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The thermodynamic limits of magnetic recording

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Thermal stability of the recorded information is generally thought to set the limit of the maximum possible density in magnetic recording. It is shown that basic thermodynamics always cause the probability of success of the write process to be less than 100%. This leads to a thermally induced error rate, which eventually limits the maximum possible density beyond that given by the traditional thermal stability limit. While the thermally induced error rate is negligible for recording of simple single domain particles, it rapidly increases in the presence of a write assist, in particular if the write assist is accomplished by an increased recording temperature. For the ultimate recording system that combines thermally assisted writing with a recording scheme that uses one grain per bit, the upper bound for the maximum achievable density is 20 Tbit/inch^2 for a bit error rate target of 10^{-2}. © *2012 American Institute of Physics*. [doi:10.1063/1.3681297]

I. INTRODUCTION

The limit of magnetic recording is generally considered to be determined by the onset of the superparamagnetic effect.[1–4] Superparamagnetism appears in small magnetic particles when the thermal energy kT ($k =$ Boltzmann's constant, $T =$ absolute temperature in Kelvin) becomes comparable to the magnetic energy of the particles. The magnetization curve of an assembly of superparamagnetic grains shows no hysteresis and behaves like paramagnetic material with giant magnetic moments, which lends the effect its name. It is worth mentioning that the magnetic energy of the particles is associated with the Zeeman energy of the particle's magnetic moment μ in the applied field H: $E_m = -\mu\mu_0$H. The strength of the superparamagnetic effect is controlled by the competition between the field energy and the thermal energy: $\mu\mu_0 H/kT$.

For magnetic recording applications, an additional aspect becomes of central importance: hysteresis. Useful information storage can only occur if there are at least two stable magnetization states, say "up" or "down." Magnetic recording media consist of small particles with uniaxial anisotropy and therefore exhibit the two desired magnetization states. These states are sufficiently stable if the magnetic energy is much greater than the thermal energy. In contrast to classical superparamagnetism, the magnetic energy has to be associated with the energy barrier that the magnetization keeps in its current state, which is given by KV where K is the anisotropy energy density and V is the particle volume. The standard Arrhenius-Néel theory[1,2] shows that an energy barrier of KV about 40 kT is required to keep the magnetization stable for roughly 10 years at room temperature. For practical applications, the minimum required KV/kT has to be increased to at least 60, because some margin has to be allowed for external demagnetization fields and other factors.

The thermal stability requirement leads to what has been termed the "trilemma of magnetic recording".[3] Achieving a sufficient signal-to-noise ratio with increasing density obviously calls for smaller magnetic grains (and thus volumes V), which then demands a higher anisotropy K to maintain thermal stability. The higher K goes along with a higher write field requirement which is difficult to meet with standard head technologies, where the write field is limited by the available magnetic materials that can be used in a recording head. There exist two main approaches to postpone the trilemma. The first approach is to address the write-ability problem by creating a write assist, where the switching field during the recording process is reduced without sacrificing thermal stability at information storage. The two most important write assist schemes are thermally assisted recording (TAR)[5,6] and the use of exchange-coupled composite (ECC) media.[7–9] In thermally assisted recording, the recording temperature is increased until the anisotropy of the medium is so low that it can be written with the available fields. In the case of ECC media, a clever design of the medium stack induces incoherent magnetization reversal mechanisms during the write process, which accordingly lowers the switching fields.

The current recording scheme uses many magnetic grains to define a bit (currently 10–20). Therefore, the second approach to postpone the trilemma is to change the recording scheme so that it uses 1 grain per bit.[10,11] Such a scheme is called bit-patterned recording (BPR). Ultimately, the write assist schemes and the BPR schemes can be combined to yield the ultimate densities.[12] A practical example for such a combination was given in Ref. 13, where a density close to 1 Tb/inch^2 was demonstrated.

Previous estimates of the ultimate recording densities have concentrated on the aspect of thermal stability. In this paper, it is worked out that thermodynamics also affect the write process, leading to another limitation of the ultimately achievable areal density. This limitation is particularly important for all write assist mechanisms. The physics is

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studied using the example of a bit-patterned recording system. The paper outline is as follows: Sec. II introduces the fundamental mechanism of thermally induced write errors; Secs. III and IV discuss the effects of a non-thermal write assist and a thermal write assist, respectively. Section V discusses several scenarios for ultimately achievable areal densities. Section VI elaborates on the role of the applied field angle and the findings are discussed and summarized in Secs. VII and VIII.

II. THERMALLY INDUCED WRITE ERRORS

In the following, we will assume a bit-patterned recording system, that is, each grain is considered to be a single domain and represents one bit. The grain has uniaxial anisotropy and for the time being, the recording field is assumed to point along the easy axis as sketched in Fig. 1. The energy of the single domain particle E normalized to the thermal energy is given as,

$$\frac{E}{kT} = -\frac{\mu\mu_0 H}{kT}\cos\vartheta + \frac{KV}{kT}\sin^2\vartheta, \tag{1}$$

where μ is the magnetic moment (saturation magnetization M_s times volume) of the particle, H the applied field, K the uniaxial anisotropy energy density, V the particle volume, k Boltzmann's constant, T the temperature in Kelvin, $\mu_0 = 4\pi \times 10^{-7}$ Vs/Am, and ϑ the angle between magnetization and applied field.

At this point it is always assumed that the applied field switches the magnetization if it is greater than the anisotropy field H_A. In a thermodynamic sense, however, the probability of a perfect alignment of the magnetization with the applied field is always less than 100%, even if the applied field exceeds the anisotropy field $\mu_0 H_A = 2K/M_s$. The presence of uniaxial anisotropy favors magnetization orientations near $\vartheta = 0$ and 180° and the equilibrium magnetization m_e can be expressed by the following simple expression,[14]

$$m_e = \tanh\frac{\mu\mu_0 H}{kT}. \tag{2}$$

It was pointed out by West that Eq. (2) is valid if the anisotropy energy KV is much greater than the thermal energy kT.[15] In an ensemble of single domain particles, $m_e < 1$ means that there exists a fraction of grains in which the magnetization opposes the applied field. If this particle ensemble constitutes a bit patterned medium, each grain with an opposing magnetization potentially represents an error. Therefore, the equilibrium magnetization is directly related to the bit error rate BER in patterned media recording,

$$BER = \frac{1-m_e}{2} = \frac{1}{1+\exp\dfrac{2\mu\mu_0 H}{kT}}. \tag{3}$$

The factor of 1/2 arises because each grain with opposing magnetization introduces a magnetization change that is exactly twice its contribution. The result is shown in Fig. 2.

The assumption that the magnetization can attain its thermal equilibrium during the recording process is a best case scenario and therefore *puts an upper bound to all areal density estimates that follow*. To illuminate this a bit further, consider a counterexample where the magnetization is not in thermal equilibrium during the write process. If indeed the error rate is found to be lower than that given by Eq. (3), this must imply that the initial magnetization state of the grains must have been closer to the desired state, which would have been created by recording in thermal equilibrium. This can, of course, happen by the luck of the draw, but it is not physically reasonable to assume that there would be any bias of the initial state toward the targeted state. In a practical situation, we do not have any control over the initial magnetization state of the grains and the thermal equilibrium therefore represents the optimum bit error rate. Assuming thermal equilibrium considerably facilitates the treatment of the problem, because magnetization dynamics do not have to be considered further. It is re-iterated that the effects of magnetization dynamics can only increase the amount of thermally induced errors and does not invalidate our claim of an upper bound for the areal density.

The assumption that thermal equilibrium can be achieved during the recording process implies that the recording speed is sufficiently low to give the magnetization enough time to reach its thermal equilibrium. In principle, Eq. (3) is valid for all fields, but for technical applications, a record field that is smaller than the anisotropy field yields definitely unreasonably long response times, and it is demanded that the record field is at least $H = H_A$. For

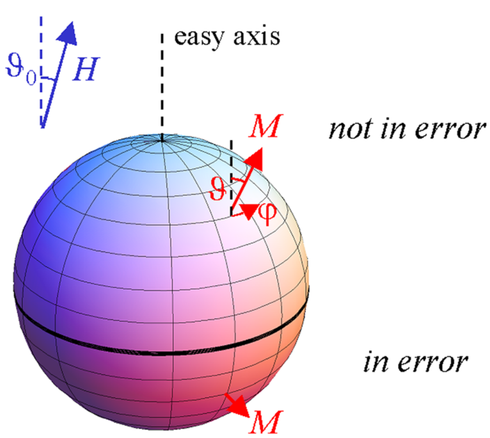

FIG. 1. (Color online) Angle definitions and illustration of the calculation of the equilibrium magnetization: An error occurs when the magnetization is in the lower hemisphere.

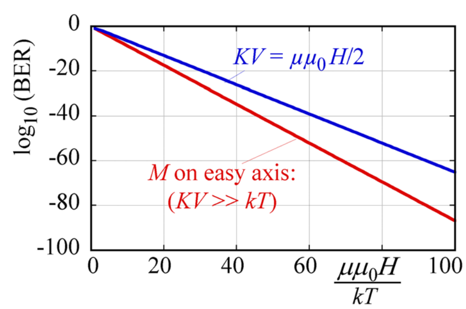

FIG. 2. (Color online) Estimated bit error rates as a function of $\mu\mu_0 H/kT$. The lower curve gives the result if the magnetization is confined to the easy axis (Eq. (3)), the upper curve holds for a Stoner-Wohlfarth particle (Eq. (4)). In both cases, the applied field is along the easy axis and equal to the anisotropy field.

$H = H_A$, the ratio between anisotropy energy and field energy $KV/\mu\mu_0 H$ is 1/2 and it does not appear legitimate to tacitly assume the approximation $KV/kT \gg 1$, especially for small field energies. A more accurate solution involves the calculation of a partition function,

$$BER = \frac{1}{2}\frac{\int_{\pi/2}^{\pi}\sin\vartheta\exp\left[-\frac{KV}{kT}\sin^2\vartheta + \frac{\mu\mu_0 H}{kT}\cos\vartheta\right]d\vartheta}{\int_0^{\pi}\sin\vartheta\exp\left[-\frac{KV}{kT}\sin^2\vartheta + \frac{\mu\mu_0 H}{kT}\cos\vartheta\right]d\vartheta}. \quad (4)$$

For the calculation it is assumed that the record field is applied sufficiently long to attain thermal equilibrium, but then the temperature is instantaneously removed. After the temperature removal, all magnetization vectors will snap back to the easy axis on the shortest trajectory, whereby those that come to lie in the lower hemisphere represent the grains in error (see Fig. 1).

Figure 2 shows the error rate as a function of $\mu\mu_0$H/kT. Traditional thermal stability requirements mandate that the grains have energy barriers of at least $\xi = KV/kT = 60$, which translates to $\mu\mu_0$H/kT = 120, due to the condition that the applied field is at least $H = H_A$. Figure 2 shows that the thermally induced error rates are extremely small and can be safely neglected under normal circumstances. As will be seen in the next section, this is not necessarily true if write assists are involved. Figure 2 also shows that the error rate obtained using the approximation that the magnetization is confined to the easy axis $KV \gg kT$ (Eq. (3)) works remarkably well.

III. NON-THERMAL WRITE ASSIST

Recording media with very small grains must have a very high magnetic anisotropy to provide adequate thermal stability for information storage. These high anisotropies, e.g., $L1_0$ FePt with an anisotropy field H_A well above 10 T,[16] cannot be written with conventional write heads made of FeCo alloys, which have a maximum saturation polarization of 2.35 T. To write these media, some form of a write assist is needed, and the required write assist factor W_A is,

$$W_A = \frac{H_A}{H}, \quad (5)$$

where H is the available write field. The goal is, of course, to achieve write assist factors greater than one. It is noted that an adequate write assist can be achieved in different ways, and here we will distinguish a thermal write assist from a non-thermal write assist. The non-thermal write assist is the focus of this section.

The essential effect of a write assist can be studied with the help of Fig. 2. Let us start from a recording system with no write assist that has an areal density AD_{ref} with grains of volume V_{ref} and an available write field H. Suppose now that the areal density is increased by a factor α, $AD = \alpha \cdot AD_{ref}$. Consequently, the grain volume has to be reduced to V_{ref}/α. Maintaining thermal stability calls for an increased anisotropy, which means that $H_A = \alpha \cdot H_{Aref}$. It is reasonable to presuppose that the write field remains constant during the scaling and consequently the write assist needed to make the new recording system work would have to be $W_A = \alpha$. To simplify matters, we have assumed that the thickness of the medium and its saturation magnetization M_s remain constant during scaling. These assumptions will be dropped in the forthcoming parts of the paper.

This means that the field energy of the scaled recording system is reduced by precisely the write assist factor W_A:

$$\frac{\mu\mu_0 H}{kT} = \frac{1}{W_A}\frac{(\mu\mu_0 H)_{ref}}{kT}. \quad (6)$$

From Fig. 2, it is seen that a write assist deteriorates the thermally induced error rates; and it is evident that sufficiently strong write assists will limit recording performance at some point.

The fact that the field energy during the recording process is reduced by the write assist factor is independent of the detailed nature of the write assist mechanism. However, it is to be expected that the details of the magnetization reversal process should have an effect on the thermally induced error rate.

A simple way to gain more insight into thermally induced error rates in write assist systems is to study two macro-spins that are exchange coupled to one another. The behavior of the two macro-spins is compared to that of a reference macro-spin. As illustrated in the inset of Fig. 3, the reference macro-spin has an anisotropy energy of $(KV)_{ref}$ and its field energy is $(\mu\mu_0 H)_{ref}$. Each of the spins has half the moment of the reference spin, $\mu_{ref}/2$, and one of them has an anisotropy energy $(KV)_{ref}$ while the other one has zero anisotropy. This implies that the material of the macro-spin with non-vanishing anisotropy has $2K_1 = K_{ref}$. We further assume that the saturation magnetizations of all spins under consideration are identical. Therefore, the reference macro-spin and the dual-spin particle present the same magnetic moment and the same anisotropy energy to the outside world. The energy of the dual-spin particle is given as,

$$\frac{E}{kT} = -\frac{(\mu\mu_0 H)_{ref}}{2kT}\cos\vartheta_1 - \frac{(\mu\mu_0 H)_{ref}}{2kT}\cos\vartheta_2 + \frac{(KV)_{ref}}{kT}\sin^2\vartheta_1 - \frac{x}{kT}[\sin\vartheta_1\sin\vartheta_2\cos(\varphi_1 - \varphi_2) - \cos\vartheta_1\cos\vartheta_2] \quad (7)$$

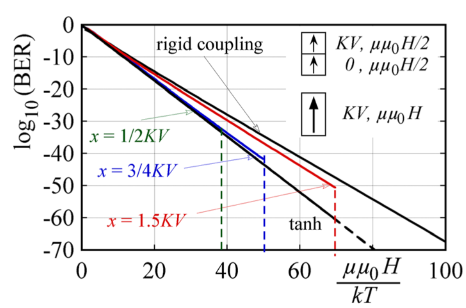

FIG. 3. (Color online) Thermally induced error rates in a dual-spin system with write assist. The inset shows that the total field energy, $\mu\mu_0 H$, and anisotropy energy, KV, of the dual spins is identical to a reference system. The exchange coupling between the dual spins is x, and write assist occurs if the coupling is less than rigid. The dashed lines indicate the reduction of the field energy after the write assist has been used to lower the particle volume.

where the exchange coupling energy between the spins is given as x. In Eq. (7), the ϑ_i are the polar angles and the φ_i are azimuth angles of the magnetization.

The write assist effect of the dual-spin particle depends on the exchange coupling strength x, which is given in units of the anisotropy energy $(KV)_{ref}$. The write assist factor is readily determined by finding the switching field of the two exchange coupled spins (see, e.g. Ref. 17) and is not repeated here. The derivation of the thermally induced error rate is analogous to that leading to Eq. (4),

$$BER = \frac{1}{2}\frac{\int_{\vartheta_1=\pi/2}^{2\pi}\int_{\vartheta_2=0}^{\pi}\int_{\varphi_1=0}^{2\pi}\int_{\varphi_2=0}^{2\pi}\exp\left[-\frac{E}{kT}(\vartheta_1,\vartheta_2,\varphi_1,\varphi_2)\right]\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}\varphi_1\mathrm{d}\varphi_2}{\int_{\vartheta_1=0}^{2\pi}\int_{\vartheta_2=0}^{\pi}\int_{\varphi_1=0}^{2\pi}\int_{\varphi_2=0}^{2\pi}\exp\left[-\frac{E}{kT}(\vartheta_1,\vartheta_2,\varphi_1,\varphi_2)\right]\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}\varphi_1\mathrm{d}\varphi_2}. \quad (8)$$

The result is shown in Fig. 3, where the starting point for the reference particle is assumed to be $KV = 50\ kT$, which leads to $\mu\mu_0 H = 100\ kT$ if the magnetization reverses coherently and $H = H_A$. For rigid coupling, $x \to \infty$, one expects that the dual-spin particle behaves exactly the same as the reference spin (compare Figs. 2 and 3). For weaker coupling, a write assist effect exists as is well known from the theory of ECC media. For $x = 1/2\ KV$, $3/4\ KV$, and $3/2\ KV$, the switching field is reduced by 2.618, 2, and 1.434, respectively. These reduction factors are the respective write assist factors W_A. Consider now a recording system that uses these dual-spin particles as opposed to standard particles, which switch magnetization coherently. Because the write assist lowers the switching field, the anisotropy field of the dual-spin particle can be increased by exactly the write assist factor W_A while maintaining write-ability ($H = const$). Consequently, the magnetic energy of the grains, KV, is increased by W_A and then the grains possess more thermal stability margin than needed. The additional thermal stability margin allows us to reduce the grain volume, which translates directly to an areal density gain. It is important to realize that the reduction of the grain volume implies that the magnetic moment μ is also reduced, and therefore the term $\mu\mu_0 H/kT$ for the dual-spin grains as it appears on the x-axis of Fig. 3 is reduced by the write assist factor W_A. As an example, the initial value $\mu\mu_0 H = 100\ kT$ becomes $50\ kT$ for $x = 3/4KV$. The vertical dashed lines in Fig. 3 indicate the corresponding field energies after the reduction of the grain volumes that restore the original KV's, thus highlighting that any form of write assist reduces the field energy by the write assist factor as shown in Eq. (6).

Figure 3 indicates that the write induced bit error rates in the presence of write assist fall in between the two curves given by Eqs. (3) and (4). The write assist increases the probability that the magnetization of macro-spin 1 is near the easy axis which drives the solution closer to the case described in Eqs. 2 and 3. It is believed that all other non-thermal write assist cases will also fall in the area between the two curves and that Eq. (3) is the limiting best case, that is, it gives the best possible error rate for a given field energy $\mu\mu_0 H/kT$.

IV. THERMALLY ASSISTED RECORDING

ECC media represent an effective way to achieve a write assist in the sense of Sec, III. In principle, materials with arbitrarily high anisotropies can be switched with the ECC approach, but practical limitations, such as the inability to accommodate very thick recording media, limit the potential of ECC media to write assist factors of 3–5.[3,18,19] It appears that the only way to advance into the regime of write assists greater than 5 is by thermally assisted recording.

The following discussion concentrates on $L1_0$ FePt as hard magnetic material. FePt is the material of choice, because it has a high anisotropy and does not suffer from severe corrosion problems as rare earth compounds. Fully ordered $L1_0$ FePt has a saturation magnetization $M_{s0} = M_s(T = 0\ \mathrm{K}) = 1150$ kA/m and an anisotropy field $\mu_0 H_A(T = 0\ \mathrm{K}) = \mu_0 H_{A0} = 14$ T, which translates to $K_0 = K\ (T = 0\ \mathrm{K}) = 8.05\ \mathrm{MJ/m^3}$.[16]

The fundamental idea of thermally assisted recording is to make the medium writeable by increasing the recording temperature[6] until the head field is strong enough to switch the magnetization. For FePt, the temperature dependence of the anisotropy is known from theory and experiment,[20–22]

$$\frac{K(T)}{K_0} = \left(\frac{M_s(T)}{M_{s0}}\right)^n, \quad (9)$$

where the exponent n was reported to be about 2.1. For the remainder of this paper, we will use $n = 2$. FePt has an effective spin of 3/2 and the saturation magnetization M_s can be calculated using the Brillouin function.[23] For mathematical convenience, we approximate $M_s(T)$ by the following expression,

$$\frac{M_s(T)}{M_{s0}} = \frac{H_A(T)}{H_{A0}} = \sqrt{1 - \left(\frac{T}{T_c}\right)^{2.5}}, \quad (10)$$

where T_c is the Curie temperature, 750 K for FePt. Equation (10) agrees almost perfectly with the Brillouin function, in particular near the Curie point where it matters most for our purposes. As a consequence of Eq. (9), the temperature dependence of the anisotropy field $H_A(T)$ is identical to that of the magnetization. It is noted that this is true for FePt and will generally differ in other material systems.

If we assume a write field of $\mu_0 H = 2$ T and fully ordered FePt with an anisotropy field of 14 T, the required write assist $W_{A,T}$ is 7. (It is noted that the thermal write assist factor has to be defined as $W_{A,T} = H/H_{A0}$). Using Eq. (10) with $H = H_A(T = T_R)$, the required recording temperature T_R

is 0.9918 T_c. As already worked out in Sec. III, after successful harvesting of the increased areal density, the write assist reduces the magnetic field energy $\mu\mu_0 H$ by the write assist factor due to the reduced volume of the grain. For thermal assist, however, there is an additional reduction because the magnetization M_s is also reduced. For the FePt system, this additional reduction is once again equal to the write assist factor, as seen from Eq. (10). Finally, increasing the temperature from T_S (storage temperature) to the recording temperature T_R augments the thermal energy and once again reduces the ratio $\mu\mu_0 H/kT$ by T_S/T_R,

$$\left.\frac{\mu\mu_0 H}{kT}\right|_{T_R} = \frac{1}{W_{A,T}^2}\frac{T_S}{T_R}\frac{(\mu\mu_0 H)_{ref}}{kT_S}. \tag{11}$$

Taking again $KV/kT = 60$ at storage temperature (say 300 K or $0.4T_c$), the ratio of field energy to thermal energy of the reference system $(\mu\mu_0 H)_{ref}/kT_S$ amounts to 120 in the absence of a write assist. With thermal write assist, however, $\mu\mu_0 H/KT$ melts down to less than 1. From Fig. 2 or Eq. (3) it is readily seen that the error rate is of the order of 0.1—the seemingly insignificant effect of thermally induced write errors becomes a major factor for thermally assisted recording!

V. AREAL DENSITY ESTIMATIONS

The previous results enable us to make estimations for the maximum possible areal densities for a bit patterned recording system. Equation (3) links the bit error rate to the magnetic properties of the medium, that is, the magnetic moment μ and the anisotropy field H_A, which is set to be identical to the recording field H. In the present context, the error rate is regarded as a design target, BER_0, and is therefore an input parameter,

$$\frac{\mu(T_S)\mu_0 H}{kT_S} = \frac{1}{2}\ln\left(\frac{1}{BER_0} - 1\right). \tag{12}$$

For simplicity, it is assumed that the recording temperature is identical to the storage temperature T_S. Although Eq. (12) is based on the assumption that the magnetization is confined to point along the easy axis, it can generally be used if the target error rate is modified accordingly. Figure 2 shows that it is straightforward to find a correction factor for the error rates at any given $\mu\mu_0 H/kT$.

The maximum areal density can be derived from Eq. (12) by substituting the magnetic moment μ with $M_s A_{Gr}\delta$, where δ is the thickness (height) of the grain and A_{Gr} is the area of the grain. Solving for $1/A_{Gr}$ yields the areal density,

$$AD_{Gr} = \frac{2\delta M_s \mu_0 H}{kT_S \ln\left(\frac{1}{BER_0} - 1\right)}. \tag{13}$$

The areal density given in Eq. (13) is the maximum possible density for the case in which all grains are densely packed without any gaps. In any practical bit patterned recording system, some space needs to be allocated to isolate the bits and the achievable areal density is reduced by the ratio of the grain area A_{Gr} divided by the bit area A_{Bit}. It is reasonable to expect that the ratio A_{Gr}/A_{Bit} will be between 25% and 50% for realistic systems.[24] Another important refinement to Eq. (13) concerns the medium thickness δ, which is limited to a maximum value of $\delta_{max} = 4\sqrt{A/K}$, where A is the exchange constant of the material.[3] If the medium thickness is greater than that, the reversal mechanism in zero field is no longer coherent and the volume increase of thicker media does not translate into higher energy barriers. In other words, there is no advantage in thicker media as far as thermal stability is concerned. With these modifications, Eq. (13) becomes,

$$AD = \frac{16\sqrt{AK}}{kT_S \ln\left(\frac{1}{BER_0} - 1\right)}\frac{B}{B_A}\frac{A_{Gr}}{A_{bit}} = \frac{16\sqrt{AK}}{kT_S \ln\left(\frac{1}{BER_0} - 1\right)}\frac{1}{W_A}\frac{A_{Gr}}{A_{bit}}. \tag{14}$$

Evidently the (non-thermal) write assist factor W_A appears in Eq. (14), indicating that the equation also holds in the presence of a write assist. To first order, the reduction of the field energy due to the write assist translates directly into the achievable areal density. It should be noted that the details of the reversal mechanism alter the result somewhat, as shown in Fig. 3. Formally, the effect of the reversal mechanism can be absorbed in an adjustment of the target error rate as already discussed. It is also interesting to find that the areal density scales with $\sqrt{AK}$, i.e., the same way as the domain wall energy density.

Equation (14) can be extended to the case of thermally assisted recording by taking into account that the writing temperature is T_R rather than T_S, which affects the material parameters A, K, and H_A. Classical theory suggests that the exchange constant A is proportional to the effective spin S^2 (Ref. 25) and, therefore, A will have the same temperature dependence as M_s^2. In view of Eq. (9), the domain wall width is then independent of temperature and the domain wall energy density depends on the temperature as M_s^2. More recent work specific to the FePt system has shown that there is a very weak temperature dependence of the domain wall width, indicating that $A(T)$ does not scale exactly as $M_s(T)^2$.[26] For the present paper, the deviation from the classical scaling is neglected and $\sqrt{AK}$ at the write temperature T_R becomes $\sqrt{A_0 K_0}/W_{A,T}^2$,

$$\begin{aligned} AD &= \frac{16\sqrt{A_0 K_0}}{kT_R \ln\left(\frac{1}{BER_0} - 1\right)}\frac{1}{W_{A,T}^2}\frac{H}{H_A(T_R)}\frac{A_{Gr}}{A_{bit}} \\ &= \frac{16\sqrt{A_0 K_0}}{kT_R \ln\left(\frac{1}{BER_0} - 1\right)}\frac{1}{W_{A,T}^2}\frac{1}{W_A}\frac{A_{Gr}}{A_{bit}}, \end{aligned} \tag{15}$$

where A_0 is the exchange constant at zero Kelvin and K_0 is the anisotropy energy density at zero Kelvin. As Eq. (15) illustrates, a thermal write assist can, in principle, coexist with a non-thermal write assist. For thermally assisted recording without an additional non-thermal write assist ($W_A = 1$), Eq. (15) can be simplified to,

$$AD \approx \frac{16\sqrt{A_0 K_0}}{kT_c \ln\left(\frac{1}{BER_0}\right)} \frac{1}{W_{A,T}^2} \frac{A_{Gr}}{A_{bit}}, \quad (16)$$

where it has been assumed the target error rate is reasonably small and that T_R is close to T_c.

It is instructive to study the effect of different material parameters on the ultimate recording density. We assume the temperature dependencies of the magnetic properties from $L1_0$ FePt with the understanding that the full range of the material parameter space studied here will not be available in practice by changing material composition and the like. The calculations are done for a bit-patterned recording system with no additional write assist. Using Eq. (15), the data are represented as curves $M_{s0}(H_{A0})$ with the areal density AD as a parameter. It is noted that the curves hold for 100% packing fraction and the numbers have to be multiplied with A_{Gr}/A_{bit} to be translated to a practically achievable density. The result is shown as full lines in Figs. 4(a)–4(d) for different target error rates BER_0. In all designs with material parameters below the respective full line, the desired bit error rate will not be achieved. More stringent bit error rate requirements cause the design curves to move upward, which means that higher saturation magnetizations are more favorable, as they increase the field energy. The effect of using Eq. (3) versus Eq. (4) is equivalent to a more stringent BER requirement and the comparison between Figs. 4(a) and 4(b) show how much difference this makes.

In addition to the error rate target, the stability of the recorded information remains a necessary condition for any recording system. As usual, it is demanded that the energy barrier $K(T_S)V = \delta A_{Gr} K(T_S)$ is ξ times the thermal energy kT_S, which translates to an areal density as,

$$AD_S = \frac{4\sqrt{A(T_S)K(T_S)}}{\xi k T_S} \frac{A_{Gr}}{A_{bit}} = \frac{4\sqrt{A_0 K_0}}{\xi k T_S} \frac{A_{Gr}}{A_{bit}} \eta, \quad (17)$$

where $\eta = 1 - \left(\frac{T_S}{T_c}\right)^{2.5}$ takes into account the temperature change of K between zero Kelvin and the storage temperature, which is about 0.89 for the FePt system. Using Eq. (17), the dashed lines in Figs. 4(a)–4(d) give $M_{s0}(H_{A0})$ with the areal density as parameter. Stable designs require combinations (H_{A0}, M_{s0}) above the respective areal density lines. Valid designs must fulfill both the error rate and the stability criterion; consequently they appear as the shaded areas illustrated in Fig. 4. Assuming $A_{Gr}/A_{bit} = 0.5$, and an error rate target of 0.01, a density of 20 Tbit/in.[2] appears to be an optimistic estimate for the ultimately achievable density. This estimate entails that the material properties of FePt may be fine-tuned to yield a somewhat higher saturation magnetization or lower anisotropy field as the nominal $L1_0$ FePt alloy. Alternatively, the combination of a non-thermal write assist together with the thermal write assist might justify the optimistic estimate. This maximum obtainable density is clearly less than that obtained using the stability criterion alone; it should also be recalled that it represents an upper bound because of the assumption that thermal equilibrium is attained during writing.

The maximum density occurs at the intersection of the two design curves, $AD = AD_S$. Interestingly, Figs. 4 show that the intersections fall on a vertical line, which means that there is an optimum anisotropy field for each targeted error rate. Solving $AD = AD_S$ for $W_{A,T}$ yields,

$$W_{A,T} = 2\sqrt{\frac{\xi}{\eta} \frac{T_S}{T_R} \frac{1}{\ln\left(\frac{1}{BER_0} - 1\right)}}. \quad (18)$$

There are no material parameters on the right-hand side of Eq. (18). This illustrates that the intersection of the design curves give an upper bound for the allowable thermal write assist and therefore the recording field sets an upper bound to the anisotropy field. This summarizes the key finding of this paper: *even though recordings on very high anisotropy materials can be made sufficiently stable, it is not always*

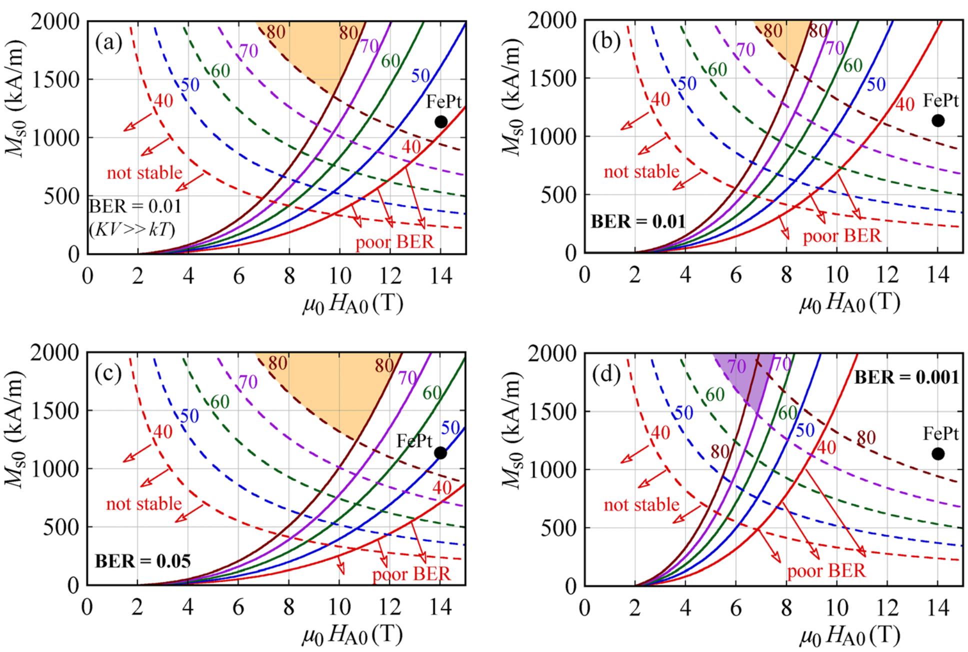

FIG. 4. (Color online) (a)–(d) Design space for materials with a temperature dependence of the magnetic properties as $L1_0$ FePt with various target error rates BER_0. M_{s0} and H_{A0} refer to zero Kelvin. The full curves give the thermally induced written-in error rate at the respective areal densities in tera grains per square inch. In all cases except (a), the BER_0 target was determined using Eq. (4). The dashed curves show the stability limit for $\xi = 60$ at room temperature. Valid designs must be located above both curves.

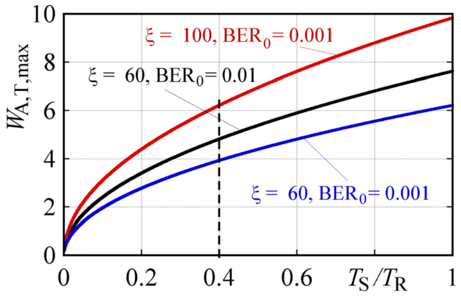

FIG. 5. (Color online) Maximal allowable thermal write assist as a function of the ratio of recording to storage temperature with different target error rates BER_0 and stability factors ξ as parameter for the FePt system ($\eta = 0.89$).

possible to record them well enough with available fields. The maximal allowable thermal write assist is shown in Fig. 5. It is straightforward to include a non-thermal write assist in this argument.

VI. FIELD ANGLE EFFECTS

The previous section presupposed that the record field is aligned with the easy axis of the grains. In practical recording systems, the field is typically at an angle to the easy axis. Therefore, it has to be investigated what difference the field angles make to the thermally induced error rates. We concentrate on a bit patterned system with thermal assist.

At first sight it might be obvious that a tilted record field is beneficial, because it lowers the switching field[27] and consequently the required write assist. A closer look, however, shows that the field direction significantly changes the equilibrium distribution of the magnetization orientations. To understand the consequences of this, one can think about a reduction of selectivity with increasing field angle. In the limit for a record field perpendicular to the easy axis, $\vartheta_0 = 90°$, there is no selectivity at all because both final magnetization orientations are equally likely to occur. Clearly, the thermal forces will reduce the selectivity with increasing field angle and the combined effect with the angle dependence of the switching field must lead to an optimum ϑ_0 somewhere between 0° and 45°. The calculation of the recorded magnetization again involves a partition function,

$$BER = \frac{1}{2}\frac{\int_{\vartheta=\pi/2}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\cos\vartheta_0\exp\left(-\frac{E}{kT}\right)\mathrm{d}\vartheta\mathrm{d}\varphi}{\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\vartheta\cos\vartheta_0\exp\left(-\frac{E}{kT}\right)\mathrm{d}\vartheta\mathrm{d}\varphi}, \quad (19)$$

where the energy is given as: $E = -\mu\mu_0 H(\sin\vartheta\sin\vartheta_0\cos\varphi + \cos\vartheta\cos\vartheta_0) + KV\sin^2\vartheta$, and ϑ and φ are the polar and azimuth angles of the magnetization with the easy axis. To ensure fast switching, we again evoke the boundary condition that the applied field is identical to the switching field $H = H_{SW}(\vartheta_0)$, where $H_{SW}(\vartheta_0) = H_A/(\cos^{2/3}\vartheta_0 + \sin^{2/3}\vartheta_0)^{3/2}$, see Ref. 27. It is noted that the term $\mu\mu_0 H$ is $\mu\mu_0 H_A$ for $\vartheta_0 = 0$ and does not change when the field is applied at a different angle, because the lower switching field due to the angle effect leads to a smaller required write assist $W_{A,T}(\vartheta_0) = W_{A,T}(0) \cdot h_{SW}(\vartheta_0)$ and, consequently, the magnetization will be increased by precisely the inverse of the same factor due to the scaling of the magnetization with temperature in the FePt system. For the same reason, the second term in the energy equation, KV, will increase by $1/h_{SW}(\vartheta_0)$.[2] Figure 6 shows the bit error rate as a function of field angle with the write assist $W_{A,T}$ as parameter for fully ordered L1$_0$ FePt. There exists an optimum at about 20° that depends on the strength of the write assist. At large angles, the curves tend to 0.25.

This shows that an inclination of the record field can improve the thermally induced error rates to some degree. Consider, for example, ideal L1$_0$ FePt, which achieves an areal density of 30.9 tera grains per square inch according to Eq. (15) at a bit error rate target of 0.01. When the storage temperature is assumed to be 300 K, the stability factor at room temperature is then $\xi = 171$. This stability factor is much greater than that necessary for conventional thermal stability, because the only way to increase the field energy at the required write assist is to increase the grain volume (note that FePt is far away from the optimum (H_{A0},M_{S0}) combination in Fig. 4(b)). If the field makes an angle of 20° with the easy axis, the error rate becomes 0.0035. This additional margin can be used to reduce the grain volume again, and the target bit error rate of 0.01 is retrieved with $\xi = 157$ at room temperature, yielding an areal density of about 37.6 tera grains per square inch. The example shows that tilting the field can be somewhat beneficial, but the overall effect remains relatively small, especially in the most important regime of strong write assists.

VII. DISCUSSION

In Sec. II, it was assumed that thermal equilibrium can be achieved during the write process and that it represents the best case scenario. It is instructive to revisit this aspect a bit more in detail, in particular for the case of a thermal write assist. In a strict sense, the calculations apply to the case where the recording medium is subjected to a constant write field H and a constant recording temperature T_R, where the temperature is instantaneously reduced to T_S *before* the applied field is switched off. Here we want to take the considerations one step further to include dynamic aspects in a very crude way.

In a recording device, the total time t_{rec} for which the write temperature is applied is of the order of a few ns. Unfortunately, given the current state of knowledge of

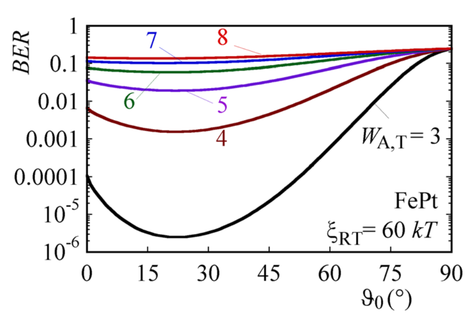

FIG. 6. (Color online) Bit error rate as a function of field orientation angle with respect to the easy axis. The various curves are for different ratios of the zero Kelvin anisotropy field to the write field. The applied field is equal to the Stoner-Wohlfarth switching field at the respective angles.

magnetization reversal dynamics near the Curie point of magnetic recording media, it is impossible to say with reasonable certainty whether this time is sufficient for the magnetization to reach thermal equilibrium.

The assumption of an instantaneous temperature reduction of T_R to T_S artificially introduces a sharp separation between a recording regime and a storage regime. The recording regime occurs for temperatures $T_R \le T < T_c$ where T_R is defined by the *ad hoc* assumption that the write field H has to be at least the Stoner-Wohlfarth switching field $H_{SW}(T,\vartheta_0)$. In traditional magnetic recording, the recorded magnetization is determined at the point where the head field is larger than the switching field for the last time (trailing edge of the head field). For thermally assisted recording this occurs at $T = T_R$ while the temperature is falling, i.e., the trailing edge of the heat spot. If the recording time t_{rec} is much longer than the relaxation time τ, that is, thermal equilibrium can be reached, the magnetization is $m_e(T_R)$, as derived in this paper. On the other hand, if t_{rec} is much shorter than $m_e(T_R)$, the present formalism can be kept if the recording temperature T_{R*} is redefined as the temperature at which the equilibrium magnetization can be reached within a specified degree. (It is known that the system becomes faster with increasing temperature.[28])

Finally we turn to the storage regime, where the temperature is less than T_R and the applied field is less than $H_{SW}(T,\vartheta_0)$. In this regime there are two possible stable magnetization states: one where the magnetization aligns with the write field and another where it opposes the write field. The calculation of the magnetization equilibrium (Eq. 4) remains valid here. However, standard theory[1,2] predicts that the time to reach thermal equilibrium increases exponentially as a function of the ratio $H/H_A(T)$. For example, for the case where the easy axis is aligned with the field, the relaxation time is $\tau \propto \exp(-KV(1 - H/H_A)^2)$. Of particular interest is the case where equilibrium magnetization was reached at $T = T_R$. On physical grounds, the relaxation time cannot be discontinuous at $T = T_R$ and the recording must take place in the storage regime. The relaxation time will rise quickly with reducing temperature until it is so long that no appreciable fraction of the grains can be switched. Similar to the analogous case in the recording regime $T > T_R$, the present formalism can be kept by a proper redefinition of the recording temperature T_{R*} where $T_{R*} < T_R$.

Consequently, as a first approximation, the present formalism can be extended to include dynamic effects by redefining the recording temperature as the lowest temperature for which equilibrium magnetization can be reached within a given time interval t_{rec}. This then defines the separation between the recording and the storage regimes, which implies that the required write field is given by $H/H_{SW}(T_{R*},\vartheta_0) = \alpha$. In principle, α can be greater or smaller than 1, but most likely $\alpha < 1$. Unfortunately, a quantification of α is beyond the scope of the current work.

VIII. SUMMARY

It has been demonstrated that thermodynamics impose a previously unrecognized fundamental limit to the achievable bit error rate in a recording system. During the recording process, the thermal forces strive to randomize the magnetization, which leads to errors in the write process. In current recording systems, the probability of these thermally induced write errors is so small that they can be safely neglected.

The new limit becomes practically important when the recording process involves a write assist. When the density of a storage system is increased, the volume of the magnetic units to be switched is naturally reduced. In the absence of a write assist, the requirement of thermal stability mandates that the anisotropy of the recording material is increased and, consequently, the applied field has to be increased and the field energy of the magnetic units remains unchanged. Owing to material limitations, the recording field cannot be increased at will and a write assist is needed to be able to switch the media. This write assist inevitably reduces the ratio between field energy and thermal energy and leads to an increase of the thermal error rate.

The highest recording densities are expected by combining the technology of bit-patterned media with that of thermally assisted recording. An upper bound of the ultimate recording density was obtained by assuming that the magnetization can reach its thermal equilibrium during the recording process. In this way, an ultimate density of about 20 Tbit/in.[2] for thermally assisted recording on bit patterned media may be conceivable. This assumes an error rate of 0.01, a high packing density of 50%, and a recording material based on an optimized FePt alloy. The estimate was obtained with a recording field of 2 T and it was shown that the results do not appreciably change if the field is inclined up to 45°.

This fundamental limit is also important for other recording systems. Recording on traditional granular media involves the writing of bits and magnetization transitions between these bits. If the recording is done with a thermal assist, the fraction of grains given in Eqs. (3) and (4) do not directly represent an error rate. Within the bits, where the record field is identical to the maximum head field, an error rate of 0.01 translates into an incompletely written bit with 99% of the grains being switched correctly. For the writing of the transitions, however, the direction of the applied field is switched during the record process. Because the magnetic field cannot be reversed instantaneously, this inevitably means that less field energy is available to combat the thermal energy and the number of incorrectly recorded grains must increase. Therefore, the thermodynamics during writing affect the quality of both the written transitions and the bits.

Last, but not least, the new fundamental limit equally applies to magnetoresistive random access memory (MRAM), where the MRAM cells can be either switched by a magnetic field or by the spin-torque effect. The essential physics remain the same; where obviously the field energy would have to be replaced by the energy associated with the write current in the case of a spin-torque driven device.

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