Joint Iterative Receiver Design and Multi-Segmental Channel Estimation for OFDM systems over Rapidly Time-Varying Channels

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Abstract—Rapidly time-varying channels introduce a significantly detrimental effect on conventional OFDM systems, which results in inter-carrier interference (ICI) and degrades the bit error rate (BER) performance, and makes channel estimation more difficult. In this paper, we propose a simple iterative receiver (MF-PIC) with multi-segmental channel estimation (MSCE) to improve the channel estimation and data detection performances over such high mobility scenarios. A matched-filter (MF) with parallel interference cancellation is employed to combat the ICI, and the symbol estimates are fed back for iterative channel estimation (MSCE). Simulation results demonstrate that the proposed receiver design can achieve a better BER performance over a wide range of normalised Doppler frequencies.

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has been applied widely to digital communication systems such as DVB, WiMAX, and LTE, and other 4G standards. Its popularity is largely due to the simple equalizer design (single tap) and the effective elimination of inter-symbol interference (ISI) by means of cyclic-prefix (CP). However, high mobility is a major issue for such systems. This is because the rapidly time-varying channels will destroy the orthogonality between subcarriers, and the performance of receivers will be significantly degraded by inter-carrier interference (ICI). Many receiver design strategies for such high mobility scenarios have been proposed, some of which can be categorized as: 1)Preprocessing and reduced complexity equalizer (single tap or banded channel matrix approximation) [1]–[4].

In [1], [2], the authors exploit the banded structure of the channel matrix in the frequency domain by maximizing SINR or SIR inside the band, and design the equalizer in serial or block form. In order to design a single tap equalizer in the following stage, [3], [4] proposed two pre-equalizers to mitigate the effects of time variations. One has developed a partial FFT method to take advantage of relatively static channels in each interval for the pre-equalizer, and the other is to formulate the pre-equalizer by minimizing ICI power. 2) Interference cancellation with or without iterative processing [5]–[8]. The ICI is modelled using derivatives of the channel amplitude, and iterative decision feedback equalizer (DFE) is

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performed to obtain a single tap equalizer in frequency domain [5]. A similar idea is implemented in [6] to obtain the diagonal matrix using mean values of transmit symbols based on LLR values from the channel decoder. In [7], the authors propose a group of subcarriers based successive interference cancellation (SIC) scheme with iterative SIC-based single-burst channel estimation (SBCE) and channel decoding, which utilizes the banded structure of channels in the frequency domain. Another method is to approximate the symbol estimates using the sequential LSQR algorithm with selective parallel interference cancellation (PIC), which is based on the banded structure of the modified channel matrix in the time or the frequency domain [8]. Some other iterative processing techniques are also presented in [9], [10] employing a novel LLR criterion or cancellation order. Besides these cancellation techniques, a low-complexity MAP based detection for mobile OFDM can also be found in [11] with successively reduced search space, which can be treated as a variation of interference cancellation with a MAP criterion.

Motivated by [1], [3], [7], we propose a joint iterative receiver design with matched filter (MF)-PIC and multisegmental channel estimation (MSCE). We exploit the banded structure in the frequency domain, and a MF to initially estimate the transmit symbols. With the aid of the channel decoder, the transmit symbol estimates can be improved continuously using PIC. A number of small-matrix inversions are replaced by the conjugate transpose of the banded matrix, the process of which can also take advantage of significant ICI signals to estimate symbols. Additionally, PIC can be performed to suppress most ICI signals with lower processing delay and a better performance compared to [1], [7]. In [3], the partial Fast Fourier transform (PFFT) was introduced to design the reduced dimension equalizer, because of the relatively static channels in each interval. In our case, we split the receive signals into several segments and estimate the channel impulse response in each segment. Hence, more accurate channel estimates can be obtained with linear interpolation, and the proposed method outperforms the SIC-MAP of [12] within a wide range of normalized Doppler frequency scenarios.

This paper is organized as follows. Section II states that

the system model and block diagram of the joint receiver structure. Section III formulates the MF-PIC receiver and LLR calculation. The development of iterative MSCE is then discussed as well as symbol selection strategy for iterative MSCE in Section IV. Simulation results are presented in Section V, and Section VI draws the conclusions.

II. SYSTEM MODEL

We consider a coded OFDM system with N_s subcarriers as illustrated in Fig. 1. The information bits are encoded as b_m , and then interleaved as u_m , where the subscript m denotes the mth bit in a bit sequence of length M. Each group of c bits are modulated onto one symbol s_k on kth subcarrier at the ith OFDM symbol, and $N_{cp} \leq N_s$. After an IFFT the signals can be written as

$$a_n^i = \frac{1}{\sqrt{N}} \sum_{k=0}^{N_s - 1} s_k^i e^{j\frac{2\pi}{N_s}kn},$$
(1)

where the quantity a_n^i is transmitted over a time-varying multipath channel. The received signals during the *i*th OFDM symbol are represented as

$$r_n^i = \sum_{l=0}^{N_h - 1} h_{tl}^i(n, l) a_{n-l}^i + z_n^i, \quad 0 \le n < N_s$$
 (2)

where $N_{cp} \ge N_h$. The length of the CP is greater than the length of the channel impulse response, and the quantity z_n^i denotes samples of additive white Gaussian noise (AWGN) with variance σ_n^2 . The receiver performs the FFT of r_n^i :

$$y_d^i = \frac{1}{\sqrt{N}} \sum_{n=0}^{N_s - 1} r_n^i e^{-j\frac{2\pi}{N_s}dn}.$$
 (3)

Here, we rewrite Eq. (3) in a matrix form:

$$\mathbf{y}^i = \mathbf{H}^i_f \mathbf{s}^i + \mathbf{w}^i, \tag{4}$$

where the AWGN noise vector after the FFT is denoted by \mathbf{w} , and the quantity \mathbf{H}_{f}^{i} is the channel frequency response matrix. The vectors $\mathbf{s}^{i} = [s_{0}^{i}, \ldots, s_{N_{s}-1}^{i}]$, $\mathbf{y}^{i} = [y_{0}^{i}, \ldots, y_{N_{s}-1}^{i}]$ and $\mathbf{w}^{i} = [w_{0}^{i}, \ldots, w_{N_{s}-1}^{i}]$ denote the transmitted symbol vector, the received data vector and the noise vector respectively. In the following parts, the OFDM symbol index will be omitted unless otherwise specified.

III. MATCHED FILTER PARALLEL INTERFERENCE CANCELLATION

In this section, we present an MF-PIC approach to mitigate the ICI in the OFDM systems. The banded structure is employed to further reduce the complexity of MF-PIC in the matched filter part and the cancellation part. The LLR calculation of MF-PIC is also discussed as follows. According to the banded structure of the channel matrix H_f , the ICI signals are mostly contributed from $2N_d$ adjacent subcarriers [1]. Hence, the residual ICIs outside the band are considered as noise. The processed signals by MF are expressed as follows.

$$\widetilde{\mathbf{y}} = \mathbf{H}_D^H \mathbf{y}
= \mathbf{H}_D^H \mathbf{H}_f \mathbf{s} + \mathbf{H}_D \mathbf{w}
= \mathbf{R}_{Df} \mathbf{s} + \widetilde{\mathbf{w}},$$
(5)

where the vector $\tilde{\mathbf{y}} = [\tilde{y}_0, \dots, \tilde{y}_{N_s-1}]^T$ represents the MF outputs, and the banded channel matrix can be represented as

$$\mathbf{H}_{D} = \begin{bmatrix} H_{D^{(0,0)}} & H_{D^{(0,1)}} & \cdots & H_{D^{(0,N_{s}-1)}} \\ H_{D^{(1,0)}} & H_{D^{(1,1)}} & H_{D^{(1,2)}} & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ H_{D^{(N_{s}-1,0)}} & \cdots & H_{D^{(N_{s}-1,N_{s}-2)}} & H_{D^{(N_{s}-1,N_{s}-1)}} \end{bmatrix}$$
(6)

The remaining elements (\cdots) of the matrix in (6) are set to zeros due to the band assumption. We consider BPSK for simplicity, so the LLR values of u_k and the initial soft symbol estimates can be computed as

$$L(u_k|\tilde{y}_k) = \ln \frac{\Pr(\tilde{y}_k|u_k = +1)\Pr(u_k = +1)}{\Pr(\tilde{y}_k|u_k = -1)\Pr(u_k = -1)}$$

$$= \ln \frac{exp(-\frac{(\tilde{y}_k - u_k^+)^2}{2\sigma_n^2})\Pr(+1)}{exp(-\frac{(\tilde{y}_k - u_k^-)^2}{2\sigma_n^2})\Pr(-1)}$$

$$= \ln \frac{\frac{exp(\operatorname{Re}(\tilde{y}_k))}{\sigma_n^2}}{\frac{exp(-\operatorname{Re}(\tilde{y}_k))}{\sigma_n^2}}$$

$$= \frac{2\operatorname{Re}(\tilde{y}_k)}{\sigma_n^2},$$
(7)

According to the Bayes's theorem and (7), the soft symbol estimate is given by

$$\hat{s}_{k} = \sum_{i+1,-1} u_{k} \Pr(u_{k} | \tilde{y}_{k}) \\
= \Pr(u_{k} = +1 | \tilde{y}_{k}) - \Pr(u_{k} = -1 | \tilde{y}_{k}) \\
= \frac{\Pr(\tilde{y}_{k} | u_{k} = +1) - \Pr(\tilde{y}_{k} | u_{k} = -1)}{\Pr(\tilde{y}_{k} | u_{k} = +1) + \Pr(\tilde{y}_{k} | u_{k} = -1)} \\
= \frac{\Pr(\tilde{y}_{k} | u_{k} = +1) / \Pr(\tilde{y}_{k} | u_{k} = -1) - 1}{\Pr(\tilde{y}_{k} | u_{k} = +1) / \Pr(\tilde{y}_{k} | u_{k} = -1) + 1} \\
= \tanh(L(u_{k} | \tilde{y}_{k})/2),$$
(8)

Subsequently, $L(u_m)$ from (7) is deinterleaved into $L_A(b_m)$, and then feed to the channel decoder as the apriori LLRs. The extrinsic LLRs $L_E(b_m)$ can be obtained from the channel decoder, and then \hat{s}_k is fed back for soft interference cancellation after the interleaver as given in (8). The received data vector after cancellation is given by

$$\bar{\mathbf{y}} = \tilde{\mathbf{y}} - \tilde{\mathbf{R}}_{Df} \hat{\mathbf{s}},\tag{9}$$

where \mathbf{R}_{Df} is the matrix \mathbf{R}_{Df} with zero diagonal elements, and the new LLR value $L(u_k)$ can be re-computed by (7)



Fig. 1: Joint iterative receiver structure with MSCE

according to $\bar{\mathbf{y}}$. The ICIs, except those outside the band, can be largely removed by this soft interference cancellation . Compared to [7], the complexity of MF-PIC is almost the same as SIC-MAP in terms of multiplication and division operations $(2(2N_d)^2+2N_d \text{ v.s. } 2(2N_d+1)^2+1)$ per subcarrier per iteration. However the MF-PIC can outperform SIC-MAP in terms of BER at high Doppler frequency scenarios and can achieve a lower processing latency. Additionally, it has a lower complexity and achieves a similar performances as opposed to [1], [2], which is well discussed in [7].

IV. MULTI-SEGMENTAL CHANNEL ESTIMATION

In this section, we develop the MSCE in detail, and discuss the soft symbol selection for channel estimation as well as the complexity of MSCE. Furthermore, the MSCE can be terminated in an early iteration by comparing with previous estimates.

For MSCE, we split the received signal in (2) into T segments, each having N_t samples. The (3) can be rewritten in the following form:

$$y_{t}(d) = \frac{1}{\sqrt{N_{s}}} \sum_{n=(t-1)N_{t}}^{tN_{t}-1} r_{n}e^{-j\frac{2\pi}{N_{s}}dn}$$

$$= \frac{1}{N_{s}} \sum_{n=(t-1)N_{t}}^{tN_{t}-1} \sum_{l=0}^{N_{h}-1} h_{tl}(n,l)e^{-j\frac{2\pi}{N_{s}}kl}$$

$$\cdot \sum_{k=0}^{N_{s}-1} s_{k}e^{-j\frac{2\pi}{N_{s}}nk}e^{-j\frac{2\pi}{N_{s}}nd}$$

$$= \frac{1}{N_{s}} \sum_{k=0}^{N_{s}-1} s_{k} \sum_{n=(t-1)N_{t}}^{tN_{t}-1} h_{f}(n,k)e^{-j\frac{2\pi}{N_{s}}n(d-k)},$$
(10)

where the quantity $h_f(n, k)$ denotes the channel frequency response for the kth subcarrier at time index n. We assume that the channel remains constant during segment t, so (10)

in can be simplified as [3]

$$y_t(d) = \sum_{k=0}^{N_s - 1} s_k H(t, k) \frac{1}{N_s} \sum_{n=(t-1)N_t}^{tN_t - 1} e^{-j\frac{2\pi}{N_s}n(d-k)}$$

=
$$\sum_{k=0}^{N_s - 1} s_k h_f(t, k) \delta_t(d-k),$$
 (11)

where

$$\delta_t(d-k) = \frac{1}{N_t} e^{j2\pi(d-k)\frac{2t-1}{2N_t}} \operatorname{sinc}(\frac{\pi(d-k)}{N_t}).$$
(12)

By defining $\mathbf{y}_t = [y_t(0), \dots, y_t(N_s - 1)]^T$, $\mathbf{h}_f(t) = [h_f(t, 0), \dots, h_f(t, N_s - 1)]^T$,

$$\boldsymbol{\Delta}_{t} = \begin{bmatrix} \delta_{t}(0) & \dots & \delta_{t}(N_{s} - 1) \\ \vdots & \ddots & \vdots \\ \delta_{t}(1 - N_{s}) & \dots & \delta_{t}(0) \end{bmatrix}, \quad (13)$$

and using \mathbf{F}_{N_h} to denote the first N_h columns of the FFT matrix, then it is straightforward to show that

$$\mathbf{y}_{t} = \underbrace{\sqrt{N_{s}} \boldsymbol{\Delta}_{t} \mathcal{D}(\mathbf{s}) \mathbf{F}_{N_{h}}}_{\mathbf{A}} \mathbf{h}_{tl}(t), \tag{14}$$

where the notation \mathcal{D} represents a conversion from a vector to a diagonal matrix or vice versa, and $\mathbf{h}_f(t) = \sqrt{N_s} \mathbf{F}_{N_h} \mathbf{h}_{tl}(t)$. Using LS estimation in time domain, the channel estimates for the *t*th segment can be obtained by

$$\hat{\mathbf{h}}_{tl}(t) = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}_t.$$
 (15)

Note that there are N_p pilot symbols already known to the receiver, and the 0s can be set for the unreliable symbol estimates of \hat{s} in each iteration (soft symbol selection according to (8) by setting a threshold). In the following steps, we introduce the piece-wise linear model to approximate the channel impulse responses for the time-varying channels in $\frac{M}{c(N_s-N_p)}$ OFDM symbol periods [12]. Hence, the channel estimates of $\hat{\mathbf{h}}_{tl}^i(n) = [\hat{h}_{tl}^i(n,0),\ldots,\hat{h}_{tl}^i(n,N_h-1)]^T$ for different time indices and OFDM symbols can be obtained.

The desired channel frequency responses \mathbf{H}_D can be computed as [7]. The complexity of MSCE is not significantly increased, but more accurate channel estimates can be obtained in each iteration. The quantity Δ_t can be pre-computed. In addition, the differences of channel estimates between the p-1th and pth iteration can be measured, which can help us to terminate the MSCE at an earlier iteration. So the MSCE at most requires $\mathcal{O}(PN_tN_h^2)$ operations. The complexity of MSCE is quite moderate if the length of the channel N_h is not very large.

V. SIMULATION RESULTS

In this section, we discuss the proposed MF-PIC receiver, the MSCE channel estimation scheme and discuss the simulation results. We assume a practical scenario with the following settings: the carrier frequency $f_c = 650$ MHz, the subcarrier spacing $\Delta f = 976.5$ Hz and the OFDM symbol duration is $T = 1/\Delta f \approx 1$ ms. The number of subcarriers is $N_s = 128$, which is modulated by BPSK. Extension to other modulations is straightforward. In addition, a half-rate convolutional code with generator polynomial (7,5) is employed, and the code length is 1920 bits which suits rapidly time-varying channels. The multipath channels $(N_h = 8)$ are modelled by Rayleigh fading with unit power delay profile, and each of them has the same maximum normalized Doppler frequency ($f_d T_{\text{OFDM}}$). We also exploit the banded structure in the following simulation results, so the one side band width $N_d = \lceil f_d T_{\text{OFDM}} \rceil$. We employ $N_p = N_s/4$ and T = 2 or 4 for MSCE, the parameters of which are the same as SBCE except for the number of segments T. In Fig. 2, we present the curves of BER performance at $f_d T_{\text{OFDM}} = 0.65$ between MF-PIC and SIC-MAP with different numbers of iterations. The performance of MF-PIC and SIC-MAP will not be further enhanced beyond 3 iterations. We can observe that the MF-PIC can outperform SIC-MAP after 2 iterations in terms of BER performance, because the residual interference of the output of MF-PIC is less than that of SIC-MAP. In addition, PIC with perfect interference cancellation is employed in our simulation as a performance benchmark. However, the MF-PIC can still achieve the same performance as a perfect PIC after several iterations.

The curves of BER against $f_d T_{OFDM}$ for MF-PIC and SIC-MAP with SBCE and MSCE (T = 2, 4) are presented in Fig. 3 , which shows that the BER performance of MSCE can is still acceptable in higher Doppler frequency around $f_d T_{OFDM} =$ 0.6 - 0.7 corresponding to a speed of the transmitter relative to the receiver of 1000mph. Because the channels for the 1st and the 2nd segments can be estimated, the linear interpolation of which is more accurate than SBCE. In addition, only 3 iterations are required to acquire promising channel estimates using MSCE, as shown by the BER performance. We may also observe that the BER performances with MSCE is degraded with the increase of $f_d T_{OFDM}$. According to our observations, MSCE with T = 2 is sufficient for most practical



Fig. 2: MF-PIC v.s. SIC-MAP with perfect channel estimates

scenarios. We also plot the curve of T = 4 for comparison, but the improvement of BER is not very significant. The greater number of segments can more accurately approximates the rapidly time-varying channels at the cost of increased complexity, but the noise may also be enhanced, which has a negative effect on the channel estimates. Hence, the value of T should not be too large for the optimum tradeoff between complexity and channel estimate accuracy. For MF-PIC and SIC-MAP, the performance differences become small with channel estimates. The performance of the receivers can be slightly improved with more iterations, but the difference is not significant beyond 3-4 iterations. We therefore may say that the MSCE can also be applied to many other similar iterative schemes.

In order to compare the performance differences between perfect channel estimates, SBCE and MSCE, we plot the BER performance with different receivers employing SBCE and MSCE. From Fig. 4, the proposed method (T = 2) significantly outperform SIC-MAP with SBCE [7] after 2 iterations at high Doppler frequencies, and the gap between the proposed method and the perfect channel estimation is around 2.5dB at 10^{-4} . The SBCE cannot work in such scenarios. The curves of MF-PIC with different iterations will converge at high SNR values, because the channel estimation errors and the residual interference introduced by the banded structure.

VI. CONCLUSION

In this paper, we have proposed a novel joint iterative receiver (MF-PIC) employing iterative MSCE channel estimation techniques to mitigate ICI with the aid of the channel decoder. The complexity of the proposed receiver is lower



Fig. 3: BER performances of MF-PIC v.s. SIC-MAP with SBCE and MSCE (T = 2, 4) against $f_d T_{\text{OFDM}}$,



Fig. 4: BER performances bewtween SIC-MAP with SBCE and MF-PIC with MSCE $T=2,\,f_dT_{\rm OFDM}=0.65$

than most existing methods, and the same as SIC-MAP. Furthermore, the complexity of the proposed iterative MSCE channel estimation techniques is very moderate if the length of the multipath channel is not very large, but it has achieved satisfactory performance compared to iterative SIC-MAP with iterative SBCE at high Doppler frequency scenarios. Hence, the proposed joint receiver can work perfectly on rapidly moving vehicles in rural areas.

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