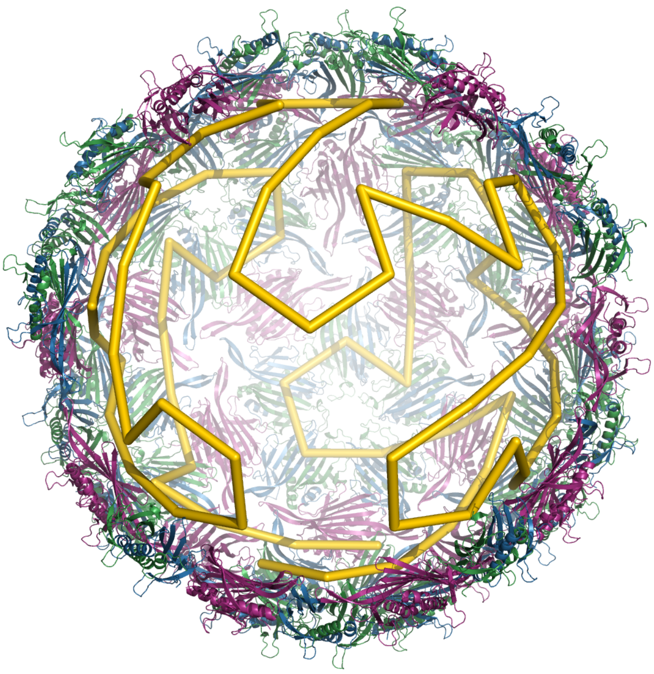
Mathematical Virology
Teacher's Resource Pack



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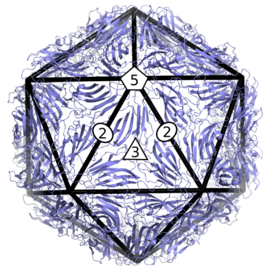
Viruses and Symmetry

Viruses are fascinating examples of symmetry in biology. They have protein containers, call viral capsids, that enclose and thus protect the genetic material. For the majority of viruses, these capsids have icosahedral symmetry, i.e. the same symmetry axes as an icosahedron.



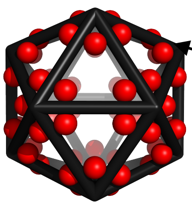
The **icosahedron** has

- 6 axes of 5-fold symmetry
- 10 axes of 3-fold symmetry
- 15 axes of 2-fold symmetry

Why do viruses have symmetry?

Crick and Watson, 1956: The principle of genetic economy

Viruses code for a small number of building blocks that are repeatedly used to form containers with symmetry. Containers with icosahedral symmetry are largest given fixed protein size, thus viruses minimise the length of the genome required to code for a protein container of sufficient volume to fit the genome.

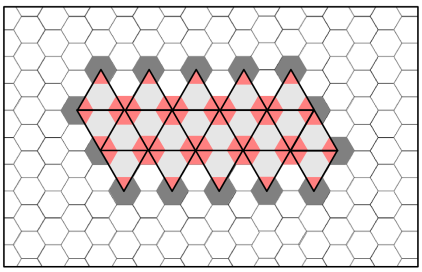


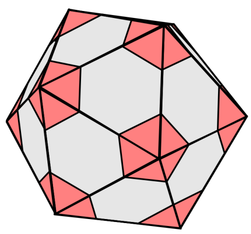
If the position of one red disk is known, then the positions of all others are implied by symmetry.

How to model virus structure?

Crick and Klug, 1962: The principle of quasiequivalence

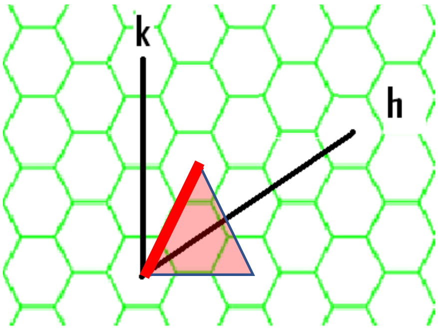
The local environments of all capsid proteins look similar. Their positions can be indicated with respect to a hexagonal net wrapped around an icosahedral surface.

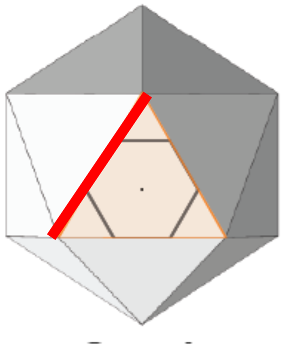


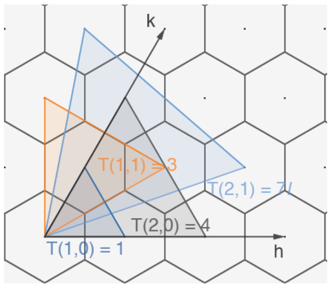


Note: Hexagonal sheets – blank and with examples – are provided in your pack.

The T-number construction





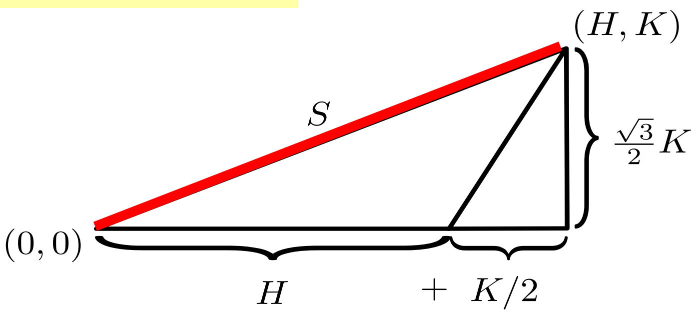


$T(h,k) = h^2 + hk + k^2$

Note:

- h and k are lines at an angle of 60° intersecting at the midpoint of one hexagon
- we count steps between the midpoints of adjacent hexagons (H steps in the direction of h and K steps in the direction of k; the example shows H=K=1)
- (H,K) labels the midpoint of the hexagon reached after H steps along h and K steps along K starting at the intersection of the lines h and k; the line connecting the start and end (red line) defines one side of an icosahedral face
- the remainder of the icosahedral surface follows from the fact that its faces are equilateral triangles; in other words: when that line is drawn the position of the entire icosahedral surface is fully determined (see attached hexagonal nets with examples)
- the edge length of an icosahedral face corresponding to (H,K) can be obtained using Pythagoras' Theorem as follows:

$$T=S^2=(H+K/2)^2+3/4K^2 = H^2+HK+K^2$$



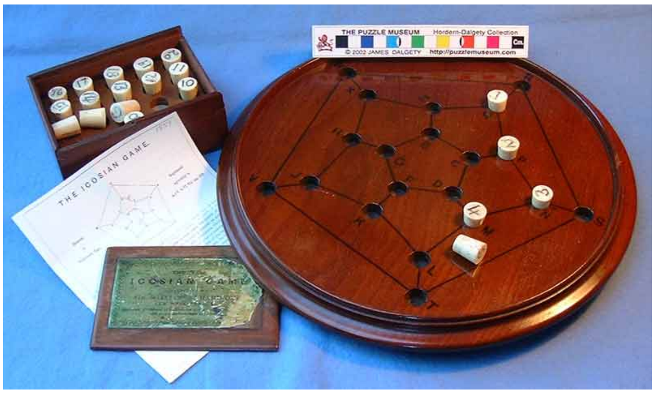
- T is called the T-number and is used to analyse virus structures.

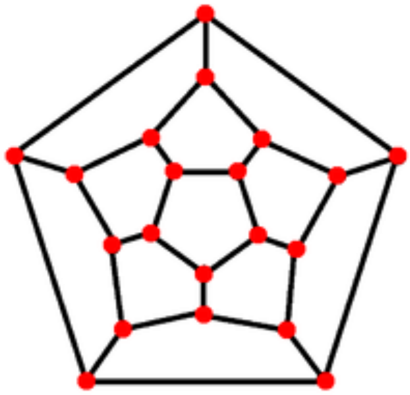
Note: Examples of viruses and their T-numbers are in your pack.

Hamiltonian Paths

The famous Irish mathematician William Rowan Hamilton invented a board game called the Icosian Game in 1856, see below. It is based on the concept of a Hamiltonian path. Hamiltonian paths on a graph are connected paths visiting every vertex precisely once. The game invites the players to explore Hamiltonian Paths on the dodecahedron, a polyhedron formed from 12 regular pentagons. The game represents the dodecahedron as a Schlegel diagram in the plane, see below right, and asks two players to alternately make moves with the aim of generating the longest uninterrupted path.

We suggest inviting students to play this game in pairs. Individual students could also be asked to find connected paths that meet every vertex (Hamiltonian paths), and paths with this property that are circular.





The novel use of Hamiltonian paths in virology has been key in the discovery of an assembly mechanism and revealed novel ways of combating viral disease (see the commentary by Peter Prevelige: Follow the Yellow Brick Road: A Paradigm Shift in Virus Assembly. *J. Mol. Biol., 428*, 416-418, 2016; see also: https://sinews.siam.org/Details-Page/follow-the-yellow-brick-road and https://www.quantamagazine.org/the-illuminating-geometry-of-viruses-20170719/). Hamilton would have been surprised to learn that viruses play the Icosian Game!

Note: The Schlegel diagram of the dodecahedronis provided in your pack.

Feedback

We would be very grateful for feedback on what your students have enjoyed about these projects, and for suggestions what else we can provide. Please visit our website on:

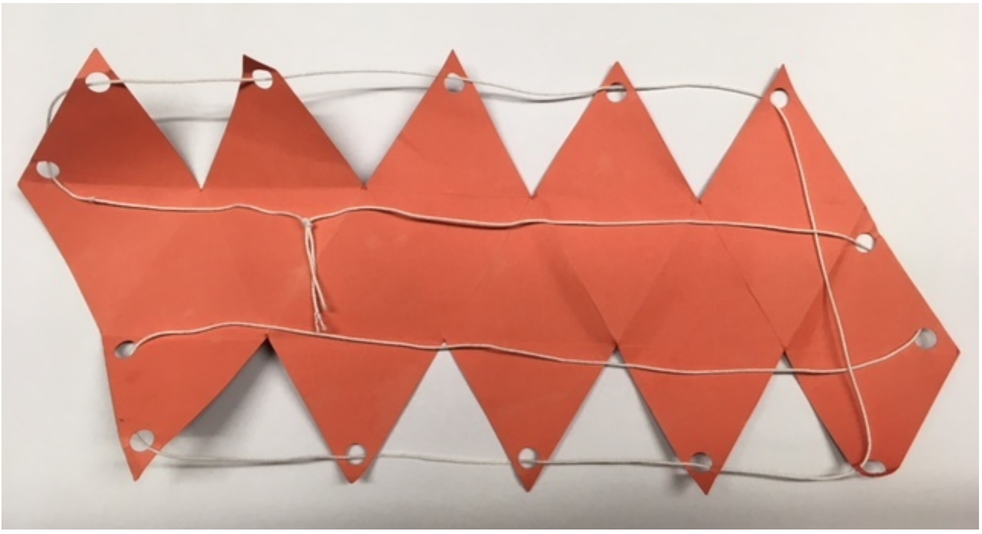
www-users.york.ac.uk/~rt507/teaching_resources.html

We are passionate about sharing our joy of mathematics with teachers and students, and your feedback is invaluable for us to provide you with what you are interested in!

APPENDIX

The icosahedral pull-up nets

Here is a picture of how to string up your pull-up net. We suggest you use a hole punch to make the holes in the indicated positions, and pre-fold the individual edges, so that it is easier for the icosahedron to fold. Then hold the two strings in the middle of the shape and pull until the icosahedron closes. **Have fun!**



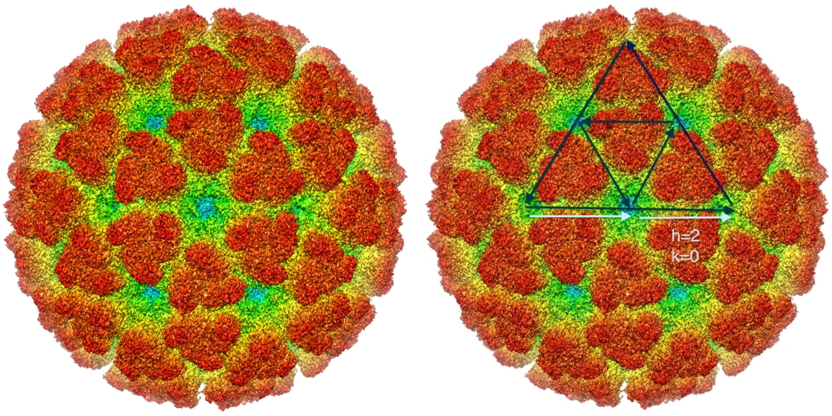
Viruses and their T-numbers

Can you answer the following questions for the viruses on the next page:

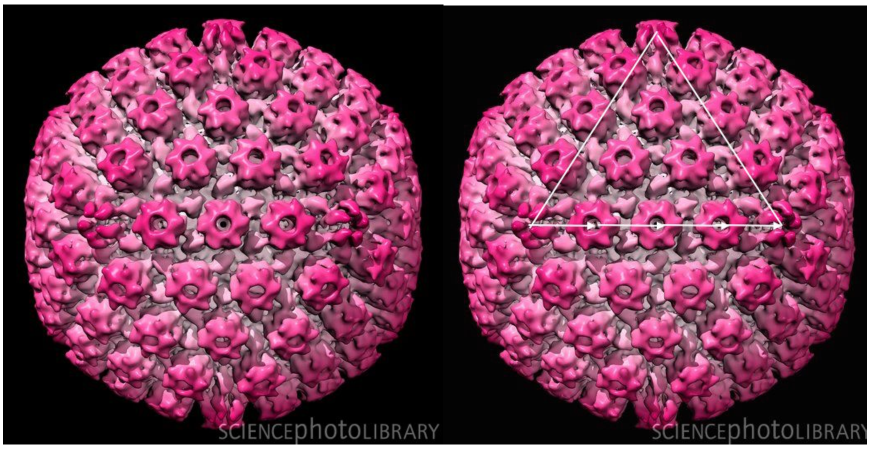
- Can you spot the icosahedron?
- What is the T-number?

Note: A Powerpoint file containing pictures of the following viruses is available for download from our website, so that you can create your own materials.

Chikungunyavirus: T=4 (H,K)=(2,0)



Herpes Simplex Virus: T=16 (H,K)=(4,0)



Rotavirus: T=13 (H,K)=(3,1)