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UNIVERSAL QUANTUM GATES FOR SINGLE COOPER PAIR BOX BASED QUANTUM COMPUTING

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1 Introduction

Several schemes have been proposed for implementing quantum computer hardware in solid state quantum electronics. These schemes use electric charge[1,2,3], magnetic flux[4,5,6], superconducting phase[7,8,9,10], electron spin[11,12,13,14], or nuclear spin[15,16] as the information bearing degree of freedom. Each scheme has various pros and cons but that based on harnessing quantized charge is especially appealing because the necessary superconducting circuitry for such a qubit can be fabricated using present day e-beam lithography equipment, and quantum coherence, essential for creating superposed and entangled states, has been demonstrated experimentally[17]. Moreover, the fidelity and leakage of such gates is understood[18]. These qualities make the SCB-based qubit a strong contender for the basic element of a quantum computer. Indeed, today's e-beam fabrication technology is sufficiently mature that it would be a simple matter to create a quantum circuit having thousands of quantum gates within a matter of a few hours! Of course, it remains to be seen whether such large-scale quantum circuits could be operated coherently en masse. Nevertheless, the relative ease of fabricating SCB-based quantum gates leads one to consider computer architectural issues related large scale SCB-based quantum circuits.

From an architectural perspective, the existing proposals for SCB-based qubits and quantum gates are sub-optimal. For example, the scheme of Schön et al.[19] uses the time at which a gate operation begins as one of the parameters that determine the unitary operation the gate is to perform. While this is certainly allowed physically, and could even be argued to be ingeniously efficient, it is not a good decision from the perspective of building reliable and scaleable quantum computers. If the starting time is a parameter, a given quantum gate would need different implementations at different times. Moreover, as the computation progressed, timing errors would accumulate leading to worsening gate fidelity. Furthermore, Schön et al. also use the duration of the gate operation as a free parameter that determines the unitary transformation the gate is to perform. Again, this is a poor decision from a computer architecture perspective, as it means that different gates would take different times making it difficult to synchronize parallel quantum gate operations in large circuits. To address both of these problems we have developed an approach to universal quantum computation in SCB-based quantum computing that specifically avoids using time as a free parameter. Instead, our gates operate by varying only voltages of magnetic fluxes in a controlled fashion.

To make a practical design for a quantum computer, one must specify how to decompose any valid quantum computation into a sequence of elementary 1- and 2-qubit quantum gates that can be realized in physical hardware that is feasible to fabricate. The set of these 1- and 2-qubit gates is arbitrary provided it is *universal*, i.e., capable of achieving any valid quantum computation from a quantum circuit comprising only gates from this set. Traditionally the set of universal gates has been taken to be the set of all 1-qubit quantum gates in conjunction with a single 2-qubit gate called controlled-NOT. However, many equally good universal gate sets exist[20] and there might be an advantage in using a non-standard universal gate set if certain gate designs happen to be easier to realize in one hardware context than another[21,22]. Certainly it has been known for some time that the simple 2-qubit exchange interaction (i.e., the SWAP gate) is as powerful as CNOT as far as computational universality is concerned. It makes sense therefore, to see what gates are easy to make and then extend them into a universal set. This is the strategy pursued in this paper. In particular, we show, in the context of SCB-based qubits, that we can implement any 1-qubit operation and a special (new) 2-qubit operation called “the square root of complex SWAP” (or “$\sqrt{iSWAP}$” for short). We then prove that, taken together, $\sqrt{iSWAP}$ and all 1-qubit gates is universal for quantum computation.

2 SCB-based Qubits

A Single Cooper Pair Box is an artificial two-level quantum system comprising a nanoscale superconducting electrode connected to a reservoir of Cooper pair charges via a Josephson junction. The logical states of the device, $|0\rangle$ and $|1\rangle$, are implemented physically as a pair of charge-number states differing by $2e$ (where e is the charge of an electron). Typically, some 10^9 Cooper pairs are involved. Transitions between the logical states are accomplished by tunneling of Cooper pairs through the Josephson junction. Although the two-level system contains a macroscopic number of charges, in the superconducting regime they behave collectively, as a Bose-Einstein condensate, allowing the two logical states to be superposed coherently. This property makes the SCB a candidate for the physical implementation of a qubit.

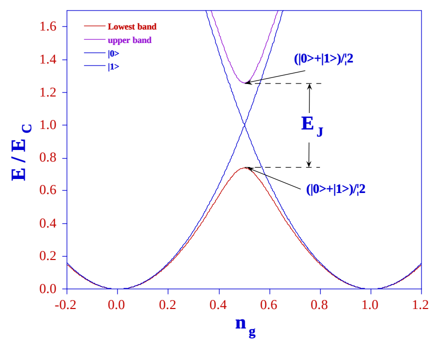

Figure 1. The level diagram for an SCB-based qubit.

The SCB-qubit gained prominence in 1999 when Nakamura et al. demonstrated coherent oscillations between the $|0\rangle$ and $|1\rangle$ states[17]. This was the first time such macroscopic coherent phenomena had been seen experimentally and distinguishes the SCB approach from other solid state schemes in which similar macroscopic coherences have recently been demonstrated[5,6].

Our qubit consists of a *split* tunnel junction as this allows us to control the Josephson: V controls the number of excess Cooper pairs—by varying the externally applied magnetic flux according to:

$$E_J(\Phi_{ext}) = 2E_J^{\text{intrinsic}} \cos\left(\frac{p\,\Phi_{ext}}{\Phi_0}\right) \tag{1}$$

where Φ_0 is the quantum of magnetic flux and $E_J^{\text{intrinsic}}$ is given by the Ambegaokar-Baratoff relation in the low temperature approximation: $E_J^{\text{intrinsic}} = \frac{h\Delta}{8e^2 R_N}$ (in which *h*, Δ, and R_N are Planck's constant, the superconducting energy gap and the normal tunneling resistance of the junction respectively). Figure 2 shows a schematic diagram of our qubit.

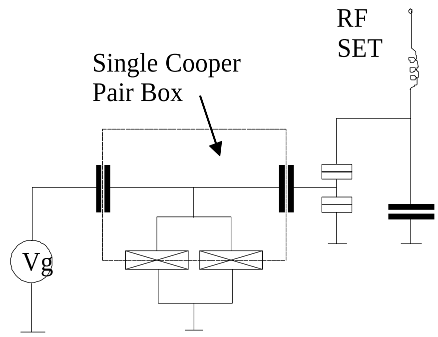

Figure 2. Schematic diagram of a single SCB-based qubit with an adjoining RF SET readout.

The Hamiltonian for the qubit is $H = 4E_C \sum_n (n - n_C)^2 - E_J(\Phi_{ext})\cos(f)$, where n is the number of excess Cooper pairs on the island, $n_C = \frac{C_g V}{2e}$, $E_C = \frac{(2e)^2}{2C_\Sigma}$, $C_\Sigma = C_J + C_g$, and f is the difference in phase of the superconducting state across the junction. In the basis of excess Cooper pair number states, $|n\rangle$, restricting the gate charge interval to be $0 \le n_C \le 1$, and choosing the zero of energy to be at $E_o = E_C(1/2 - n_c)^2$, the Hamiltonian reduces to:

$$\hat{H}_1 = \begin{pmatrix} -\frac{1}{2}E(V) & -\frac{1}{2}E_J(\Phi_{ext}) \\ -\frac{1}{2}E_J(\Phi_{ext}) & +\frac{1}{2}E(V) \end{pmatrix} \tag{2}$$

where $E(V) = E_C\left(1 - \frac{C_G V}{e}\right)$ and $E_J(\Phi_{ext}) = 2E_J^{\text{intrinsic}}\left|\cos\left(\frac{p\,\Phi_{ext}}{\Phi_0}\right)\right|$. The two parameters V and Φ_{ext} can be adjusted to achieve different Hamiltonians and hence different 1-qubit quantum gates.

3 One-Qubit Gates

The 1-qubit Hamiltonian, $\hat{H}_1$, acting for a time Δt induces a 1-qubit quantum gate operation given by:

$$\hat{U}_1 = \exp\left(-\frac{i\hat{H}_1 \Delta t}{\eta}\right) \tag{3}$$

We assume that the Hamiltonian can be switched on and off quickly so that the interval Δt is sharp. The fact that $\hat{H}_1$ has a symmetric structure means that we are only able to implement a limited set of primitive unitary transformations. Nevertheless, it turns out that these primitive transformations can be composed to achieve *arbitrary* 1-qubit gates. The proof is via a factorization of an arbitrary 2×2 unitary matrix into a product of rotation matrices. Specifically, the matrix for an arbitrary 1-qubit gate is described mathematically by[23]

$$\hat{U}(a,q,b)=\begin{pmatrix} e^{i\left(\frac{a+b}{2}\right)}\cos\left(\frac{q}{2}\right) & e^{i\left(\frac{a-b}{2}\right)}\sin\left(\frac{q}{2}\right) \\ -e^{-i\left(\frac{a-b}{2}\right)}\sin\left(\frac{q}{2}\right) & e^{-i\left(\frac{a+b}{2}\right)}\cos\left(\frac{q}{2}\right) \end{pmatrix} \tag{4}$$

Such a matrix can be factored into the product of rotations about just the *z*- and *x*-axes.

$$\hat{U}(a,q,b)=\hat{R}_z\left(a-\frac{p}{2}\right)\cdot\hat{R}_x(q)\cdot\hat{R}_z\left(b+\frac{p}{2}\right) \tag{5}$$

where $\hat{R}_z(x)=\exp(ixs_z/2)$ is a rotation[§] about the *z*-axis through angle x, $\hat{R}_x(x)=\exp(ixs_x/2)$ is a rotation about the *x*-axis through angle x, and $s_i : i\in\{x,y,z\}$ are Pauli spin matrices.

It is therefore sufficient to configure the parameters in $\hat{H}_1$ to perform rotations about just the *z*- and *x*-axes to achieve an arbitrary 1-qubit gate. From equations (2) and (3), we find that $\hat{R}_z(x)$ can be achieved within time Δt by setting $\Phi_{ext}=\Phi_0/2$, and $V=\frac{x\eta e}{C_G E_C \Delta t}-\frac{e}{C_G}$. Similarly, $\hat{R}_x(x)$ can be achieved within time Δt by setting $\Phi_{ext}=\frac{\Phi_0}{p}\cos^{-1}\left(\frac{x\eta}{2E_J^{\text{intrinsic}}\Delta t}\right)$, and $V=\frac{e}{C_G}$. These settings cause, within time Δt, $\hat{U}_1$ to take the form $\hat{R}_z(x)=\begin{pmatrix} e^{ix/2} & 0 \\ 0 & e^{-ix/2}\end{pmatrix}$ or $\hat{R}_x(x)=\begin{pmatrix}\cos(x/2) & i\sin(x/2) \\ i\sin(x/2) & \cos(x/2)\end{pmatrix}$ respectively.

Thus, by the factorization given in equation (5), an arbitrary 1-qubit gate can be achieved in the SCB-based approach to quantum computing in a time of $3\Delta t$.

Note that the only free parameters used to determine the action of the 1-qubit gate are the external flux Φ_{ext} and the voltage V. The time interval, Δt, over which the Hamiltonian needs to act to bring about an *x*- or *z*-rotation, is fixed by the physics of the particular substrate, e.g., Aluminium or Niobium, used for the qubit. Although we could also have used Δt as an additional control parameter, such a choice would complicate integration of quantum gates into parallel, synchronous, quantum circuits.

4 Qubit Gates

To achieve a 2-qubit gate, it is necessary to couple pairs of qubits. In our scheme, two qubits are coupled using two tunnel junctions connected in parallel. This allows the coupling to be turned on or off as necessary. A schematic for the 2-qubit gate is shown in Figure 3.

[§] The doubling of the angle arises because of the relationship between operations in SO(3) (rigid-body rotations) to operations in SU(2).

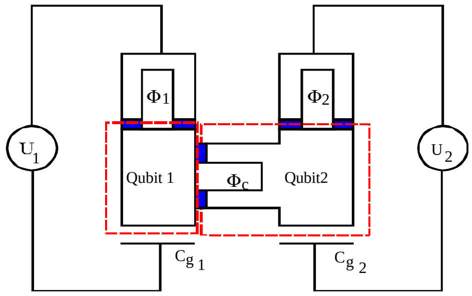

Figure 3. Schematic diagram of a pair of coupled qubits.

The Hamiltonian for the coupled pair of qubits is given by:

$$\hat{H}_2 = E_{C_1}(n_1 - n_{C_1})^2 + E_{C_2}(n_2 - n_{C_2})^2 - E_{J_1}(\Phi_1)\cos(f_1) - \\ E_{J_2}(\Phi_2)\cos(f_2) - E_{J_C}(\Phi_C)\cos(f_1 - f_2) \tag{10}$$

where the subscripts 1, 2, and *C*, refer to parameters of qubit 1, qubit 2 and the coupling between them respectively. Assuming again that the zero of energy is at $E_o=E_{c1}(1/2-n_{c1})+E_{c2}(1/2-n_{c2})^2$

$$\hat{H}_2 = \begin{pmatrix} -\frac{E_1}{2}-\frac{E_2}{2} & -\frac{1}{2}E_{J_2}(\Phi_2) & -\frac{1}{2}E_{J_1}(\Phi_1) & 0 \\ -\frac{1}{2}E_{J_2}(\Phi_2) & -\frac{E_1}{2}+\frac{E_2}{2} & -\frac{1}{2}E_{J_C}(\Phi_C) & -\frac{1}{2}E_{J_1}(\Phi_1) \\ -\frac{1}{2}E_{J_1}(\Phi_1) & -\frac{1}{2}E_{J_C}(\Phi_C) & \frac{E_1}{2}-\frac{E_2}{2} & -\frac{1}{2}E_{J_2}(\Phi_2) \\ 0 & -\frac{1}{2}E_{J_1}(\Phi_1) & -\frac{1}{2}E_{J_2}(\Phi_2) & \frac{E_1}{2}+\frac{E_2}{2} \end{pmatrix} \tag{11}$$

where $E_1=E_{c1}(1-2n_{c1})$ and $E_2=E_{c2}(1-2n_{c2})$. The 2-qubit quantum gate induced by this Hamiltonian is

$$\hat{U}_2 = \exp\left(-\frac{i\hat{H}_2 t}{\eta}\right) \tag{12}$$

We can specialize $\hat{U}_2$ to a particular form by setting $n_{C_1} = n_{C_2} = \frac{1}{2}$, $E_{J_1} = E_{J_2} = 0$. These values induce the 2-qubit gate

$$\hat{U}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\frac{E_{J_C}\Delta t}{2\eta}\right) & i\sin\left(\frac{E_{J_C}\Delta t}{2\eta}\right) & 0 \\ 0 & i\sin\left(\frac{E_{J_C}\Delta t}{2\eta}\right) & \cos\left(\frac{E_{J_C}\Delta t}{2\eta}\right) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$

Specializing further by setting $E_{J_C} = \eta p/(2\Delta t)$ we achieve a 2-qubit gate that we call the "square root of complex SWAP", $\sqrt{iSWAP}$:

$$\sqrt{iSWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{14}$$

5 Universal Quantum Computation

The set of all 1-qubit gates together with controlled-NOT is known to be universal for quantum computation. As we have already shown that it is possible to implement any 1-qubit gate in the SCB context, we can prove that all 1-qubit gates and $\sqrt{iSWAP}$ is also a universal set by exhibiting a construction for *CNOT* using only 1-qubit gates and $\sqrt{iSWAP}$. The following gate sequence achieves *CNOT* up to an unimportant overall phase factor of $\exp(i\,3\pi/4)$:

$$CNOT \equiv e^{i\frac{3\pi}{4}} \left(I \otimes R_y\left(\frac{\pi}{2}\right)\right) \cdot (NOT \otimes NOT) \cdot \left(R_z\left(\frac{\pi}{2}\right) \otimes R_z\left(\frac{\pi}{2}\right)\right) \cdot \exp\left(i\frac{\pi}{4}(\sigma_z \otimes \sigma_z)\right) \cdot \ldots$$
$$\ldots \cdot \sqrt{iSWAP} \cdot \left(R_z(\pi) \otimes I\right) \cdot \sqrt{iSWAP} \cdot (NOT \otimes NOT) \cdot \left(I \otimes R_y\left(-\frac{\pi}{2}\right)\right) \tag{12}$$

Thus a controlled-NOT operation can be implemented within the SCB-based approach to quantum computing. If each primitive gate operation, i.e., each 1-qubit rotation or a square root of complex SWAP, takes time Δt then controlled-NOT is implementable in time $9\Delta t$.

6 Experimental program

We have fabricated SCB-based qubits using electron beam lithography and a standard shadow mask technique with double angle evaporation of Aluminum. The resulting junctions were 100nmx50nm in size. To couple the voltage pulses necessary for manipulation of the qubits we use a coplanar wave guide structure designed to have a 50 Ohm impedance and therefore minimize unwanted reflections. The magnetic field to generate the external flux will be created by a wire in close proximity to the SCB. This wire is a short circuit termination of another coplanar waveguide structure.

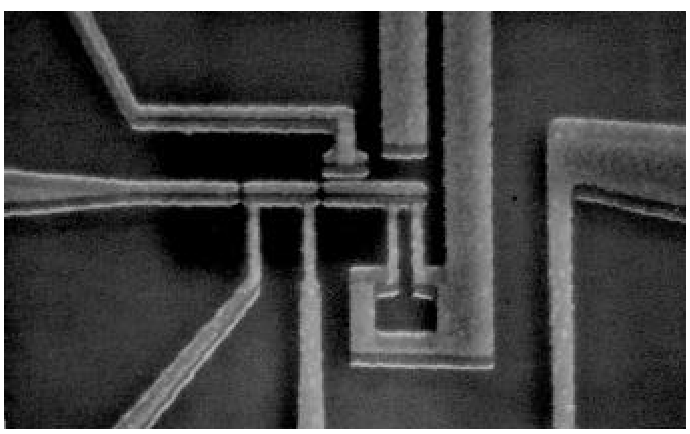

Figure 4. An SCB-based qubit fabricated in Aluminum using e-beam lithography.

In close proximity to the SCB sits a Single electron transistor which will be operated in the RF mode. The resonant circuit needed for operation of the RF-SET is provided by an inductor and capacitor defined by optical lithography. Testing of the fabricated structures are in the preliminary stages. We have equipped our dilution refrigerator with the necessary microwave equipment to perform the experiments. On a first run, we were able to cool the mixing chamber down to 50 mK. Initial tests are concentrating on observing the resonances of the on-chip resonant circuits. We have observed resonances close to the design frequencies with Q values up to 150. Our next step is to operate the SETs as RF-SETs and characterize its performance. We will then test the SCB and will attempt to measure the coherence times for various operating points.

7 Conclusions

We have designed a realizable set of quantum gates to support universal quantum computation in the context of SCB-based quantum computing. In selecting our universal gate set we paid special attention to two principles of good computer design, namely, that each gate operation should take a fixed and predictable length of time, and that the operations needed to bring about the action of a particular gate should not depend upon the time at which the gate operation begins. Earlier proposals for SCB-based universal quantum computation did not satisfy these criteria. We have fabricated SCB-based qubits using existing state-of-the-art e-beam lithography at the JPL Microdevices Laboratory. There appears to be no impediment to fabricating large-scale quantum circuits that manipulate SCB-based qubits.

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