# Adaptive Minimum BER Reduced-Rank Interference Suppression for Very Large Multiuser MIMO Systems

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Abstract—In this paper, we propose a novel adaptive reducedrank strategy for very large multiuser multi-input multi-output (MIMO) systems. The proposed reduced-rank scheme is based on the concept of joint iterative optimization (JIO) of filters according to the minimization of the bit error rate (BER) cost function. The proposed optimization technique adjusts the weights of a projection matrix and a reduced-rank filter jointly. We develop stochastic gradient (SG) algorithms for their adaptive implementation and introduce a novel automatic rank selection method based on the BER criterion. Simulation results for multiuser MIMO systems show that the proposed adaptive algorithms significantly outperform existing schemes.

Index Terms- Multiuser MIMO systems, massive MIMO, reduced-rank methods, adaptive algorithms, BER cost function.

# I. INTRODUCTION

Wireless communication research has recently focused on multi-input multi-output (MIMO) systems in order to exploit the increased capacity offered by the use of multiple antennas, and improve the quality and reliability of wireless links [1]. In MIMO systems, two configurations can be employed, namely, diversity and spatial multiplexing, which exploit spatial diversity to combat fading and increase the data rates by transmitting independent data streams, respectively. In particular, spatial multiplexing can be used for multiuser MIMO systems to transmit multiple data streams that can be separated using signal processing techniques at the receiver. More recently, multiuser detection has been considered in conjunction with MIMO techniques, which is widely believed to play an important role in future communication systems [2], [3], [4]. A recent trend has been introduced with the concept of massive MIMO [5] and the investigation of algorithms for very large MIMO systems [6], [7], which present key technical challenges for designers. Central problems in very large multiuser MIMO systems are the tasks of detection and parameter estimation that are required for interference suppression and must deal with a large number of parameters.

In this context, reduced-rank signal processing is a very promising technique due to its ability to deal with a large number of parameters. It has received significant attention in the past several years, since it provides faster convergence speed, better tracking performance and an increased robustness against interference as compared to conventional schemes operating with a large number of parameters. A number of reduced-rank techniques have been developed to design the subspace projection matrix and the reduced-rank filter [8]-[15]. Among the first schemes are eigendecomposition-based (EIG) algorithms [8], [9]. The multistage Wiener filter (MWF) has been investigated in [10] and [11], whereas the auxiliary vector filtering (AVF) algorithm has been considered in [12]. EIG, MWF and AVF have faster convergence speed with a much smaller filter size, but their computational complexity is very high. A strategy based on the joint and iterative optimization (JIO) of a subspace projection matrix and a reduced-rank filter has been reported in [13], [14], whereas algorithms with switching mechanisms have been considered in [15] for DS-CDMA systems.

Most of the contributions to date are either based on the minimization of the mean square error (MSE) and/or the minimum variance criteria [8]-[15], which are not the most appropriate metric from a performance viewpoint in digital communications. Design approaches that can minimize the bit error rate (BER) have been reported in [16], [17], [18] and are termed adaptive minimum bit error rate (MBER) techniques. The work in [18] appears to be the first approach to combine a reduced-rank algorithm with the BER criterion. However, the scheme is a hybrid between an EIG or an MWF approach, and a BER scheme in which only the reduced-rank filter is adjusted in an MBER fashion.

In this paper, we propose adaptive reduced-rank techniques based on a novel JIO strategy that minimizes the BER cost function for very large multiuser MIMO systems. The proposed strategy adjusts the weights of both the rank-reduction matrix and the reduced-rank filter jointly in order to minimize the BER. We develop stochastic gradient (SG) algorithms for their adaptive implementation and present an automatic rank selection method with the BER as a metric. Simulation results for large multiuser MIMO systems show that the proposed algorithms significantly outperform existing schemes.

The paper is structured as follows. Section II briefly describes the multiuser MIMO system model. The derivation of the MBER reduced-rank algorithm is described in section III. The complexity analysis of the proposed algorithm and the automatic rank selection scheme are introduced in section IV. The simulation results are presented in section V. Finally, section VI draws the conclusions.

## **II. SYSTEM MODEL**

Let us consider the uplink of an uncoded synchronous multiuser MIMO system with K users and one base station (BS), where each user is equipped with a single antenna and the BS is with M uncorrelated receive antennas,  $K \leq M$ . We assume that the channel is a MIMO time-varying flat fading channel. The M-dimensional received vector is given by

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k \mathbf{h}_k(i) b_k(i) + \mathbf{n}(i), \qquad (1)$$

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where  $b_k(i) \in \{\pm 1\}$  is the *i*-th symbol for user k, and the amplitude of user k is  $A_k$ , k = 1, ..., K. The  $M \times 1$  vector  $\mathbf{h}_k(i)$  is the channel vector of user k, which is given by

$$\mathbf{h}_k(i) = [h_{k,1}(i)\dots h_{k,M}(i)]^T, \qquad (2)$$

whose elements  $h_{k,f}(i)$ , f = 1, ..., M, are independent and identically distributed complex Gaussian variables with zero mean and unit variance,  $\mathbf{n}(i) = [n_1(i) ... n_M(i)]^T$ is the complex Gaussian noise vector with zero mean and  $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$ , where  $\sigma^2$  is the noise variance,  $(.)^T$  and  $(.)^H$  denote transpose and Hermitian transpose, respectively.

## III. DESIGN OF MBER REDUCED-RANK SCHEMES

In this section, we detail the design of reduced-rank schemes which minimize the BER. In a reduced-rank algorithm, an  $M \times D$  projection matrix  $\mathbf{S}_D$  is applied to the received data to extract the most important information of the processed data by performing dimensionality reduction, where  $1 \le D \le M$ . A  $D \times 1$  projected received vector is obtained as follows

$$\bar{\mathbf{r}}(i) = \mathbf{S}_D^H \mathbf{r}(i),\tag{3}$$

where it is the input to a  $D \times 1$  filter  $\bar{\mathbf{w}}_k = [\bar{w}_1, \bar{w}_2, \dots, \bar{w}_D]^T$ . The filter output is given by

$$\bar{x}_k(i) = \bar{\mathbf{w}}_k^H \bar{\mathbf{r}}(i) = \bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{r}(i).$$
(4)

The estimated symbol of user k is given by

$$\hat{b}_k(i) = \operatorname{sign}\{\Re[\bar{\mathbf{w}}_k^H \bar{\mathbf{r}}(i)]\},\tag{5}$$

where the operator  $\Re[.]$  retains the real part of the argument and sign $\{.\}$  is the signum function. The probability of error for user k is given by

$$P_{e} = P(\tilde{x}_{k} < 0) = \int_{-\infty}^{0} f(\tilde{x}_{k}) d\tilde{x}_{k}$$
$$= Q\left(\frac{\operatorname{sign}\{b_{k}(i)\}\Re[\bar{x}_{k}(i)]}{\rho(\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k})^{\frac{1}{2}}}\right),$$
(6)

where  $\tilde{x}_k = \text{sign}\{b_k(i)\}\Re[\bar{x}_k(i)], f(\tilde{x}_k)$  is the single point kernel density estimate [16] which is given by

$$f(\tilde{x}_k) = \frac{1}{\rho \sqrt{2\pi \bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k}} \times \exp\left(\frac{-(\tilde{x}_k - \operatorname{sign}\{b_k(i)\}\Re[\bar{x}_k(i)])^2}{2\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k \rho^2}\right),$$
(7)

where  $\rho$  is the radius parameter of the kernel density estimate, Q(.) is the Gaussian error function. The parameters of  $\mathbf{S}_D$ and  $\bar{\mathbf{w}}_k$  are designed to minimize the probability of error. By taking the gradient of (6) with respect to  $\bar{\mathbf{w}}_k^*$  and after further mathematical manipulations we obtain

$$\frac{\partial P_{e}}{\partial \bar{\mathbf{w}}_{k}^{*}} = \frac{-\exp\left(\frac{-|\Re[\bar{x}_{k}(i)]|^{2}}{2\rho^{2}\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k}}\right)}{\sqrt{2\pi}} \times \frac{\partial\left(\frac{\operatorname{sign}\{b_{k}(i)\}\Re[\bar{x}_{k}(i)]}{\rho(\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k})^{\frac{1}{2}}}\right)}{\partial \bar{\mathbf{w}}_{k}^{*}} = \frac{-\exp\left(\frac{-|\Re[\bar{x}_{k}(i)]|^{2}}{2\rho^{2}\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k}}\right)\operatorname{sign}\{b_{k}(i)\}}{2\sqrt{2\pi}\rho} \times \left(\frac{\mathbf{S}_{D}^{H}\mathbf{r}}{(\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k})^{\frac{1}{2}}} - \frac{\Re[\bar{x}_{k}(i)]\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k}}{(\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k})^{\frac{3}{2}}}\right).$$
(8)

By taking the gradient of (6) with respect to  $S_D^*$  and following the same approach we have

$$\frac{\partial P_{e}}{\partial \mathbf{S}_{D}^{*}} = \frac{-\exp\left(\frac{-|\Re[\bar{x}_{k}(i)]|^{2}}{2\rho^{2}\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k}}\right)}{\sqrt{2\pi}} \times \frac{\partial\left(\frac{\operatorname{sign}\{b_{k}(i)\}\Re[\bar{x}_{k}(i)]}{\rho(\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k})^{\frac{1}{2}}}\right)}{\partial \mathbf{S}_{D}^{*}} = \frac{-\exp\left(\frac{-|\Re[\bar{x}_{k}(i)]|^{2}}{2\rho^{2}\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k}}\right)\operatorname{sign}\{b_{k}(i)\}}{2\sqrt{2\pi}\rho} \times \left(\frac{\mathbf{r}\bar{\mathbf{w}}_{k}^{H}}{(\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k})^{\frac{1}{2}}} - \frac{\mathbf{S}_{D}\bar{\mathbf{w}}_{k}\bar{\mathbf{w}}_{k}^{H}\Re[\bar{x}_{k}(i)]}{(\bar{\mathbf{w}}_{k}^{H}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\bar{\mathbf{w}}_{k})^{\frac{3}{2}}}\right).$$
(9)

# IV. PROPOSED MBER ADAPTIVE ALGORITHMS

In this section, we firstly describe the proposed scheme and MBER adaptive SG algorithms to adjust the weights of  $S_D(i)$  and  $\bar{w}(i)$  based on the minimization of the BER criterion. Then, a method for automatically selecting the rank of the algorithm using the BER criterion is presented.



Fig. 1. Structure of the proposed reduced-rank scheme

# A. Adaptive Estimation of Projection Matrix and Receiver

The proposed scheme is depicted in Fig. 1, the projection matrix  $\mathbf{S}_D(i)$  and the reduced-rank filter  $\mathbf{\bar{w}}_k(i)$  are jointly optimized according to the BER criterion. The algorithm has been devised to start its operation in the training (TR) mode, and then to switch to the decision-directed (DD) mode. The proposed SG algorithm is obtained by substituting the gradient terms (8) and (9) in the expressions  $\mathbf{\bar{w}}_k(i+1) = \mathbf{\bar{w}}_k(i) - \mu_w \frac{\partial P_e}{\partial \mathbf{\bar{w}}_k^*}$  and  $\mathbf{S}_D(i+1) = \mathbf{S}_D(i) - \mu_{S_D} \frac{\partial P_e}{\partial \mathbf{S}_D^*}$  [19] subject to the constraint of  $\mathbf{\bar{w}}_k^H(i)\mathbf{S}_D^H(i)\mathbf{S}_D(i)\mathbf{\bar{w}}_k(i) = 1$ . Based on [16], we can see that, with respect to the product  $\mathbf{S}_D\mathbf{\bar{w}}_k$ , there are only global minimum solutions, and all the solutions form a half hyperplane. In this work, we pick one of the MBER solutions

for  $\mathbf{S}_D \overline{\mathbf{w}}_k$ , which is with the unit length. At each time instant, the weights of the two quantities are updated in an alternating way by using the following equations

$$\bar{\mathbf{w}}_{k}(i+1) = \bar{\mathbf{w}}_{k}(i) + \mu_{w} \frac{\exp\left(\frac{-|\Re[\bar{x}_{k}(i)]|^{2}}{2\rho^{2}}\right) \operatorname{sign}\{b_{k}(i)\}}{2\sqrt{2\pi\rho}} \times \left(\mathbf{S}_{D}^{H}(i)\mathbf{r}(i) - \Re[\bar{x}_{k}(i)]\mathbf{S}_{D}^{H}(i)\mathbf{S}_{D}(i)\bar{\mathbf{w}}_{k}(i)\right)$$
(10)

$$\mathbf{S}_{D}(i+1) = \mathbf{S}_{D}(i) + \mu_{S_{D}} \frac{\exp\left(\frac{-|\Re[\bar{x}_{k}(i)]|^{2}}{2\rho^{2}}\right) \operatorname{sign}\{b_{k}(i)\}}{2\sqrt{2\pi}\rho} \times \left(\mathbf{r}(i)\bar{\mathbf{w}}_{k}^{H}(i) - \mathbf{S}_{D}(i)\bar{\mathbf{w}}_{k}(i)\bar{\mathbf{w}}_{k}^{H}(i)\Re[\bar{x}_{k}(i)]\right)$$
(11)

where  $\mu_w$  and  $\mu_{S_D}$  are the step-size values. Expressions (10) and (11) need initial values,  $\bar{\mathbf{w}}_k(0)$  and  $\mathbf{S}_D(0)$ , and we scale the reduced-rank filter by  $\bar{\mathbf{w}}_k = \frac{\bar{\mathbf{w}}_k}{\sqrt{\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k}}$  at each iteration. The scaling has an equivalent performance to using a constrained optimization with Lagrange multipliers although it is computationally simpler. The proposed adaptive JIO-MBER algorithm is summarized in table I.

	TABLE I PROPOSED ADAPTIVE JIO-MBER ALGORITHMS
1	Initialize $\bar{\mathbf{w}}_k(0)$ and $\mathbf{S}_D(0)$ .
<b>2</b>	Set step-size values $\mu_w$ and $\mu_{S_D}$
3	for each time instant $i = 0, 1, \cdots$ do
4	Compute $\bar{\mathbf{w}}_k(i+1)$ and $\mathbf{S}_D(i+1)$ using (10) and (11).
5	Scale the $\bar{\mathbf{w}}_k$ using $\bar{\mathbf{w}}_k = \frac{\bar{\mathbf{w}}_k}{\sqrt{1-1}}$ .
	$\sqrt{\mathbf{\bar{w}}_{k}^{H}}\mathbf{S}_{D}^{H}\mathbf{S}_{D}\mathbf{\bar{w}}_{k}$
6	Obtain $\bar{\mathbf{w}}_k(i+1)$ and $\mathbf{S}_D(i+1)$ for the next time instant.

The joint optimization of  $\bar{\mathbf{w}}_k$  and  $\mathbf{S}_D$  has been shown to converge to the global minimum when the MSE is employed as the cost function [14]. The proposed scheme promotes an iterative exchange of information between the transformation matrix and the reduced-rank filter, which leads to improved convergence and tracking performance. However, when the BER is used as the cost function, there are local minima associated with the optimization.

#### B. Computational Complexity of Algorithms

We describe the computational complexity of the proposed JIO-MBER adaptive algorithm in multiuser MIMO systems. In Table II, we compute the number of additions and multiplications to compare the complexity of the proposed JIO-MBER algorithm with the conventional adaptive reduced-rank algorithms, the adaptive least mean squares (LMS) full-rank algorithm based on the MSE criterion [19] and the SG full-rank algorithm based on the BER criterion [16]. Note that the MWF-MBER algorithm corresponds to the use of the procedure in [10] to construct  $S_D(i)$  and (10) to compute  $\bar{w}_k(i)$ . In particular, for a configuration with M = 32 and D = 6, the number of multiplications for the MWF-MBER and the proposed JIO-MBER algorithms are 7836 and 1225, respectively. The number of additions for them are 5517 and 933, respectively. Compared to the MWF-MBER algorithm,

the JIO-MBER algorithm reduces the computational complexity significantly.

TABLE II COMPUTATIONAL COMPLEXITY OF ALGORITHMS.

	Number of operations per symbol		
Algorithm	Multiplications	Additions	
Full-Rank-LMS	2M + 1	2M	
Full-Rank-MBER	4M + 1	4M - 1	
MWF-LMS [11]	$DM^2 - M^2$	$DM^2 - M^2$	
	+2DM + 4D + 1	+3D - 2	
<b>EIG</b> [9]	$O(M^3)$	$O(M^3)$	
<b>JIO-LMS</b> [13]	$3D\dot{M} + M$	$2D\dot{M} + M$	
	+3D + 6	+4D - 2	
MWF-MBER [18]	$(D+1)M^2$	$(D-1)M^2$	
	+(3D+1)M+3D	+(2D-1)M	
	+M+10	+2D + M + 1	
JIO-MBER	6MD + 5D	5MD + D	
	+M + 11	-M - 1	

## C. Automatic Rank Selection

The performance of reduced-rank algorithms depends on the rank D, which motivates automatic rank selection schemes to choose the best rank at each time instant [10], [14], [15]. Unlike prior methods for rank selection, we develop a rank adaptation algorithm based on the probability of error, which is given by

$$P_D(i) = Q\left(\frac{\operatorname{sign}\{b_k(i)\}\Re[\bar{x}_k^D(i)]}{\rho}\right)$$
(12)

where the receiver is subject to  $\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k = 1$ . For each time instant, we adapt a reduced-rank filter  $\tilde{\bar{\mathbf{w}}}_k(i)$  and a projection matrix  $\tilde{\mathbf{S}}_D(i)$  with the maximum allowed rank  $D_{\text{max}}$ , which can be expressed as

$$\tilde{\mathbf{w}}_k(i) = [\tilde{w}_1(i), \dots, \tilde{w}_{D_{\min}}(i), \dots, \tilde{w}_{D_{\max}}(i)]^T$$
(13)

$$\tilde{\mathbf{S}}_{D}(i) = \begin{bmatrix} \tilde{s}_{1,1}(i) & \dots & \tilde{s}_{1,D_{\min}}(i) & \dots & \tilde{s}_{1,D_{\max}}(i) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{s}_{M,1}(i) & \dots & \tilde{s}_{M,D_{\min}}(i) & \dots & \tilde{s}_{M,D_{\max}}(i) \end{bmatrix}$$
(14)

where  $D_{\min}$  and  $D_{\max}$  are the minimum and maximum ranks allowed for the reduced-rank filter, respectively. For each symbol, we test the value of rank D within the range, namely,  $D_{\min} \leq D \leq D_{\max}$ . For each tested rank, we substitute the filter  $\tilde{\mathbf{w}}_k(i) = [\tilde{w}_1(i), \dots, \tilde{w}_D(i)]^T$  and the matrix

$$\tilde{\mathbf{S}}_{D}^{\prime}(i) = \begin{bmatrix} \tilde{s}_{1,1}(i) & \dots & \tilde{s}_{1,D}(i) \\ \vdots & \vdots & \vdots \\ \tilde{s}_{M,1}(i) & \dots & \tilde{s}_{M,D}(i) \end{bmatrix}$$
(15)

into (12) to obtain the probability of error  $P_D(i)$ . The optimum rank can be selected as

$$D_{\text{opt}}(i) = \arg \min_{D \in \{D_{min}, \dots, D_{max}\}} P_D(i).$$
(16)

The proposed MBER automatic rank selection requires the operation with  $D_{\rm max}$  to calculate

$$\bar{x}_{k}^{D}(i) = \bar{\mathbf{w}}_{k}^{H} \left(\sum_{d=1}^{D_{\min}} \mathbf{s}_{d}^{H} \mathbf{r}(i) \mathbf{v}_{d} + \ldots + \mathbf{s}_{D_{\mathrm{opt}}}^{H} \mathbf{r}(i) \mathbf{v}_{D_{\mathrm{opt}}} + \sum_{d=D_{\mathrm{opt}}+1}^{D_{\max}} \mathbf{s}_{d}^{H} \mathbf{r}(i) \mathbf{v}_{d}\right),$$

$$(17)$$

where  $\mathbf{v}_d$  is a zero vector with a one in the *d*th position and  $\mathbf{s}_d = [\tilde{s}_{1,d}(i), \ldots, \tilde{s}_{M,d}(i)]^T$ . A simple search over the values of  $\bar{x}_k^D(i)$  and the selection of the terms corresponding to  $D_{\text{opt}}$  and  $P_{D_{\text{opt}}}(i)$  are performed.

# V. SIMULATIONS

In this section, we evaluate the performance of the proposed JIO-MBER reduced-rank algorithms and compare them with existing full-rank and reduced-rank algorithms. Monte-carlo simulations are conducted to verify the effectiveness of the JIO-MBER adaptive reduced-rank SG algorithms. The number of receive antennas at the BS is M = 32. The channel coefficient  $h_{k,f}(i)$  is computed according to the Jakes model [20]. We optimized the parameters of the JIO-MBER adaptive reduced-rank SG algorithms with step sizes model adaptive reduced-rank SG algorithms with step sizes  $\mu_w = 0.01$  and  $\mu_{S_D} = 0.025$ . The step sizes for LMS adaptive full rank, SG adaptive MBER full rank and the conventional adaptive reduced-rank techniques are 0.085, 0.05 and 0.035, respectively. The initial full rank and reduced-rank filters are all zero vectors. The initial projection matrix is given by  $\mathbf{S}_D(0) = [\mathbf{I}_D, \mathbf{0}_{D\times(M-D)}]^T$ . The algorithms process 250 symbols in TR and 1500 symbols in DD. We set  $\rho = 2\sigma$ .



Fig. 2. BER performance versus the number of received symbols for the JIO-MBER reduced-rank algorithms and the conventional schemes.  $(D_{min} = 3, D_{max} = 20, K = 7)$ 

Figs. 2 and 3 show the BER performance of the desired user versus the number of received symbols for the JIO-MBER adaptive SG algorithms and the conventional schemes. We set the rank D = 8 for the reduced-rank schemes, and the normalized Doppler frequency is  $f_d T_s = 1 \times 10^{-5}$ . We use 15 dB for the input signal to noise ratio (SNR). From Fig. 2 and 3, we can see that the proposed JIO-MBER SG algorithm



Fig. 3. BER performance versus the number of received symbols for the JIO-MBER reduced-rank algorithms and the conventional schemes. ( $D_{min} = 3$ ,  $D_{max} = 20$ , K = 17)

with the automatic rank selection mechanism achieves the best performance. Although the full-rank MBER SG algorithm has a better performance compared to the proposed JIO-MBER SG algorithm with D = 8 for a system with a low load, the proposed JIO-MBER SG algorithm with D = 8 outperforms the full-rank MBER SG algorithm for a highly-loaded system. We also can see that the JIO-MBER reduced-rank algorithms converge much faster than the conventional reduced-rank algorithms, and the MBER eigen-decomposition reduced-rank method with D = 8 does not work well in time-varying MIMO fading channels. For the group of JIO-MBER adaptive algorithms, the auto-rank selection algorithms outperform the fixed rank algorithms.



Fig. 4. BER performance versus SNR for the JIO-MBER reduced-rank algorithms and the conventional schemes. 1500 symbols are transmitted and 250 symbols in TR.  $(D_{min} = 3, D_{max} = 20, K = 10)$ 

Figs. 4 and 5 illustrate the BER performance of the desired user versus SNR and number of users K, where we set  $f_dT_s = 1 \times 10^{-5}$  and D = 8. We can see that the best performance is achieved by the proposed JIO-MBER algorithm



Fig. 5. BER performance versus number of users for the JIO-MBER reducedrank algorithms and the conventional schemes. 1500 symbols are transmitted and 250 symbols in TR. ( $D_{min} = 3$ ,  $D_{max} = 20$ , SNR = 15 dB)

with the automatic rank selection mechanism. The proposed JIO-MBER algorithm with D = 8 outperforms the MWF-MBER reduced-rank algorithm. For the low-SNR region and the high-load case, the proposed JIO-MBER algorithm with D = 8 outperforms the full-rank MBER SG algorithm. In particular, the JIO-MBER algorithm using the automatic rank selection mechanism can save up to over 5dB and support up to six more users in comparison with the full rank MBER SG algorithm, at the BER level of  $6 \times 10^{-3}$ .



Fig. 6. BER performance versus the (cycles/symbol) for the JIO-MBER reduced-rank algorithms and the conventional schemes. 1500 symbols are transmitted and 250 symbols in TR. ( $D_{min} = 3$ ,  $D_{max} = 20$ , SNR = 15 dB, K = 17)

We show the BER performance of the analyzed algorithms as the fading rate of the channels vary. In this experiment, we set the number of users K = 17 and SNR = 15dB. In Fig. 6, we can see that, as the fading rate increases the performance gets worse, and the proposed JIO-MBER algorithm with the automatic rank selection mechanism achieves the best performance, followed by the proposed JIO-MBER algorithm with D = 8, the full-rank MBER SG algorithm, the conventional MWF-MBER algorithm and the full-rank LMS algorithm. It shows the ability of the proposed JIO-MBER algorithms to deal with dynamic channels.

### VI. CONCLUSIONS

In this paper, we have proposed a novel adaptive MBER reduced-rank scheme based on joint iterative optimization of filters for multiuser MIMO systems. We have developed SGbased algorithms for the adaptive estimation of the reducedrank filter and the projection matrix, and proposed an automatic rank selection scheme using the BER as a criterion. The simulation results have shown that the proposed JIO-MBER adaptive reduced-rank algorithms significantly outperform the existing full-rank and reduced-rank algorithms at a low cost.

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