et al.* in [35] perform orthogonal decompositions in order to compute its parameters, leading to very good performance and a relatively low complexity. Another technique that resembles the MWF is the auxiliary-vector filtering (AVF) with orthogonal auxiliary vectors (AV) [38], [42]. In this regard, the equivalence between the MWF and the AVF with orthogonal AVs was established in [41]. An AVF structure with non-orthogonal auxiliary vectors

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(AV) was reported in [43] and was shown to slightly outperform the MWF at the cost of a higher computational complexity. Existing work on blind reduced-rank parameter estimation is very limited and relies on an MWF [36] or an AVF [43], [44] version of the constrained minimum variance (CMV) design criterion, which are very sensitive to signature mismatch. In addition, prior work on blind constrained parameter estimators with the constant modulus criterion [48]–[50] has shown improved performance and increased robustness against signature mismatch over the CMV approach and a reduced-rank version of the CCM is still not available in the literature.

In this paper, we develop a multistage decomposition for blind adaptive parameter estimation in the Krylov subspace with the constrained constant modulus (CCM) design criterion. Based on constrained optimization of the constant modulus cost function and utilizing the Lanczos algorithm and Arnoldi-like iterations, an efficient multistage decomposition is developed for blind parameter estimation. Based on the Krylov subspace projection, we also devise a family of computationally efficient blind adaptive reduced-rank stochastic gradient (SG) and RLS type algorithms along with an automatic rank selection procedure. An analysis of the convergence properties of the method is carried out and shows some mathematical conditions for the method to be globally convergent. We also establish the convergence conditions for the reduced-rank adaptive algorithms. The proposed batch and adaptive algorithms are then extensively studied in simulation experiments for CDMA interference suppression.

This paper is structured as follows. Section II describes a DS-CDMA System Model. Section III presents a framework for the CCM linear receiver design, briefly reviews linearly constrained receivers and blind adaptive constrained algorithms. Section IV introduces the reduced-rank version of the CCM design for linear receivers and details the multistage decomposition used to compute the reduced-rank projection matrix. Section V is devoted to the derivation of blind adaptive constrained reduced-rank estimation algorithms and the automatic rank selection mechanism based on the CCM criterion. Section VI presents and discusses the numerical simulation results, while Section VII gives the conclusions.

II. DS-CDMA System Model

Let us consider the uplink of a symbol synchronous binary phase-shift keying (BPSK) DS-CDMA system with K users, N chips per symbol and L_p propagation paths. It should be remarked that a synchronous model is assumed for simplicity, although it captures most of the features of more realistic asynchronous models with small to moderate delay spreads. Although BPSK modulation was adopted in the system model for the sake of simplicity, the techniques presented in this work can be easily extended to other constant modulus modulation formats. The baseband signal transmitted by the kth active user to the base station is given by

$$x_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k[i] s_k(t - iT) \tag{1}$$

where $b_k[i] \in \{\pm 1\}$ denotes the ith symbol for user k, the real valued spreading waveform and the amplitude associated with user k are $s_k(t)$ and A_k, respectively. The spreading waveforms are expressed by $s_k(t) = \sum_{i=1}^{N} a_k[i]\phi(t - iT_c)$, where $a_k[i] \in \{\pm 1/\sqrt{N}\}, \phi(t)$ is the chip waverform, T_c is the chip duration and $N = T/T_c$ is the processing gain. Assuming that the receiver is synchronized with the main path, the coherently demodulated composite received signal is

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L_p-1} h_{k,l}(t) x_k(t - \tau_{k,l}) + n(t) \tag{2}$$

where $h_{k,l}(t)$ and $\tau_{k,l}$ are, respectively, the channel coefficient and the delay associated with the lth path and the kth user and $n(t)$ represents the noise at the receiver. Assuming that $\tau_{k,l} = lT_c$, that the channel is time-varying but constant during each symbol interval and the spreading codes are repeated from symbol to symbol, the received signal $r(t)$ after filtering by a chip-pulse matched filter and sampled at chip rate yields the M-dimensional received vector

$$\begin{aligned}\mathbf{r}[i] &= \sum_{k=1}^{K} \mathbf{H}_k[i] A_k \mathbf{S}_k \mathbf{b}_k[i] + \mathbf{n}[i] \\ &= \sum_{k=1}^{K} A_k b_k[i] \mathbf{C}_k \mathbf{h}_k[i] + \boldsymbol{\eta}_k[i] + \mathbf{n}[i]\end{aligned} \tag{3}$$

where $M = N + L_p - 1, \mathbf{n}[i] = [n_1[i] \cdots n_M[i]]^T$ is the complex Gaussian noise vector with zero mean and $E[\mathbf{n}[i]\mathbf{n}^H[i]] = \sigma^2 \mathbf{I}$ whose components are independent and identically distributed, where $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively, and $E[\cdot]$ stands for expected value. The user symbol vector is $\mathbf{b}_k[i] = [b_k[i + L_s - 1] \cdots b_k[i] \cdots b_k[i - L_s + 1]]^T$, the amplitude of user k is $A_k, \boldsymbol{\eta}_k[i]$ is the ISI for user k and it should be noted that it is already contained in the first term of the first line of (3) and L_s is the ISI span. The $(2 \cdot L_s - 1) \cdot N \times (2 \cdot L_s - 1)$ diagonal matrix $\mathbf{S}_k$ with N-chips shifted versions of the signature of user k is given by

$$\mathbf{S}_k = \begin{bmatrix} \mathbf{s}_k & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{s}_k & \ddots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{s}_k \end{bmatrix} \tag{4}$$

where $\mathbf{s}_k = [a_k(1) \cdots a_k(N)]^T$ is the signature sequence for the kth user, the $M \times L_p$ constraint matrix $\mathbf{C}_k$ that contains one-chip shifted versions of the signature sequence for user k and the $L_p \times 1$ vector $\mathbf{h}_k[i]$ with the multipath components are described by

$$\begin{aligned}\mathbf{C}_k &= \begin{bmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{bmatrix} \\ \mathbf{h}_k[i] &= \begin{bmatrix} h_{k,0}[i] \\ \vdots \\ h_{k,L_p-1}[i] \end{bmatrix}.\end{aligned} \tag{5}$$

The $M \times (2 \cdot L_s - 1) \cdot N$ channel matrix $\mathbf{H}_k[i]$ for user k is given by

$$\mathbf{H}_k[i] = \begin{bmatrix} \mathbf{h}_k^T[i] & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \mathbf{h}_k^T[i] \end{bmatrix}. \tag{6}$$

The MAI comes from the non-orthogonality between the received signature sequences, whereas the ISI span L_s depends on the length of the channel response and how it is related to the length of the chip sequence. For $L_p = 1, L_s = 1$ (no ISI), for $1 < L_p \leq N, L_s = 2$, for $N < L_p \leq 2N, L_s = 3$, and so on. This means that at time instant i we will have ISI coming not only from the previous $L_s - 1$ time instants but also from the next $L_s - 1$ symbols.

III. The CCM Linear Receiver Design

Consider the received vector $\mathbf{r}[i]$, the $M \times L_p$ constraint matrix $\mathbf{C}_k$ that contains one-chip shifted versions of the signature sequence for user k and the $L_p \times 1$ vector $\hat{\mathbf{h}}_k[i] = [\hat{h}_{k,0}[i] \cdots \hat{h}_{k,L_p-1}[i]]^T$ with the multipath components to be estimated. The CCM linear receiver design is equivalent to determining an FIR filter $\mathbf{w}_k[i]$ with M coefficients that provide an estimate of the desired symbol $\hat{b}_k[i] = \mathrm{sgn}(\Re(\mathbf{w}_k^H[i]\mathbf{r}[i]))$, where $\mathrm{sgn}(\cdot)$ is the signum function, $\Re(\cdot)$ selects the real component, and $\mathbf{w}_k[i]$ is optimized according to the CM cost function

$$J_{\mathrm{CM}}(\mathbf{w}_k[i]) = E\left[\left(\left|\mathbf{w}_k^H[i]\mathbf{r}[i]\right|^2 - 1\right)^2\right] \tag{7}$$

subject to the constraints given by $\mathbf{w}_k^H[i]\mathbf{p}_k[i] = \nu$, where $\mathbf{p}_k[i] = \mathbf{C}_k\mathbf{h}_k[i], \mathbf{h}_k[i]$ is the vector that contains the multipath gains and ν is a constant to ensure the convexity of (7), as will be discussed in Appendix I. This approach assumes the knowledge of the channel. However, when multipath is present these parameters are unknown and time-varying, requiring channel estimation. The CCM filter expression that iteratively solves the constrained optimization problem in (7) is

$$\begin{aligned}\mathbf{w}_k[i+1] = \mathbf{R}_k^{-1}[i]\Big[\mathbf{d}_k[i] - \left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{p}_k[i]\right)^{-1} \\ \cdot\left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{d}_k[i] - \nu\right)\mathbf{p}_k[i]\Big], \quad i = 1, 2, \dots\end{aligned} \tag{8}$$

where $z_k[i] = \mathbf{w}_k^H[i]\mathbf{r}[i], \mathbf{R}_k[i] = E[|z_k[i]|^2\mathbf{r}[i]\mathbf{r}^H[i]], \mathbf{d}_k[i] = E[z_k^*[i]\mathbf{r}[i]]$. A detailed derivation of the CCM estimation approach is given in Appendix II. It should be remarked that the expression in (8) is a function of previous values of the filter $\mathbf{w}_k[i]$ and therefore must be iterated in order to reach a solution. In addition to this, the iterative method in (8) assumes the knowledge of the channel parameters. Since there is a large number of applications that have to deal with unknown multipath propagation, it is also important to be able to blindly estimate the multipath components. In this regard, it should be remarked that the approach in (8) can also work without the channel information by employing the inverse filtering criterion of [51], i.e., by simply setting $\mathbf{h}_k[i] = [10 \cdots 0]^T$. However, by exploiting the signal copies of the received signal through some knowledge of the channel it is possible to achieve superior performance.

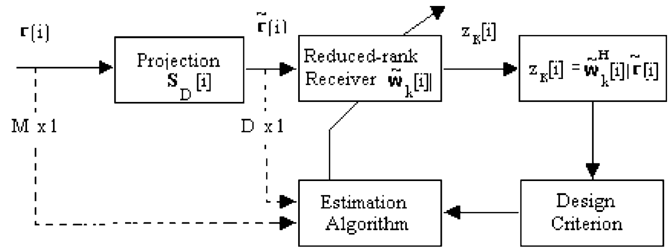


Fig. 1. Reduced-rank processing and receiver design.

In order to blindly estimate the channel, we adopt here the blind channel estimation procedure based on the subspace approach proposed in [51], [47], [53] which is described by

$$\hat{\mathbf{h}}_k[i] = \arg\min_{\mathbf{h}_k[i]} \mathbf{h}_k^H[i]\mathbf{C}_k^H\mathbf{R}^{-1}[i]\mathbf{C}_k\mathbf{h}_k[i] \tag{9}$$

subject to $||\mathbf{h}_k[i]|| = 1$, where $\mathbf{R}[i] = E[\mathbf{r}[i]\mathbf{r}^H[i]]$. The solution is the eigenvector of the $L_p \times L_p$ matrix corresponding to the minimum eigenvalue of $\mathbf{C}_k^H\mathbf{R}^{-1}[i]\mathbf{C}_k$ through singular value decomposition (SVD). Here, we use $\mathbf{R}_k[i]$ in lieu of $\mathbf{R}[i]$ to avoid the estimation of both $\mathbf{R}[i]$ and $\mathbf{R}_k[i]$, and which shows no performance loss as verified in our studies and explained in Appendix III.

IV. The Reduced-Rank CCM Linear Receiver Design

In this section, we describe the reduced-rank CCM design based on a multistage decomposition of the expression obtained in the previous section.

A. Reduced-Rank Receiver Design

The filter expression for the CCM design can be estimated by either computing the matrix inversion and the remaining operations in (8) or resorting to SG and RLS type adaptive algorithms. However, whenever the dimension M of received data $\mathbf{r}[i]$ is large, the convergence performance is slow. In this section, we describe a reduced-rank algorithm that reduces the number of adaptive coefficients by projecting the received signal onto a lower dimension subspace. An illustration of reduced-rank signal processing dimensionality reduction and corresponding receiver design is depicted in Fig. 1.

Specifically, let $\mathbf{S}_D[i]$ be a $M \times D$-dimensional matrix that accomplishes the dimensionality reduction as given by

$$\tilde{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i] \tag{10}$$

where, in what follows, all D-dimensional quantities incorporate a "tilde". The reduced-rank CCM linear receiver design is equivalent to computing an FIR filter $\tilde{\mathbf{w}}_k[i]$ with D elements that yield the desired symbol as

$$\hat{b}_k[i] = \mathrm{sgn}\left(\Re\left[\tilde{\mathbf{w}}_k^H[i]\tilde{\mathbf{r}}[i]\right]\right) = \mathrm{sgn}(\Re[z_k[i]]) \tag{11}$$

where the optimization criterion, i.e., the CM cost function is $J_{\mathrm{CM}}(\tilde{\mathbf{w}}_k[i]) = E[(|\tilde{\mathbf{w}}_k^H[i]\mathbf{S}_D^H[i]\mathbf{r}[i]|^2 - 1)^2]$ and the set of constraints is $\tilde{\mathbf{w}}_k^H[i]\tilde{\mathbf{p}}_k[i] = \nu$, where $\tilde{\mathbf{p}}_k[i] = \mathbf{S}_D^H[i]\mathbf{C}_k\mathbf{h}_k[i]$. The reduced-rank CCM filter expression is

$$\tilde{\mathbf{w}}_k[i+1] = \tilde{\mathbf{R}}_k^{-1}[i]\Big[\tilde{\mathbf{d}}_k[i] - \left(\tilde{\mathbf{p}}_k^H[i]\tilde{\mathbf{R}}_k^{-1}[i]\tilde{\mathbf{p}}_k[i]\right)^{-1} \cdot \left(\tilde{\mathbf{p}}_k^H[i]\tilde{\mathbf{R}}_k^{-1}[i]\tilde{\mathbf{d}}_k[i] - \nu\right)\tilde{\mathbf{p}}_k[i]\Big] \quad (12)$$

where $z_k[i] = \tilde{\mathbf{w}}_k^H[i]\tilde{\mathbf{r}}[i] = \tilde{\mathbf{w}}_k^H[i]\mathbf{S}_D^H[i]\mathbf{r}[i], \tilde{\mathbf{R}}_k[i] = \mathbf{S}_D^H[i]\mathbf{R}_k[i]\mathbf{S}_D[i] = \mathbf{S}_D^H[i]E[|z_k[i]|^2\mathbf{r}[i]\mathbf{r}^H[i]]\mathbf{S}_D[i], \mathbf{d}_k[i] = E[z_k^*[i]\tilde{\mathbf{r}}[i]] = \mathbf{S}_D^H[i]E[z_k^*[i]\mathbf{r}[i]]$.

B. Multistage Decomposition and Projection Matrix Design

Here we detail the procedure to compute the projection matrix $\mathbf{S}_D^H[i]$ and the multistage decomposition. Let us rewrite the CCM expression of (8) in the following alternative form

$$\mathbf{w}_k[i+1] = \mathbf{R}_k^{-1}[i]\mathbf{q}_k[i] \quad (13)$$

where

$$\mathbf{q}_k[i] = \mathbf{d}_k[i] - \left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{p}_k[i]\right)^{-1} \cdot \left(\mathbf{p}_k^H[i]\mathbf{R}_k^{-1}[i]\mathbf{d}_k[i] - \nu\right)\mathbf{p}_k[i]. \quad (14)$$

Following the schematic of Fig. 2, we wish to develop a multistage decomposition of the expression in (13) that computes the projection matrix $\mathbf{S}_D^H[i]$. Specifically, the first filter of the structure in Fig. 2, namely $\mathbf{f}_k^{(1)}[i]$, is the normalized version of $\mathbf{q}_k[i]$, i.e., $\mathbf{f}_k^{(1)}[i] = (q_k[i]/\|\mathbf{q}_k[i]\|)$. In this proposed multistage decomposition, the dth filter $\mathbf{f}_k^{(d)}[i]$ maximizes the real part of the correlation between its output $z^{(d)}[i]$ and the output of the previous filters $z^{(d-1)}[i]$. This optimization problem first appeared in [54], although, in the context of reduced-rank estimation, a similar approach was firstly employed in [39]. Both approaches lead to the computation of Krylov subspaces. By restricting the filters to be orthonormal, the dth filter can be computed via the following optimization:

$$\begin{aligned}\mathbf{f}_k^{(d)}[i] &= \arg\max_{\mathbf{f}_k^{(d)}[i]} E\left[\Re\left(z^{(d)}[i]z^{(d-1)*}[i]\right)\right] \\ &= \arg\max_{\mathbf{f}_k^{(d)}[i]} E \\ &\quad \cdot \left[\Re\left(\mathbf{f}_k^{(d),H}[i]\mathbf{r}[i]\mathbf{r}^H[i]\mathbf{f}_k^{(d-1)}[i]\right)\right] \end{aligned} \quad (15)$$

subject to $\mathbf{f}_k^{(j)H}[i]\mathbf{f}_k^{(m)}[i] = 1, m = j$ and $\mathbf{f}_k^{(j)H}[i]\mathbf{f}_k^{(m)}[i] = 0$, for $m \neq j$.

A general solution to the optimization problem in (15) can be computed via the Arnoldi iteration [54], [52], which is a numerical optimization algorithm to solve linear systems problems, and is described by

$$\mathbf{f}_k^{(d)}[i] = \frac{\left(\Pi_{l=1}^{d-1}\mathbf{P}_l[i]\right)\mathbf{R}_k^{-1}[i]\mathbf{f}_k^{(d-1)}[i]}{\left\|\left(\Pi_{l=1}^{d-1}\mathbf{P}_l[i]\right)\mathbf{R}_k^{-1}[i]\mathbf{f}_k^{(d-1)}[i]\right\|}, \in \mathcal{C}^M \quad (16)$$

where the matrix $\mathbf{P}_l[i] = \mathbf{I}_M - \mathbf{f}_k^{(l)}[i]\mathbf{f}_k^{(l),H}[i]$ has the role of projecting the signal onto the space orthogonal to the filter $\mathbf{f}_k^{(l)}[i]$ and $\mathbf{I}_M$ is the $M \times M$ identity matrix. Because $\mathbf{R}_k[i]$ is Hermitian, the designer can resort to the Lanczos algorithm, a simpler technique than the Arnoldi recursion and that can be used to solve symmetric systems of linear equations [52]. The reader is referred to [52] for further details on the method. The method generates a sequence of tridiagonal matrices and parameters that are gradually better estimates of the desired solution as given by

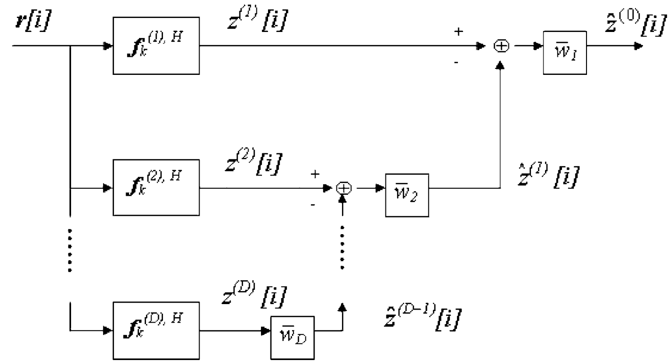


Fig. 2. Proposed reduced-rank CCM filterbank structure.

$$\mathbf{f}_k^{(d)}[i] = \frac{\mathbf{P}_{d-1}[i]\mathbf{P}_{d-2}[i]\mathbf{R}_k^{-1}[i]\mathbf{f}_k^{(d-1)}[i]}{\left\|\mathbf{P}_{d-1}[i]\mathbf{P}_{d-2}[i]\mathbf{R}_k^{-1}[i]\mathbf{f}_k^{(d-1)}[i]\right\|}, \in \mathcal{C}^M. \quad (17)$$

The Lanczos method above leads to a tridiagonal covariance matrix $\mathbf{R}_k^{(d)}[i]$ for the projected vector $\mathbf{z}[i] = [z^{(1)}[i]\cdots z^{(d)}[i]]^T$, where the parameters $z^{(d)}[i]$ are obtained after the application of each filter $\mathbf{f}_k^{(l)}[i]$. The scalar filters $\tilde{w}_d[i]$, where $d = 1,\ldots,D$, are utilized to estimate the output $z^{(d-1)}[i]$ of the previous filter from an error signal $\epsilon[i]$. Following this procedure and Fig. 2, the reduced-rank CCM filter with rank D can be obtained by neglecting the signal $z^{(D)}[i]$. In this respect, the procedure described above details the computation of the projection matrix $\mathbf{S}_D[i]$ for user k, which has the following structure

$$\begin{aligned}\mathbf{S}_D[i] &= \left[\mathbf{f}_k^{(1)}[i],\ldots,\mathbf{f}_k^{(D)}[i]\right], \quad \in \mathcal{C}^{M\times D} \\ &= \left[\mathbf{q}_k[i], \mathbf{R}_k[i]\mathbf{q}_k[i],\ldots,\mathbf{R}_k^{(D-1)}[i]\mathbf{q}_k[i]\right]. \end{aligned} \quad (18)$$

At this point, we remark the main differences between the eigendecomposition techniques and the Krylov subspace approaches. Specifically, in the work of Goldstein and Reed [35] an eigendecomposition approach would require an SVD on the full-rank covariance matrix and the selection of the D eigenvectors associated with the D largest eigenvalues. In contrast to that, the Krylov-based approach does not require eigendecomposition and selects the D basis vectors which minimize the desired cost function and will form the projection matrix (see also Honig and Goldstein [36]). By using the projection matrix $\mathbf{S}_D[i]$, the D-dimensional observation vector is expressed by

$$\tilde{\mathbf{r}}[i] = \mathbf{z}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i] \quad (19)$$

and the reduced-rank CCM filter $\tilde{\mathbf{w}}_k^D[i] = [\tilde{w}_1[i]\cdots\tilde{w}_D[i]]^T$ with rank D is

$$\begin{aligned}\tilde{\mathbf{w}}_k^D[i+1] &= \left(\mathbf{S}_D^H[i]\mathbf{R}_k[i]\mathbf{S}_D[i]\right)^{-1}\mathbf{S}_D^H[i]\mathbf{q}_k[i] \\ &= \tilde{\mathbf{R}}_k^{-1}[i]\tilde{\mathbf{q}}_k[i]. \end{aligned} \quad (20)$$

The reduced-rank solution with rank D above projects the received signal onto a lower dimensional subspace, which cor-

TABLE I
BATCH ITERATIVE ALGORITHM FOR THE DESIGN OF THE PROPOSED CCM REDUCED-RANK FILTER SCHEME

Algorithm :
Initialize $\tilde{\mathbf{w}}_k[0] = [0\ldots0]^T$, $z_k[0]=1$, $\hat{\mathbf{h}}_k[0] = [1\,0\ldots0]^T$, $\mathbf{p}_k[0] = \mathbf{C}_k\hat{\mathbf{h}}_k[0]$
Choose rank D
for each time instant [i] do
Compute Estimate of $\mathbf{R}_k$: $\hat{\mathbf{R}}_k[i] = \sum_{j=1}^{i}\lambda^{i-l}\lvert z_k[i]\rvert^2\tilde{\mathbf{r}}[j]\tilde{\mathbf{r}}^H[j]$
Estimate Channel $\hat{\mathbf{h}}_k[i] = \arg\min_{\mathbf{h}_k}\ \mathbf{h}_k^H\mathbf{C}_k^H\hat{\mathbf{R}}_k^{-1}[i]\mathbf{C}_k\mathbf{h}_k$
Compute $\mathbf{S}_D$: $\mathbf{S}_D[i] = \left[\mathbf{q}_k[i],\ \hat{\mathbf{R}}_k[i]\mathbf{q}_k[i],\ \ldots,\ \hat{\mathbf{R}}_k^{(D-1)}[i]\mathbf{q}_k[i]\right]$
Compute reduced-rank received vector: $\tilde{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i]$
Compute estimate of $\mathbf{q}_k[i]$
Reduced-rank CCM filter: $\tilde{\mathbf{w}}_k^D[i+1] = \left(\mathbf{S}_D^H[i]\mathbf{R}_k[i]\mathbf{S}_D[i]\right)^{-1}\mathbf{S}_D^H[i]\mathbf{q}_k[i]$
Reduced-rank estimate: $z_k[i] = \tilde{\mathbf{w}}_k^{D,\,H}[i]\tilde{\mathbf{r}}[i]$

responds to the D-dimensional Krylov subspace $\mathcal{K}^D(\mathbf{R}_k,\mathbf{q}_k)$, where $\mathcal{K}^D(\mathbf{A},\mathbf{b}) = \mathrm{span}[\mathbf{b},\mathbf{A}\mathbf{b},\ldots,\mathbf{A}^{(D-1)}\mathbf{b}]$ and was defined for the problem at hand in (18). A summary of the batch CCM reduced-rank algorithm is shown in Table I.

V. BLIND ADAPTIVE REDUCED-RANK ESTIMATION ALGORITHMS

In this section, we derive blind adaptive reduced-rank estimation algorithms based on the reduced-rank decomposition of the previous section. Specifically, we develop SG and RLS type blind constrained algorithms for reduced-rank parameter estimation. The complexity in terms of arithmetic operations of the new algorithms and the existing techniques is included as a function of the number of adaptive elements for comparison purposes.

A. SG Reduced-Rank Algorithm

In order to develop an SG reduced-rank estimation algorithm based on the CCM design, let us consider the following unconstrained cost function

$$J'_{\mathrm{CM}}(\mathbf{w}_k[i]) = E\left[\left(\lvert\tilde{\mathbf{w}}_k^{(D),H}[i]\tilde{\mathbf{r}}[i]\rvert^2 - 1\right)^2\right] + \Re\left[\lambda\left(\tilde{\mathbf{w}}_k^{(D),H}[i]\tilde{\mathbf{p}}_k[i] - \nu\right)\right] \quad (21)$$

where $\tilde{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i], \tilde{\mathbf{p}}_k[i] = \mathbf{S}_D^H[i]\mathbf{p}_k[i]$ and $\mathbf{S}_D[i] = [\mathbf{q}_k[i],\mathbf{R}_k[i]\mathbf{q}_k[i],\ldots,\mathbf{R}_k^{(D-1)}[i]\mathbf{q}_k[i]]$.

By taking the gradient terms of (21) with respect to $\tilde{\mathbf{w}}_k^*[i]$, where $*$ stands for complex conjugate, and using instantaneous estimates of all parameters and $\mathbf{S}_D[i]$, we seek to adaptively minimize J_{CM}. If we consider the set of constraints $\tilde{\mathbf{w}}_k^{(D),H}[i]\tilde{\mathbf{p}}_k[i] = \nu$, we arrive at the update equations for the estimation of $\mathbf{w}_k^{(D)}[i]$

$$\tilde{\mathbf{w}}_k^D[i+1] = \mathbf{\Pi}_k[i]\left(\mathbf{w}_k^{(D)}[i] - \mu_w e_k[i]z_k^*[i]\tilde{\mathbf{r}}[i]\right) + \nu\tilde{\mathbf{p}}_k[i] \quad (22)$$

where $z_k[i] = \tilde{\mathbf{w}}_k^{(D),H}[i]\tilde{\mathbf{r}}[i], e_k[i] = (\lvert z_k[i]\rvert^2 - 1)$ and $\mathbf{\Pi}_k[i] = \mathbf{I} - \tilde{\mathbf{p}}_k[i]\tilde{\mathbf{p}}_k^H[i]$ is a matrix that projects the reduced-rank receiver's parameters onto another hyperplane in order to ensure the constraints. The SG channel estimation procedure described in [53] is employed for estimating $\mathbf{h}_k[i]$ and constructing $\tilde{\mathbf{p}}_k[i] = \mathbf{S}_D^H[i]\mathbf{C}_k\hat{\mathbf{h}}_k[i]$. A summary of the proposed CCM reduced-rank SG algorithm is shown in Table II.

TABLE II
SG ITERATIVE ALGORITHM FOR THE DESIGN OF THE PROPOSED CCM REDUCED-RANK FILTER SCHEME

Algorithm :
Initialize $\tilde{\mathbf{w}}_k[0] = [0\ldots0]^T$, $z_k[0]=1$, $\hat{\mathbf{h}}_k[0] = [1\,0\ldots0]^T$, $\mathbf{p}_k[0] = \mathbf{C}_k\hat{\mathbf{h}}_k[0]$
Choose rank D and step size μ_w
for each time instant [i] do
Estimate Channel with SG algorithm of [53]
Compute $\mathbf{S}_D$: $\mathbf{S}_D[i] = \left[\mathbf{q}_k[i],\ \hat{\mathbf{R}}_k[i]\mathbf{q}_k[i],\ \ldots,\ \hat{\mathbf{R}}_k^{(D-1)}[i]\mathbf{q}_k[i]\right]$
Compute $\tilde{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i]$ and $\tilde{\mathbf{p}}_k[i] = \mathbf{S}_D^H[i]\mathbf{p}_k[i]$
Reduced-rank CCM filter: $\mathbf{w}_k^D[i+1] = \mathbf{\Pi}_k[i]\left(\mathbf{w}_k^{(D)}[i] - \mu_w e_k[i]z_k^*[i]\tilde{\mathbf{r}}[i]\right) + \nu\,\tilde{\mathbf{p}}_k[i]$
Reduced-rank estimate: $z_k[i] = \tilde{\mathbf{w}}_k^{D,\,H}[i]\tilde{\mathbf{r}}[i]$

B. RLS Reduced-Rank Algorithm

Given the expressions for the reduced-rank CCM linear filter $\tilde{\mathbf{w}}_k[i]$ in (12) and the projection matrix $\mathbf{S}_D[i]$, we need to estimate $\mathbf{R}_k^{-1}[i]$ and $\tilde{\mathbf{R}}_k^{-1}[i]$ recursively to reduce the computational complexity required to invert these matrices. Using the matrix inversion lemma and Kalman RLS recursions [1] we have

$$\mathbf{G}_k[i] = \frac{\alpha^{-1}\hat{\mathbf{R}}_k^{-1}[i-1]z_k^*[i]\mathbf{r}[i]}{1 + \alpha^{-1}\mathbf{r}^H[i]z_k[i]\hat{\mathbf{R}}_k^{-1}[i-1]z_k^*[i]\mathbf{r}[i]} \quad (23)$$

$$\hat{\mathbf{R}}_k^{-1}[i] = \alpha^{-1}\hat{\mathbf{R}}_k^{-1}[i-1] - \alpha^{-1}\mathbf{G}_k[i]z_k[i]\mathbf{r}^H[i]\hat{\mathbf{R}}_k^{-1}[i-1] \quad (24)$$

and

$$\tilde{\mathbf{G}}_k[i] = \frac{\alpha^{-1}\hat{\tilde{\mathbf{R}}}_k^{-1}[i-1]z_k^*[i]\tilde{\mathbf{r}}[i]}{1 + \alpha^{-1}\tilde{\mathbf{r}}^H[i]z_k[i]\hat{\tilde{\mathbf{R}}}_k^{-1}[i-1]z_k^*[i]\tilde{\mathbf{r}}[i]} \quad (25)$$

$$\hat{\tilde{\mathbf{R}}}_k^{-1}[i] = \alpha^{-1}\hat{\tilde{\mathbf{R}}}_k^{-1}[i-1] - \alpha^{-1}\tilde{\mathbf{G}}_k[i]z_k[i]\tilde{\mathbf{r}}^H[i]\hat{\tilde{\mathbf{R}}}_k^{-1}[i-1] \quad (26)$$

where $0 < \alpha < 1$ is the forgetting factor. The recursions in (23)–(26) correspond to the use of the matrix inversion lemma to reduce the complexity required for estimating the inverse of the $M\times M$ full-rank matrix $\mathbf{R}_k[i]$ and the inverse of the $D\times D$ reduced-rank matrix $\tilde{\mathbf{R}}_k[i]$. The algorithms can be initialized with $\hat{\mathbf{R}}_k^{-1}(0) = \delta\mathbf{I}$ and $\hat{\tilde{\mathbf{R}}}_k^{-1}(0) = \delta\mathbf{I}$ where δ is a scalar to ensure numerical stability. Once $\hat{\mathbf{R}}_k^{-1}[i]$ is updated, it is used for channel estimation, to obtain $\tilde{\mathbf{q}}_k[i]$ and to construct an estimate of $\mathbf{S}_D[i]$. The RLS channel estimation procedure described in [53] is employed for estimating $\mathbf{h}_k[i]$. Finally, we construct the CCM reduced-rank linear receiver as described by

$$\tilde{\mathbf{w}}_k^D[i+1] = \hat{\tilde{\mathbf{R}}}_k^{-1}[i]\hat{\tilde{\mathbf{q}}}_k[i] \quad (27)$$

where

$$\hat{\mathbf{q}}_k[i] = \hat{\mathbf{d}}_k[i] - \left(\hat{\mathbf{p}}_k^H[i]\mathbf{R}_k^{-1}[i]\hat{\mathbf{p}}_k[i]\right)^{-1} \times\left(\hat{\mathbf{p}}_k^H[i]\hat{\mathbf{R}}_k^{-1}[i]\hat{\mathbf{d}}_k[i] - \nu\right)\hat{\mathbf{p}}_k[i] \quad (28)$$

$\hat{\mathbf{d}}_k[i]$ is estimated by $\hat{\mathbf{d}}_k[i+1] = \alpha\hat{\mathbf{d}}_k[i] + (1-\alpha)z_k^*[i]\tilde{\mathbf{r}}[i], \hat{\tilde{\mathbf{p}}}_k[i] = \hat{\mathbf{S}}_D[i]\mathbf{C}_k\hat{\mathbf{h}}_k[i]$ and the reduced-rank projection matrix is $\mathbf{S}_D[i] = [\hat{\mathbf{q}}_k[i],\hat{\mathbf{R}}_k[i]\hat{\mathbf{q}}_k[i],\ldots,\hat{\mathbf{R}}_k^{(D-1)}[i]\hat{\mathbf{q}}_k[i]]$. A summary of the proposed CCM reduced-rank RLS algorithm is shown in Table III.

TABLE III
RLS ITERATIVE ALGORITHM FOR THE DESIGN OF THE PROPOSED CCM REDUCED-RANK FILTER SCHEME

Algorithm :
Initialize $\tilde{\mathbf{w}}_k[0] = [0\ldots0]^T$, $z_k[0]=1$, $\hat{\mathbf{h}}_k[0] = [1\,0\ldots0]^T$, $\mathbf{p}_k[0] = \mathbf{C}_k\hat{\mathbf{h}}_k[0]$
Choose rank D and forgetting factor α
for each time instant [i] do
Compute estimate of $\mathbf{R}_k^{-1}$ and $\tilde{\mathbf{R}}_k^{-1}[i]$ with Matrix Inversion Lemma
Estimate channel with RLS algorithm of [53] and $\mathbf{R}_k^{-1}$
Compute $\mathbf{S}_D$: $\mathbf{S}_D[i] = \left[\mathbf{q}_k[i],\ \hat{\mathbf{R}}_k[i]\mathbf{q}_k[i],\ \ldots,\ \hat{\mathbf{R}}_k^{(D-1)}[i]\mathbf{q}_k[i]\right]$
Compute reduced-rank received vector: $\tilde{\mathbf{r}}[i] = \mathbf{S}_D^H[i]\mathbf{r}[i]$
Compute estimate $\hat{\tilde{\mathbf{q}}}_k[i]$
Reduced-rank CCM filter: $\tilde{\mathbf{w}}_k^D[i+1] = \hat{\tilde{\mathbf{R}}}_k^{-1}[i]\hat{\tilde{\mathbf{q}}}_k[i]$
Reduced-rank estimate: $z_k[i] = \tilde{\mathbf{w}}_k^{D,\,H}[i]\tilde{\mathbf{r}}[i]$

TABLE IV
COMPUTATIONAL COMPLEXITY OF SG ADAPTATION ALGORITHMS AND THE PROPOSED MWF-CCM SG ALGORITHM

Algorithm	Number of operations per symbol: Additions	Multiplications
Full-rank	$2M$	$2M+1$
CMV-Full-rank	M^2+ML_p $+2M+1$	M^2+ML_p $+3M$
CCM-Full-rank	M^2+ML_p $+2M+2$	M^2+ML_p $+3M+2$
MWF	$2(D-1)^2+2D(M-1)$ $+(D-1)(M-1)+M$	$D^2+3D+2DM$ $+M+1$
MWF-CMV	$2(D-1)^2+2D(M-1)$ $+(D-1)(M-1)+M$	$D^2+3D+2DM$ $+ML_p+1$
MWF-CCM	$3(D-1)^2+2D(M-1)$ $+(D+1)(M-1)+3M+3$	$2D^2+7D+2DM$ $+ML_p+2$

C. Computational Complexity of Algorithms

In this section, we illustrate the computational complexity of the proposed MWF-CCM-based SG and RLS algorithms and other existing ones, as shown in Tables IV and V. The computational requirements are described in terms of number of arithmetic operations, namely additions and multiplications.

In Fig. 3 we depict curves that describe the computational complexity in terms of the arithmetic operations (additions and multiplications) as a function of the number of parameters M. We use the same colors for the corresponding SG techniques in Fig. 3(a) and their associated RLS counterparts in Fig. 3(b). For these curves, we consider $L_p = 8$ and assume that $D = 4$ for the MWF-SG based approaches, while $D = 5$ for the MWF-RLS techniques and $D = 8$ for the AVF-based techniques, which are depicted with RLS methods. The curves in Fig. 3(a) show that there is a significant computational advantage of the reduced-rank SG recursions over the blind full-rank methods even though the LMS algorithm is still significantly less complex. For the RLS algorithms, as depicted in Fig. 3(b), we verify that the reduced-rank schemes are much simpler than any full-rank RLS algorithm due to the quadratic cost on M rather than D for the full-rank schemes operating with the RLS algorithm. Amongst the reduced-rank techniques, the blind CMV and CCM algorithms are slightly more complex than their supervised counterparts, whereas the proposed CCM recursions are comparable in complexity to the CMV-based estimators (being slightly more complex for SG algorithms). The AVF-based schemes [43], namely the blind AVF-CMV and the supervised AVF, usually imply in extra complexity as they have more operations per auxiliary vector (AV) and also require a higher number of AVs to ensure good performance. The AVF-CMV uses the effective signature sequence $\mathbf{C}_k\hat{\mathbf{h}}_k[i]$ as the initial AV, whereas the supervised AVF employs a steering vector estimated by $\hat{\mathbf{p}}[i] = \alpha\hat{\mathbf{p}}[i-1] + (1-\alpha)b_k^*[i]\mathbf{r}[i]$.

TABLE V
COMPUTATIONAL COMPLEXITY OF RLS, THE AVF-BASED RECURSIONS, AND THE PROPOSED MWF-CCM RLS-TYPE ALGORITHMS

Algorithm	Number of operations per symbol: Additions	Multiplications
Full-rank	$3(M-1)^2$ $+M^2+2M$	$6M^2$ $+2M+2$
CMV-Full-rank	$4(M-1)^2+M^2$ $+M+3(L_p-1)^2$	$7M^2+M+L_p^2$ $+ML_p+L_p+4$
CCM-Full-rank	$5(M-1)^2$ $+M^2+5M-1$ $+3(L_p-1)^2$	$8M^2+5M$ $+L_p^2+ML_p$ $+L_p+4$
MWF	$D^2+2(D-1)^2$ $+2D(M-1)+M$ $+(D-1)(M-1)$	$4D^2+3D$ $+2DM+ML_p$ $+4$
MWF-CMV	$D^2+(M-1)L_p$ $+2D(M-1)+2(D-1)^2$ $+(D-1)(M-1)$	$4D^2+3D$ $+2DM+(D-1)M$ $+ML_p$
MWF-CCM	$D^2+3(D-1)^2$ $+2DM+(M-1)L_p$ $+(D-1)(M+1)$	$5D^2+7D$ $+2DM+(D-1)M$ $+ML_p+4$
AVF	$D(M^2+3(M-1)^2)$ $+D(5(M-1)+1)$ $+2(M-1)+1$	$D(4M^2+4M+1)$ $+4M+2$
AVF-CMV	$D(M^2+3(M-1)^2)$ $+D(5(M-1)+1)$ $+(M-1)+M(L_p-1)$	$D(4M^2+4M+1)$ $+ML_p+2M+1$

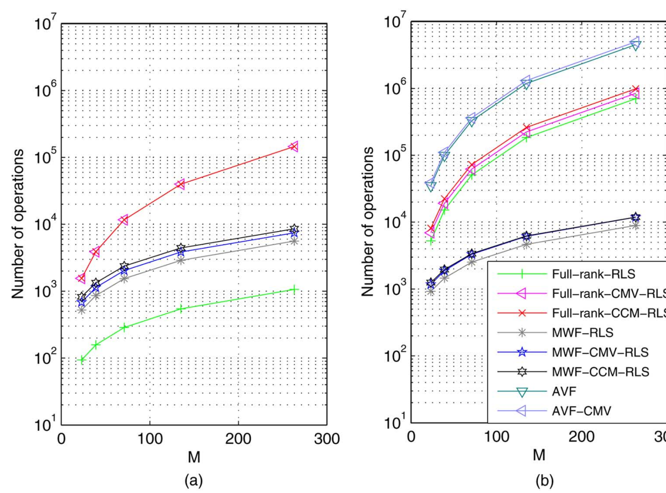


Fig. 3. Complexity in terms of arithmetic operations of (a) SG and (b) RLS algorithms and AVF-based recursions. (a) Complexity of SG Algorithms. (b) Complexity of RLS Algorithms and AVF recursions.

D. Automatic Rank Selection

The performance of the algorithms described in the previous subsections is indeed a sensitive function of the rank D. Unlike prior methods for rank selection which utilize MWF-based algorithms [36] or AVF-based recursions [44], we focus on a blind criterion based on the constant modulus criterion. In particular, we present a method for automatically selecting the rank of the algorithms based on the exponentially weighed *a posteriori* least-squares type constant modulus cost function described by

$$J_{\rm CM}\left(\tilde{\mathbf{w}}_k^D[i-1], D\right) = \sum_{l=1}^{i} \alpha^{i-l} \left(\left| \tilde{\mathbf{w}}_k^{D,H}[l-1]\tilde{\mathbf{r}}^D[l] \right|^2 - 1 \right)^2 \tag{29}$$

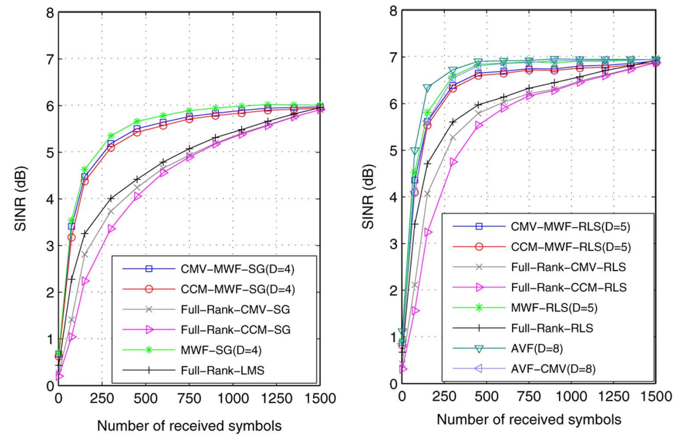
where α is the forgetting factor, $\tilde{\mathbf{w}}_k^{D,H}[i-1]$ is the reduced-rank filter for user k with rank D, and $\bar{\mathbf{r}}^D[i]$ is the reduced-rank received data with rank D. For each time interval i, we can select D which minimizes $J_{\rm CM}(\tilde{\mathbf{w}}_k^D[i-1], D)$ and the exponential weighting factor α is required as the optimal rank varies as a function of the data record. The proposed rank adaptation algorithm is given by

$$D_{\rm opt} = \arg \min_{D_{\min} \leq d \leq D_{\max}} J_{\rm CM}\left(\tilde{\mathbf{w}}_k^d[i-1], d\right) \tag{30}$$

where d is an integer, $D_{\min}$ and $D_{\max}$ are the minimum and maximum ranks allowed, respectively. Note that a smaller rank may provide faster adaptation during the initial stages of the estimation procedure and a slightly greater rank usually yields a better steady-state performance. Our studies reveal that the range for which the rank D of the proposed algorithms have a positive impact on the performance of the algorithms is very limited, being from 3 to 5 for SG algorithms and from 3 to 8 for RLS recursions. Furthermore, these values are rather insensitive to the system load (number of users), to the processing gain and work very well for all scenarios examined. In the next section, we will illustrate how the proposed rank adaptation algorithm performs.

VI. Simulation Experiments

In this section, we assess the bit error rate (BER) and the signal-to-interference-plus-noise ratio (SINR) performance of the receivers designed with the following adaptive parameter estimation criteria, i.e., the least-squares (LS), the constrained minimum variance (CMV) and the proposed CCM. We evaluate their corresponding full-rank and reduced-rank versions, as well as, adaptive implementations based on batch (that perform matrix inversions), stochastic gradient (SG), and RLS algorithms. In particular, we consider the MWF-based implementations of the CMV, LS, and RLS algorithms [36] and the AVF-based implementations of the CMV and LS methods [43], [44]. The DS-CDMA system employs randomly generated sequences of length $N = 64$. The channels experienced by the users are different since we focus on an uplink scenario and the channel coefficients $h_{k,l}[i] = p_{k,l}\alpha_{k,l}[i]$, where $\alpha_{k,l}[i] (l = 0, 1, \ldots, L_p - 1)$ are obtained with Clarke's model [55]. We show the results in terms of the normalized Doppler frequency f_dT (cycles/symbol) and use three-path channels with relative powers given by 0, -3, and -6 dB,



(a) (b)

Fig. 4. SINR convergence performance of SG and RLS recursions at $E_b/N_0 = 10$ dB when the algorithms converge to the same level of SINR. (a) $\mathrm{N} = 64, \mathrm{K} = 16$ users, $\mathrm{E_b/N_0} = 10$ dB; (b) $\mathrm{N} = 64, \mathrm{K} = 16$ users, $\mathrm{E_b/N_0} = 10$ dB.

where in each run the spacing between paths is obtained from a discrete uniform random variable between one and two chips. The channel estimator of [53] models the channel as an FIR filter and we employ a filter with eight taps as an upper bound for the experiments. The phase ambiguity derived from the blind channel estimation method in [53] is eliminated in our simulations by using the phase of $\hat{\mathbf{h}}_k(0)$ as a reference to remove the ambiguity and for fading channels we assume ideal phase tracking and express the results in terms of the normalized Doppler frequency f_dT (cycles/symbol). Alternatively, differential modulation can be used to account for the phase rotations.

The supervised estimation techniques are adjusted with the aid of a pilot sequence during the training phase, while the blind methods only rely on their knowledge of the signature sequences. For the sake of comparison, we also include the curves for supervised LMS and RLS [1] adaptive algorithms, which are trained with 200 symbols provided by a pilot channel (at $i = 1, \ldots, 200$ and $i = 1501, \ldots, 1701$) and then switch to decision-directed mode. It is assumed that the system has a power distribution amongst the users for each run that follows a log-normal distribution with associated standard deviation of 1.5 dB and all experiments are averaged over 200 runs. Note that given the performance of current power control algorithms, this power control scenario is close to a realistic situation. The step size μ_w of SG algorithms are optimized for each situation, whereas for RLS recursions we used $\alpha = 0.998$ because it leads to the best performance.

In the first experiments, depicted in Fig. 4, we assess the convergence performance of the adaptive algorithms when they converge to the same level of SINR. This allows us to effectively verify the speed of convergence of all analyzed methods. All parameters are adjusted to ensure the convergence to the same level of SINR and we employ multipath channel which has three taps with relative powers given by 0, -3, and -6 dB spaced by two chips. This is the only experiment conducted with fixed channels in order to facilitate the setting of parameters and the convergence to the same SINR value.

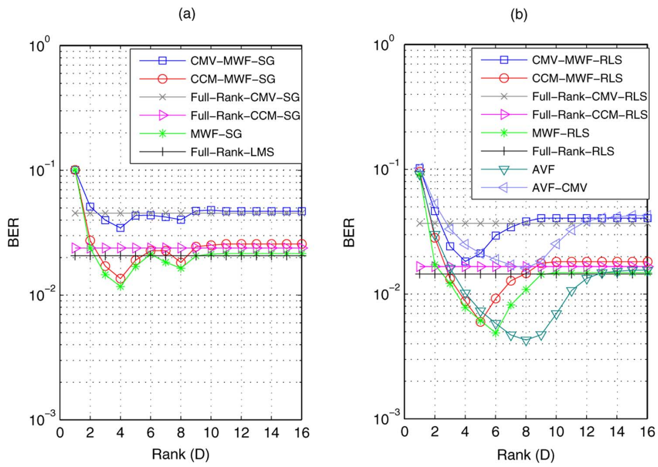


Fig. 5. BER performance versus Rank (D) for (a) SG and (b) RLS algorithms for a data record of 1500 symbols. (a) $\mathrm{N} = 64, \mathrm{f_d T} = 0.0001, \mathrm{E_b/N_0} = 12$ dB, $\mathrm{K} = 16$. (b) $\mathrm{N} = 64, \mathrm{f_d T} = 0.0005, \mathrm{E_b/N_0} = 15$ dB, $K = 24$.

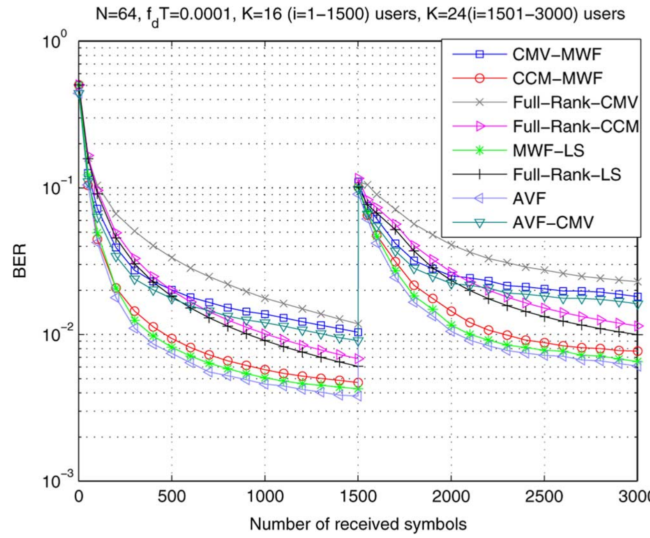


Fig. 6. BER convergence performance of LS algorithms at $E_b/N_0 = 12$ dB in a dynamic scenario.

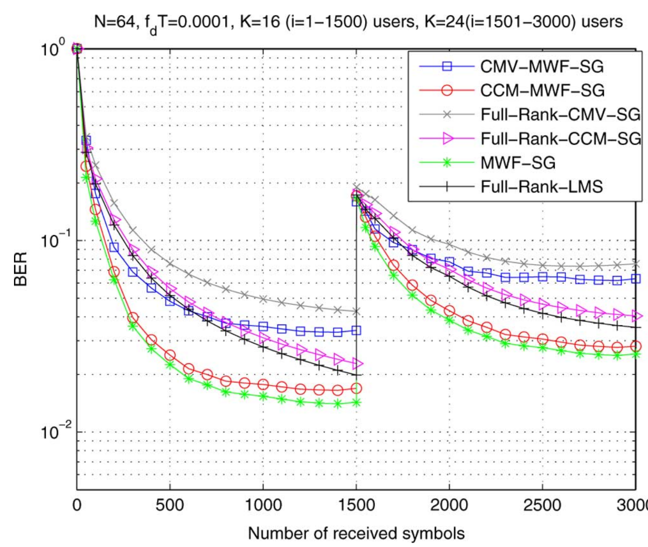


Fig. 7. BER convergence performance of SG algorithms at $E_b/N_0 = 15$ dB in a dynamic scenario.

The remaining plots employ the settings described in the previous paragraph. The results indicate that the MWF-based and AVF-based reduced-rank algorithms are substantially faster than full-rank techniques. Amongst SG and RLS recursions, it can be noticed that RLS techniques are faster than SG methods, as expected, and this is verified for both full-rank and reduced-rank schemes.

In the next experiments, we evaluate the BER performance of the proposed and analyzed reduced-rank algorithms versus their associated rank D. This experiment is intended for setting the adequate rank D of the reduced-rank schemes for the remaining assessments for a given BER and data record. The full-rank performance is also included for comparison purposes. The results shown in Fig. 5 indicate that the best performance of the proposed reduced-rank CCM scheme with SG and RLS estimation algorithms is obtained with ranks $D = 4$ and $D = 5$, respectively. For the AVF-based algorithms, the best rank was found to be $D = 8$. It is interesting to note that the best D is usually much smaller than the number of elements in the received data M, which leads to significant computational savings. These optimized parameters for D will be used for the remaining numerical results.

In what follows, we assess the average BER convergence performance of the analyzed and proposed algorithms. The BER convergence performance of the receivers is shown for batch and SG and RLS algorithms, in Figs. 6 and 7, respectively. It should be remarked that RLS techniques show a performance extremely close to the batch methods and differ basically due to the use of the matrix inversion lemma [1]. For this reason, we will only show the batch approach in Fig. 6 and in the remaining plots we will only show the SG and RLS techniques. We consider a non-stationary scenario, where the system starts with $K = 16$ users and at time $i = 1500$, eight additional users enter the system, totalling $K = 24$ users, and the blind adaptive algorithms are subject to new interferers/users in the environment.

The results show that the new reduced-rank algorithms based on the CCM design criterion can perform very close to the supervised AVF and MWF-based reduced-rank algorithms, while they do not require training data. The convergence performance of the proposed algorithms, i.e., CCM-MWF-SG and CCM-MWF, is significantly better than all existing full-rank schemes and allows a faster acquisition and tracking of the desired signals. In addition, we observe that the proposed CCM blind algorithms are significantly superior to the existing blind MWF and AVF techniques based on the CMV criterion, which are more susceptible to signature mismatches. The MWF version of the CMV approach suffers from lack of tridiagonalization of the covariance matrix, as pointed out in [36]. A comparison between the curves for SG and batch algorithms also reveals that a considerable performance degradation is verified for SG techniques, which despite being less complex have an inferior performance to batch (and RLS) techniques. This is because the performance of SG algorithms is subject to the eigenvalue spread of the covariance matrix of the received vector $\mathbf{r}[i]$.

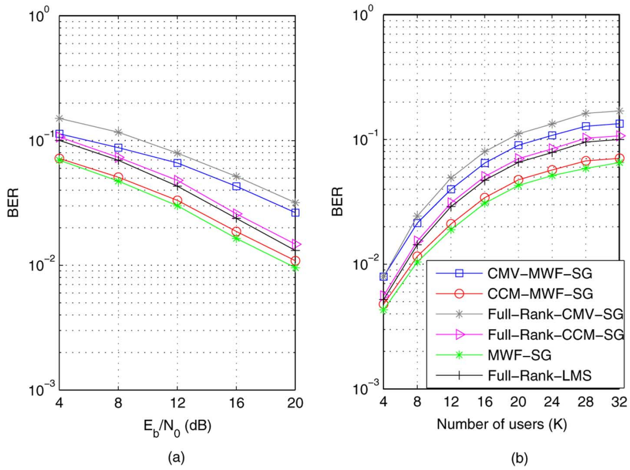


Fig. 8. BER performance of SG algorithms versus (a) E_b/N_0 and (b) number of users (K). (a) $\mathrm{N} = 64, \mathrm{K} = 16$ users, $\mathrm{f_d T} = 0.0001$. (b) $\mathrm{N} = 64, \mathrm{E_b/N_0} = 12$ dB, $\mathrm{f_d T} = 0.0001$.

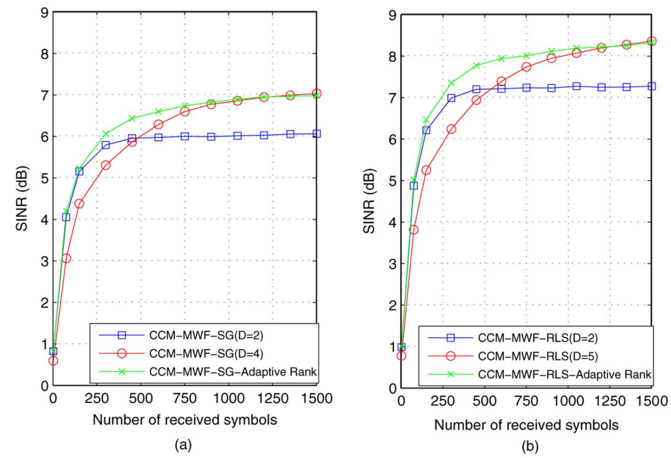


Fig. 10. SINR convergence performance of SG and RLS recursions at $E_b/N_0 = 10$ dB using the proposed automatic rank selection algorithm with $f_d T = 0.0001$. (a) $\mathrm{N} = 64, \mathrm{K} = 16, \mathrm{E_b}/N_0 = 10$ dB. (b) $\mathrm{N} = 64, \mathrm{K} = 16, \mathrm{E_b}/N_0 = 10$ dB.

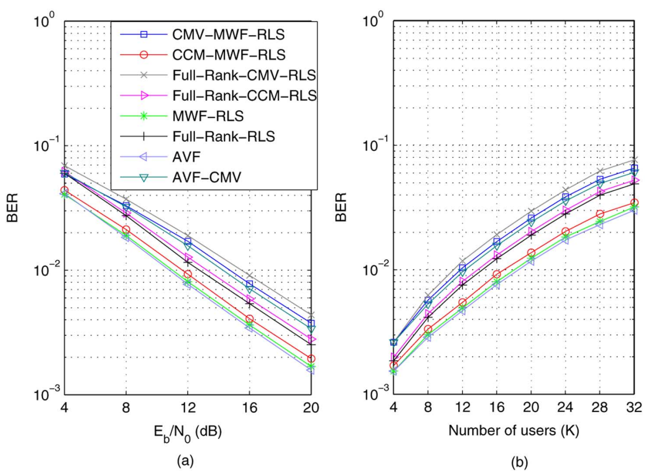


Fig. 9. BER performance of RLS algorithms versus (a) E_b/N_0 and (b) number of users (K). (a) $\mathrm{N} = 64, \mathrm{K} = 16$ users, $\mathrm{f_d T} = 0.0001$. (b) $\mathrm{N} = 64, \mathrm{E_b/N_0} = 12$ dB, $\mathrm{f_d T} = 0.0001$.

Specifically, when the eigenvalue spread of the covariance matrix of the received vector $\mathbf{r}[i]$ is large SG algorithms perform poorly, whereas the rate of convergence of batch or RLS algorithms is invariant to such situation in a stationary scenario [1]. The RLS methods were found to be less susceptible to this phenomenon than SG algorithms in the non-stationary scenario considered. Even though the impact of eigenvalue spread is much reduced in the proposed reduced-rank CCM-MWF-SG algorithm due to the dimensionality reduction (the eigenvalue spread has been verified to undergo a major reduction in most scenarios studied), for large systems or those that do not have good power control batch or RLS recursions are the most appropriate solutions.

In this part, the BER performance of the analyzed techniques is further investigated and the receivers process 1500 symbols to obtain the curves. In particular, the average BER performance of the receivers versus E_b/N_0 and number of users (K) is depicted in Figs. 8 and 9 for SG and RLS algorithms, respectively.

For the SG algorithms, the curves reveal that the proposed CCM-MWF-SG technique has a performance very close to the supervised MWF-SG method. The proposed CCM-MWF-SG algorithm can save up to 4 dB in E_b/N_0 as compared to the full-rank CCM-SG and LMS for the same BER performance, and up to 8 dB as compared to both the full-rank and MWF approaches designed with the CMV criterion. In terms of system capacity, the proposed CCM-MWF-SG algorithm can accommodate up to eight additional users as compared to the full-rank schemes designed with the MMSE and CCM criteria, and the gains are even more pronounced over CMV-based techniques. The results with RLS algorithms corroborate those obtained with SG recursions, even though the performance gap between the CMV-based algorithms, the full-rank RLS and CCM-RLS and the proposed CCM-MWF-RLS technique is slightly reduced. This is because the RLS algorithms are more powerful and even the full-rank RLS approaches were more capable to deal with the parameter estimation tasks than SG recursions. The AVF-based algorithms with non-orthogonal AVs are slightly better than the MWF-based ones, however, this comes at the expense of a higher complexity. When operating in blind mode with the CMV criterion, neither the AVF nor the MWF yields a performance close to their supervised counterparts. Conversely, the proposed CCM-MWF techniques are able to effectively approach the performance of supervised reduced-rank algorithms with a very good tradeoff between performance and complexity.

Since the performance of the reduced-rank algorithms was found in our studies to be a function of the rank D and other parameters such as step size and forgetting factor, the designer has to consider its impact on the performance of the system. In particular, we found that the rank is relatively dependent on the system size and load, however, our studies indicate that the rank does not vary significantly with system size and load. The results of Honig and Xiao [56] on large system analysis with asymptotic values conducted for the MWF support this. In order to illustrate how the problem of rank adaptation was solved we consider an experiment, shown in Fig. 10, with the automatic rank selection algorithm proposed in Section V-D. Specifically, we assume that the step size of SG-based recursions is optimized (for prior work on variable step size mechanisms, the reader is referred to [6]–[13]), the forgetting factor of LS and RLS algorithms is also adequately chosen and we focus on the proposed automatic rank selection algorithm.

In the plots depicted in Fig. 10, the reduced-rank algorithms utilize different values for their rank and also the proposed automatic rank selection mechanism. The results show that with a lower rank the reduced-rank algorithms usually converge faster, however, they achieve a lower steady-state SINR value. Conversely, with a higher rank the proposed algorithms converge relatively slower than with a lower rank but they are able to reach a higher SINR value at steady state. The proposed automatic rank selection algorithm allows the proposed reduced-rank adaptive estimators to circumvent the tradeoff between convergence and steady-state performance for a given rank, by adaptively selecting the best rank for a given data record which provides both fast convergence and excellent steady-state performance. We remark that the proposed rank adaptation mechanism should be used in a realistic environment to ensure that the best rank D is appropriately selected.

VII. Conclusion

This paper proposed a multistage decomposition for blind adaptive parameter estimation in the Krylov subspace with the CCM design criterion. Based on constrained optimization of the constant modulus cost function and utilizing the Lanczos algorithm along with Arnoldi-like iterations, we developed a multistage decomposition for blind parameter estimation, a family of computationally efficient blind adaptive reduced-rank SG and RLS type algorithms and an automatic rank adaptation technique. An analysis of the convergence properties of the method was carried out and convergence conditions for the reduced-rank adaptive algorithms were established. Simulation results considered the application of the proposed techniques to the suppression of multiaccess and intersymbol interference in DS-CDMA systems and have shown that the proposed blind algorithms achieve a performance equivalent to the best known supervised reduced-rank approaches without the need for training data.

Appendix I
Convergence Properties

Let us express the cost function in (7) as $J_{\rm CM} = (E[|z_k|^4] - 2E[|z_k|^2] + 1)$, drop the time index [i] for simplicity, assume a stationary scenario and that the $b_k, k = 1, \ldots, K$ are statistically independent i.i.d. complex random variables with zero mean and unit variance, b_k and $\mathbf{n}$ are statistically independent. Let us also define

$$\mathbf{x} = \sum_{k=1}^{K} A_k b_k \mathbf{p}_k, \quad \mathbf{p}_k = \mathbf{C}_k \mathbf{h}_k \tag{31}$$

$$\mathbf{R} = \mathbf{Q} + \mathbf{G} + \sigma^2 \mathbf{I}$$

$$\mathbf{G} = E[\mathbf{x}\mathbf{x}^H], \quad \mathbf{P} = E[\boldsymbol{\eta}\boldsymbol{\eta}^H]. \tag{32}$$

Consider user 1 as the desired one, let $\mathbf{w}_1 = \mathbf{w}$ and define $u_k = A_k^* \mathbf{p}_k^H \mathbf{w}, \mathbf{u} = \mathbf{A}^H \mathbf{P}^H \mathbf{w} = [u_1 \cdots u_K]^T$, where $\mathbf{P} = [\mathbf{p}_1 \cdots \mathbf{p}_K], \mathbf{A} = \mathrm{diag}(A_1 \cdots A_k)$ and $\mathbf{b} = [b_1 \cdots b_K]^T$. Using the constraint $\mathbf{w}^H \mathbf{p}_1 = \mathbf{w}^H \mathbf{C}_1 \mathbf{h}_1 = \nu$ and the relation between the filter, the channel and the signature $\mathbf{C}_1^H \mathbf{w} = \nu \hat{\mathbf{h}}_1$ [47], [49], [50] we have for the desired user the condition $u_1 = (A_1^* \mathbf{p}_1^H)\mathbf{w} = A_1^* \mathbf{h}_1^H \mathbf{C}_1^H \mathbf{w} = \nu A_1^* \mathbf{h}_1^H \hat{\mathbf{h}}_1$. In the absence of noise and neglecting ISI, the (user 1) cost function can be expressed as

$$\begin{aligned} J_{\rm CM}(\mathbf{w}) &= E[(\mathbf{u}^H \mathbf{b}\mathbf{b}^H \mathbf{u})^2] - 2E[(\mathbf{u}^H \mathbf{b}\mathbf{b}^H \mathbf{u})] + 1 \\ &= 8\left(\sum_{k=1}^{K} u_k u_k^*\right)^2 - 4\sum_{k=1}^{K} (u_k u_k^*)^2 - 4\sum_{k=1}^{K} u_k u_k^* + 1 \\ &= 8\left(D + \sum_{k=2}^{K} u_k u_k^*\right)^2 - 4D^2 \\ &\quad - 4\sum_{k=2}^{K} (u_k u_k^*)^2 - 4D \\ &\quad - 4\sum_{k=2}^{K} (u_k u_k^*) + 1 \end{aligned} \tag{33}$$

where $D = u_1 u_1^* = \nu^2 |A_1|^2 |\hat{\mathbf{h}}_k^H \mathbf{h}_k|^2$. To examine the convergence properties of the optimization problem in (3), we proceed as follows. Under the constraint $\mathbf{w}^H \mathbf{p} = \nu$, we have

$$\begin{aligned} \tilde{J}_{\rm CM}(\bar{\mathbf{u}}) = 8(D + \bar{\mathbf{u}}^H \bar{\mathbf{u}})^2 - 4\left(D^2 + \sum_{k=2}^{K} (u_k u_k^*)^2\right) \\ -4(D + \bar{\mathbf{u}}^H \bar{\mathbf{u}}) + 1 \end{aligned} \tag{34}$$

where $\bar{\mathbf{u}} = [u_2, \ldots, u_K]^T = \mathbf{B}\mathbf{w}, \mathbf{B} = \mathbf{A}'^H \mathbf{P}'^H, \mathbf{P}' = [\mathbf{p}_2 \ldots \mathbf{p}_K]$ and $\mathbf{A}' = \mathrm{diag}(A_2 \ldots A_K)$. To evaluate the convexity of $\tilde{J}_{\rm CM}(\cdot)$, we compute its Hessian $(\mathbf{H})$) using the rule $\mathbf{H} = (\partial/\partial \bar{\mathbf{u}}^H)(\partial(\tilde{J}_{\rm CM}(\bar{\mathbf{u}}))/\partial \bar{\mathbf{u}}$ that yields

$$\mathbf{H} = 16[(D - 1/4)\mathbf{I} + \bar{\mathbf{u}}^H \bar{\mathbf{u}} \mathbf{I} + \bar{\mathbf{u}}\bar{\mathbf{u}}^H - \mathrm{diag}(|u_2|^2 \cdots |u_K|^2)]. \tag{35}$$

Specifically, $\mathbf{H}$ is positive definite if $\mathbf{a}^H \mathbf{H} \mathbf{a} > 0$ for all nonzero $\mathbf{a} \in \boldsymbol{C}^{K-1 \times K-1}$ [52]. The second, third, and fourth terms of (12) yield the positive definite matrix

$$16\left(\bar{\mathbf{u}}\bar{\mathbf{u}}^H + \mathrm{diag}\left(\sum_{k=3}^{K} |u_k|^2 \times \sum_{k=2, k\neq 3}^{K} |u_k|^2 \cdots \sum_{k=2, k\neq K}^{K} |u_k|^2\right)\right) \tag{36}$$

while the first term provides the condition

$$\nu^2 |A_1|^2 \left|\hat{\mathbf{h}}_k^H \mathbf{h}_k\right|^2 \geq 1/4 \tag{37}$$

that ensures the convexity of $\tilde{J}_{\rm CM}(\,\cdot\,)$ in the noiseless case. Because $\bar{\mathbf{u}} = \mathbf{B}\mathbf{w}$ is a linear function of $\mathbf{w}$ then $\tilde{J}_{\rm CM}(\bar{\mathbf{u}})$ being a convex function of $\bar{\mathbf{u}}$ implies that $J_{\rm CM}(\mathbf{w}) = \tilde{J}_{\rm CM}(\mathbf{B}\mathbf{w})$ is a convex function of $\mathbf{w}$. The same reasoning applies for the reduced-rank CCM design.

Since the extrema of the cost function can be considered for small σ^2 a slight perturbation of the noise-free case [48], the cost function is also convex for small σ^2 when $\nu^2|A_1|^2|\hat{\mathbf{h}}_k^H\mathbf{h}_k|^2 \geq 1/4$. Interestingly, if we assume ideal channel estimation ($|\hat{\mathbf{h}}_k^H\mathbf{h}_k| = 1$) and $\nu = 1$, our result reduces to $|A_1|^2 \geq 1/4$, which is the same found in [57]. For larger values of σ^2, we remark that the term ν can be adjusted in order to make the cost function $J_{\rm CM}$ in (3) convex, as pointed out in [48].

Appendix II
CCM Filter Design

Here we show the optimization steps towards the filter design according to the CCM criterion. Let us consider the constraint vector $\mathbf{h}_k$ and transform the original constrained optimization problem given by (7) into an unconstrained optimization task by resorting to method of Lagrange multipliers which yield the following unconstrained cost function

$$J'_{\rm CM}(\mathbf{w}_k) = E\left[\left(\left|\mathbf{w}_k^H\mathbf{r}\right|^2 - 1\right)^2\right] + \Re\left[\lambda\left(\mathbf{w}_k^H\mathbf{p}_k - \nu\right)\right] \tag{38}$$

where $\boldsymbol{\lambda}$ is a complex Lagrange multiplier. By taking the gradient terms of $J'_{\rm CM}$ with respect to $\mathbf{w}_k^*$ and setting them to zero we have

$$\nabla J'_{\rm CM} = 2E\left[\left(\left|\mathbf{w}_k^H\mathbf{r}\right|^2 - 1\right)\mathbf{r}\mathbf{r}^H\mathbf{w}_k\right] + \mathbf{p}_k\lambda = \mathbf{0}. \tag{39}$$

Then, by rearranging the terms we obtain

$$E[|z_k|^2\mathbf{r}\mathbf{r}^H]\mathbf{w}_k = E[z_k^*\mathbf{r}] - p_k\lambda/2 \tag{40}$$

and then

$$\mathbf{w}_k = \mathbf{R}_k^{-1}[\mathbf{d}_k - \mathbf{p}_k\lambda/2] \tag{41}$$

where $z_k = \mathbf{w}_k^H\mathbf{r}, \mathbf{R}_k = E[|z_k|^2\mathbf{r}\mathbf{r}^H], \mathbf{d}_k = E[z_k^*\mathbf{r}]$. Using the constraint $\mathbf{w}_k^H\mathbf{p}_k = \nu$ and substituting (41) into it, we arrive at the expression for the Lagrange multiplier

$$\lambda = 2\left(\mathbf{p}_k^H\mathbf{R}_k^{-1}\mathbf{p}_k\right)^{-1}\left(\mathbf{p}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k - \nu\right). \tag{42}$$

By substituting λ into $\mathbf{w}_k = \mathbf{R}_k^{-1}[\mathbf{d}_k - \mathbf{p}_k\lambda]$ we obtain the expression for the CCM linear filter

$$\mathbf{w}_k = \mathbf{R}_k^{-1}\left[\mathbf{d}_k - \left(\mathbf{p}_k^H\mathbf{R}_k^{-1}\mathbf{p}_k\right)^{-1}\left(\mathbf{p}_k\mathbf{p}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k - \nu\mathbf{p}_k\right)\right]. \tag{43}$$

Appendix III
Channel and Parameter Estimation With $\mathbf{R}_k$

Here, we discuss the suitability of the matrix $\mathbf{R}_k$, that arises from the CCM method and its reduced-rank version, for use in channel estimation. From the analysis in Appendix I for the linear receiver, we have for an ideal and asymptotic case that $u_k = (A_1^*\mathbf{p}_k^H)\mathbf{w}_1 \approx 0$, for $k = 2, \ldots, K$. Then, $\mathbf{w}_1^H\mathbf{r} \approx A_1b_1\mathbf{w}_1^H\mathbf{p}_1 + \mathbf{w}_1^H\mathbf{n}$ and $|\mathbf{w}_1^H\mathbf{r}|^2 \approx A_1^2|\mathbf{w}_1^H\mathbf{p}_1|^2 + A_1b_1(\mathbf{w}_1\mathbf{p}_1)\mathbf{n}^H\mathbf{w}_1 + A_1b_1^*(\mathbf{p}_1^H\mathbf{w}_1)\mathbf{w}_1^H\mathbf{n} + \mathbf{w}_1^H\mathbf{n}\mathbf{n}^H\mathbf{w}_1$. Therefore, we have for the desired user (i.e., user 1)

$$\begin{aligned}
\mathbf{R}_1 &= E\left[\left|\mathbf{w}_1^H\mathbf{r}\right|^2\mathbf{r}\mathbf{r}^H\right]\\
&\cong A_1^2\left|\mathbf{w}_1^H\mathbf{p}_1\right|^2\mathbf{R} + A_1\mathbf{w}_1^H\mathbf{p}_1E\left[b_1\mathbf{n}^H\mathbf{w}_1\mathbf{r}\mathbf{r}^H\right]\\
&\quad + A_1\mathbf{p}_1^H\mathbf{w}_1E\left[b_1^*\mathbf{w}_1^H\mathbf{n}\mathbf{r}\mathbf{r}^H\right]\\
&\quad + E\left[\mathbf{w}_1^H\mathbf{n}\mathbf{n}^H\mathbf{w}_1\mathbf{n}\mathbf{n}^H\right] + \sigma^2\mathbf{Q}\mathbf{w}_1^H\mathbf{w}_1\\
&\cong A_1^2\left|\mathbf{w}_1^H\mathbf{p}_1\right|^2\mathbf{R} + A_1\mathbf{w}_1^H\mathbf{p}_1E\left[b_1\mathbf{n}^H\mathbf{w}_1\mathbf{r}\mathbf{r}^H\right]\\
&\quad + E\left[\left|\mathbf{w}_1^H\mathbf{n}\right|^2\mathbf{n}\mathbf{n}^H\right]\\
&\quad + A_1\mathbf{p}_1^H\mathbf{w}_1E\left[b_1^*\mathbf{w}_1^H\mathbf{n}\mathbf{r}\mathbf{r}^H\right]\\
&\quad + \sigma^2(\mathbf{R} - \sigma^2\mathbf{I})\mathbf{w}_1^H\mathbf{w}_1\\
&\cong \left(A_1^2\left|\mathbf{w}_1^H\mathbf{p}_1\right|^2 + \sigma^2\right)\mathbf{R}\\
&\quad + A_1^2\sigma^2\left(\mathbf{w}_1^H\mathbf{p}_1\right)\left(\mathbf{w}_1\mathbf{p}_1^H\right)\\
&\quad + A_1^2\sigma^2\left(\mathbf{p}_1^H\mathbf{w}_1\right)\left(\mathbf{p}_1\mathbf{w}_1^H\right)\\
&\quad + \sigma^4\left[\mathrm{diag}(|w_1|^2, \ldots, |w_N|^2)\right.\\
&\quad \left. + \mathbf{w}_1\mathbf{w}_1^H\right] - \sigma^4\mathbf{w}_1^H\mathbf{w}_1\mathbf{I}\\
&\cong A_1^4\left[\left(\frac{\left|\mathbf{w}_1^H\mathbf{p}_1\right|^2}{A_1^2} + \frac{\sigma^2}{A_1^2}\right)\mathbf{R}\right.\\
&\qquad + \frac{\sigma^2}{A_1^2}\left(\left(\mathbf{w}_1^H\mathbf{p}_1\right)\left(\mathbf{w}_1\mathbf{p}_1^H\right) + \sigma^2\left(\mathbf{p}_1^H\mathbf{w}_1\right)\left(\mathbf{p}_1\mathbf{w}_1^H\right)\right)\\
&\qquad \left. + \frac{\sigma^4}{A_1^4}\left(\left[\mathrm{diag}(|w_1|^2, \ldots, |w_N|^2) + \mathbf{w}_1\mathbf{w}_1^H\right] - \mathbf{w}_1^H\mathbf{w}_1\mathbf{I}\right)\right]\\
&\cong \alpha\mathbf{R} + \tilde{\mathbf{N}}
\end{aligned} \tag{44}$$

where $\mathbf{R} = \mathbf{Q} + \sigma^2\mathbf{I}, \mathbf{Q} = E[\mathbf{x}\mathbf{x}^H] = \sum_{k=1}^K |A_k|^2\mathbf{p}_k\mathbf{p}_k^H$. From (44), it can be seen that $\mathbf{R}_k$ can be approximated by $\mathbf{R}$ multiplied by a scalar factor α plus a noise-like term $\tilde{\mathbf{N}}$ that for sufficient E_b/N_0 has an insignificant contribution. In addition, when the symbol estimates $z_k = \mathbf{w}_k^H\mathbf{r}$ are reliable, that is the cost function in (7) is small ($J_{\rm CM} \ll 1$), then $|z_k|^2$ has small variations around unity for linear detectors, yielding the approximation

$$\begin{aligned}
E[|z_k|^2\mathbf{r}\mathbf{r}^H] &= E[\mathbf{r}\mathbf{r}^H] + E[(|z_k|^2 - 1)\mathbf{r}\mathbf{r}^H]\\
&\cong E[\mathbf{r}\mathbf{r}^H] = \mathbf{R}.
\end{aligned} \tag{45}$$

Therefore, we conclude that the channel and parameter estimation can be performed using $\mathbf{R}_k$ in lieu of $\mathbf{R}$, since the properties of the matrix $\mathbf{R}$ studied in [53] hold for $\mathbf{R}_k$.

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