Telecloning of Continuous Quantum Variables

P. van Loock and Samuel L. Braunstein
Informatics, Bangor University, Bangor LL57 1UT, United Kingdom
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We propose entangled $(M + 1)$-mode quantum states as a multiuser quantum channel for continuous-variable communication. Arbitrary quantum states can be sent via this channel *simultaneously* to M remote and separated locations with equal minimum excess noise in each output mode. For a set of coherent-state inputs, the channel realizes optimal symmetric $1 \rightarrow M$ cloning at a distance ("telecloning"). It also provides the optimal cloning of coherent states without the need of amplifying the state of interest. The generation of the multiuser quantum channel requires no more than two $10\log_{10}\{(\sqrt{M} - 1)/(\sqrt{M} + 1)\}$ dB squeezed states and M beam splitters.

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Quantum information encoded in nonorthogonal quantum states can be perfectly transferred between two distant locations that are linked by a maximally entangled state and a classical communication channel. This quantum teleportation [1] is a prime example of quantum information processing [2] where otherwise impossible cryptographic, computational, and communication tasks can be performed through the presence of shared entanglement. In principle, perfect teleportation with unit fidelity from a sender to a *single* receiver is possible in accordance with quantum mechanics. What about conveying quantum information via a "multiuser quantum channel" (MQC) simultaneously to several receivers? Of course, the no-cloning theorem [3] of quantum theory forbids *perfect* cloning (or copying) of arbitrary quantum states and does so also over a distance. This prevents the MQC from being able to produce *exact* clones of the sender's input state at all receiving stations. The MQC, however, can provide each receiver with at least a part of the input quantum information and distribute *approximate* clones [4]. This cloning at a distance or "telecloning" may be seen as the "natural generalization of teleportation to the many-recipient case" [5].

The original teleportation proposal for d-level systems ("qu$d$its") [1] was later extended to infinite-dimensional Hilbert spaces [6,7] followed by a successful demonstration of continuous-variable teleportation [8]. The initial results on quantum cloning referred to finite-dimensional systems, in particular, qubits [4,9–12]. Qubit telecloning has also been studied theoretically, first with one input sent to two receivers [10], and more generally with one input [5] and N identical inputs [13] distributed among M receivers. Later, telecloning with one input and M receivers was generalized to qu$d$its [14].

The first investigations on continuous-variable cloning led to cloning transformations enabling optimal "local" $1 \rightarrow 2$ cloning (one state mapped to two approximate copies) of coherent states with fidelity $2/3$ [15]. Subsequently, fidelity boundaries of Gaussian $N \rightarrow M$ cloners were derived, $F \leq MN/(MN + M - N) \equiv F_{\text{clon},N,M}^{\text{coh st},\infty}$ [16]. In Ref. [17], it was shown that for any Hilbert space dimension, the optimal universal local cloner that clones all possible input states equally well can be constructed from a single family of quantum circuits. In the continuous limit, the optimal universal cloner reduces to a classical probability distributor attaining $F = N/M$ [17], consistent with the limit of Werner's optimum fidelity for qu$d$its, $F = [N(d - 1) + M(N + 1)]/M(N + d) \equiv F_{\text{clon},N,M}^{\text{univ},d}$ [12]. The optimal local $N \rightarrow M$ coherent-state cloner that achieves $F_{\text{clon},N,M}^{\text{coh st},\infty}$ turns out to be a classical amplitude distributor which can be built from a phase-insensitive amplifier and beam splitters [18] (see also [19]). Here we propose a continuous-variable protocol that relies on *finite-squeezing resources* and linear optics and enables in principle an experimental realization of symmetric $1 \rightarrow M$ coherent-state telecloning with the maximum fidelities allowed by quantum theory. It is the first feasible telecloning scheme, taking into account that the entangled states of existing qu$d$it schemes are hard to produce. More generally, our MQC transfers arbitrary quantum states from a sender to M receivers with equal minimum excess noise in each output state. Further, it forms a cloning circuit with no need to amplify the input.

Clearly a telecloner needs entanglement as soon as its fidelity is greater than the maximum fidelity attainable by "classical teleportation" F_{class}. In fact, for *universal* $1 \rightarrow M$ qu$d$it cloning we have $F_{\text{clon},1,M}^{\text{univ},d} > F_{\text{class}} = 2/(1 + d)$ [9,11,12]; for $1 \rightarrow M$ cloning of *coherent states* we have $F_{\text{clon},1,M}^{\text{coh st},\infty} > F_{\text{class}} = 1/2$ [16,20]. Therefore, optimal telecloning cannot be achieved by simply measuring the input state and sending copies of the classical result to all receivers ("classical telecloning"). On the other hand, in the limit $M \rightarrow \infty$, $F_{\text{clon},1,M}^{\text{univ},d} \rightarrow F_{\text{class}} = 2/(1 + d)$ and $F_{\text{clon},1,M}^{\text{coh st},\infty} \rightarrow F_{\text{class}} = 1/2$ which implies that *no* entanglement is needed for infinitely many copies.

The most wasteful scheme would be a protocol in which the sender locally creates M optimum clones and perfectly teleports one clone to each receiver using M maximally entangled two-party states [5,14]. In fact, a much more economical strategy is that all participants share a particular multipartite entangled state as a quantum channel.

This MQC may contain maximum bipartite entanglement ($\log_2 d$ ebits) between the sender and all receivers as does the d-level telecloning state of Murao *et al.* [5,14]. Maximum entanglement for any M, however, is more entanglement than we expect from the most frugal scheme (the entanglement should become vanishingly small as $M \to \infty$). In a continuous-variable scenario based on the quadratures of single electromagnetic modes, multipartite entangled states can be generated using squeezers and beam splitters [21,22], and any maximum bipartite entanglement involved would require infinite squeezing. Having said that entanglement is the essential ingredient of a telecloner, we propose as an MQC a two-parametric family of pure entangled $(M+1)$-mode quantum states described by the Wigner function

$$W_{\mathrm{MQC}}(\mathbf{x},\mathbf{p}) = \left(\frac{2}{\pi}\right)^{M+1} \exp\Bigg\{-2e^{-2(s+r_1)}\left(\sin\theta_0 x_1 + \frac{\cos\theta_0}{\sqrt{M}}\sum_{i=2}^{M+1} x_i\right)^2 - 2e^{+2(s+r_1)}\left(\sin\theta_0 p_1 + \frac{\cos\theta_0}{\sqrt{M}}\sum_{i=2}^{M+1} p_i\right)^2 - 2e^{-2(s-r_2)}\left(\cos\theta_0 x_1 - \frac{\sin\theta_0}{\sqrt{M}}\sum_{i=2}^{M+1} x_i\right)^2 - 2e^{+2(s-r_2)}\left(\cos\theta_0 p_1 - \frac{\sin\theta_0}{\sqrt{M}}\sum_{i=2}^{M+1} p_i\right)^2 - \frac{1}{M}\sum_{i,j=2}^{M+1}[e^{-2s}(x_i - x_j)^2 + e^{+2s}(p_i - p_j)^2]\Bigg\}, \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_{M+1})$ and $\mathbf{p} = (p_1, p_2, \ldots, p_{M+1})$ are the "positions" and "momenta" corresponding to a pair of quadratures for each mode ($[\hat{a}, \hat{a}^\dagger] = 1$, $\hat{a} = \hat{x} + i\hat{p}$ or $\hbar = \frac{1}{2}$). The free parameters are s and θ_0 with

$$\frac{1}{\sqrt{M+1}} \le \sin\theta_0 \le \sqrt{\frac{M}{M+1}}, \tag{2}$$

$$e^{-2r_1} \equiv \frac{\sqrt{M}\sin\theta_0 - \cos\theta_0}{\sqrt{M}\sin\theta_0 + \cos\theta_0}, \tag{3}$$

$$e^{-2r_2} \equiv \frac{\sqrt{M}\cos\theta_0 - \sin\theta_0}{\sqrt{M}\cos\theta_0 + \sin\theta_0}. \tag{4}$$

We explain the meaning of the parameters in W_{MQC} later but first look at the potential telecloning protocol in which W_{MQC} is used. Mode 1 may be used as a "port" at the sending station and is combined at a phase-free symmetric beam splitter with mode "in," $\alpha_{\mathrm{u}} = (\alpha_{\mathrm{in}} - \alpha_1)/\sqrt{2}$, $\alpha_{\mathrm{v}} = (\alpha_{\mathrm{in}} + \alpha_1)/\sqrt{2}$, where $\alpha = x + ip$. With mode "in" in an *arbitrary* quantum state described by W_{in}, the Wigner function for the whole system after the beam splitter is $W(\alpha_{\mathrm{u}}, \alpha_{\mathrm{v}}, \alpha_2, \ldots, \alpha_{M+1}) = W_{\mathrm{in}}[\alpha_{\mathrm{in}} = (\alpha_{\mathrm{v}} + \alpha_{\mathrm{u}})/\sqrt{2}]W_{\mathrm{MQC}}[\alpha_1 = (\alpha_{\mathrm{v}} - \alpha_{\mathrm{u}})/\sqrt{2}, \alpha_2, \ldots, \alpha_{M+1}]$. The remaining steps are now the same as in the $1 \to 1$ quantum teleportation protocol [7], except for the crucial difference that *each* of the M distant and separated locations of modes 2 through $M+1$ need to be provided with the classical results of the "Bell detection," i.e., homodyne detections of x_{u} and p_{v}. Finally, after "displacing" all these modes correspondingly, $x_{2\ldots M+1} \to x_{2\ldots M+1} + \sqrt{2}x_{\mathrm{u}}$ and $p_{2\ldots M+1} \to p_{2\ldots M+1} + \sqrt{2}p_{\mathrm{v}}$, we obtain the M-mode output Wigner function

$$W_{\mathrm{out}}(\alpha_2, \ldots, \alpha_{M+1}) \propto \int d^2\alpha_{\mathrm{u}}\, d^2\alpha_{\mathrm{v}}\, W[\alpha_{\mathrm{u}}, \alpha_{\mathrm{v}}, \alpha_2 - \sqrt{2}(x_{\mathrm{u}} + ip_{\mathrm{v}}), \ldots, \alpha_{M+1} - \sqrt{2}(x_{\mathrm{u}} + ip_{\mathrm{v}})]. \tag{5}$$

The Wigner function $W_{\mathrm{out}}(\alpha_2, \ldots, \alpha_{M+1})$ is totally symmetric with respect to all M modes. We may therefore choose an arbitrary mode and trace out (integrate out) the remaining $M-1$ modes which gives us the one-mode Wigner function of each individual "clone":

$$\mathrm{Tr}_{3\ldots M+1} W_{\mathrm{out}}(\alpha_2, \ldots, \alpha_{M+1}) = W_{\mathrm{clon}}(\alpha_2) \equiv W_{\mathrm{clon}}(\alpha). \tag{6}$$

W_{clon} is a convolution of W_{in} with a bivariate Gaussian,

$$W_{\mathrm{clon}}(x,p) = \frac{1}{2\pi\sqrt{\lambda_x\lambda_p}} \int dx'\, dp'\, W_{\mathrm{in}}(x',p') \times \exp\left[-\frac{(x-x')^2}{2\lambda_x} - \frac{(p-p')^2}{2\lambda_p}\right], \tag{7}$$

with the excess noise variances $\lambda_x = e^{2s}(M-1)/2M$ and $\lambda_p = e^{-2s}(M-1)/2M$ *independent of* θ_0 [23]. Note that in our scales, a quadrature vacuum variance is $\frac{1}{4}$, i.e., $[\hat{x}, \hat{p}] = \frac{i}{2}$. Let us now consider the fidelity $F \equiv \langle\psi_{\mathrm{in}}|\hat{\rho}_{\mathrm{clon}}|\psi_{\mathrm{in}}\rangle = \pi \int d^2\alpha\, W_{\mathrm{in}}(\alpha) W_{\mathrm{clon}}(\alpha)$ for a coherent-state or squeezed-state input with coherent amplitude $x_0 + ip_0$ and squeezing parameter s',

$$W_{\mathrm{in}}(x,p) = \frac{2}{\pi}\exp[-2e^{-2s'}(x-x_0)^2 - 2e^{2s'}(p-p_0)^2]. \tag{8}$$

Since the mean amplitude is preserved through telecloning, the fidelity does not depend on x_0 and p_0. For $s = 0$, our MQC exactly realizes optimal symmetric $1 \to M$ telecloning of coherent states ($s' = 0$), $F = F_{\mathrm{clon},1,M}^{\mathrm{coh\ st},\infty}$. Furthermore, the above protocol demonstrates that our MQC is capable of transferring *arbitrary* quantum states W_{in} simultaneously to M remote and separated receivers with equal minimum excess noise in each output mode, $\lambda_x = \lambda_p = (M-1)/2M$ for $s = 0$ (less excess noise emerging at each output for arbitrary W_{in} would imply that we could also beat the optimal-cloning limit for coherent-state inputs [16]). This observation highlights the quantum character of the MQC:

for any M, it excels the classical telecloner, because the latter creates at least two units of vacuum excess noise in each output mode ("two quduties" [7]). A compelling example is one half of an Einstein-Podolsky-Rosen (EPR) state [7] as the MQC input resulting in entanglement of the other EPR half with every single output mode. The easiest way to see this is by applying Duan *et al.*'s sufficient inseparability criterion in terms of total variances [24] for sufficiently large squeezing of the EPR state and $s = 0$.

Minimum excess noise symmetrically added in phase space does not necessarily ensure optimum telecloning *fidelities* at the outputs. It does for coherent-state inputs, but squeezed-state inputs ($s' \neq 0$) require asymmetric excess noise $s = s'$, $\lambda_x = e^{2s'}(M-1)/2M$, $\lambda_p = e^{-2s'}(M-1)/2M$, according to $F = 2/[\sqrt{(4\lambda_x e^{-4s'} + 2e^{-2s'})(4\lambda_p e^{4s'} + 2e^{2s'})}]$. Adjustment of $s = s'$ in W_{MQC} maintains optimum fidelities in a nonuniversal fashion where s' must be fixed and known. We study now how to generate the state W_{MQC}.

Let us define a sequence of ideal phase-free beam splitters acting on M modes ("M-splitter" [21]) as $\mathcal{U}(M) \equiv B_{M-1\,M}(\sin^{-1}1/\sqrt{2})B_{M-2\,M-1}(\sin^{-1}1/\sqrt{3}) \times \cdots \times B_{12}(\sin^{-1}1/\sqrt{M})$, where $B_{kl}(\theta)$ is an M-dimensional identity matrix with the entries I_{kk}, I_{kl}, I_{lk}, and I_{ll} replaced by $\sin\theta$, $\cos\theta$, $\cos\theta$, and $-\sin\theta$, respectively. A single symmetric beam splitter, for example, is described by $(\hat{c}_1', \hat{c}_2')^T = \mathcal{U}(2)(\hat{c}_1, \hat{c}_2)^T$. The recipe to build an MQC is now as follows (see Fig. 1): first produce a bipartite entangled state by combining two squeezed vacua [one squeezed in p with r_1 and the other one squeezed in x with r_2 as defined in Eqs. (3) and (4)] at a phase-free beam splitter with reflectivity/transmittance parameter $\theta = \theta_0$. Then keep one half as a "port" mode (our mode 1) and send the other half together with $M-1$ ancilla modes through an M-splitter. The ancilla modes, $\hat{b}_i' = \cosh s\hat{b}_i + \sinh s\hat{b}_i^\dagger$ with $\hat{b}_3, \hat{b}_4, \ldots, \hat{b}_{M+1}$ being vacuum modes, are either vacua $s = 0$ or squeezed vacua $s \neq 0$. In the latter case, in order to obtain W_{MQC}, the squeezing of the two inputs of the first beam splitter needs to be adjusted correspondingly, $\hat{b}_1' = \cosh(s + r_1)\hat{b}_1 + \sinh(s + r_1)\hat{b}_1^\dagger$ and $\hat{b}_2' = \cosh(s - r_2)\hat{b}_2 + \sinh(s - r_2)\hat{b}_2^\dagger$, with $\hat{b}_1$ and $\hat{b}_2$ also being vacuum modes. The circuit to generate the state W_{MQC} is then simply

$$(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{M+1})^T = \mathcal{U}(M+1)B_{12}^{-1}(\sin^{-1}1/\sqrt{M+1}) \times B_{12}(\theta_0)(\hat{b}_1', \hat{b}_2', \ldots, \hat{b}_{M+1}')^T, \tag{9}$$

where $B_{12}(\theta)$ is an $(M+1)\times(M+1)$ matrix as defined above. Note that the optimality is not affected by the particular choice of θ_0. These instructions imply that, although W_{MQC} is an entangled multimode or multiparty state, it is actually *bipartite* entanglement between mode 1 and the other M modes that makes telecloning possible [the symmetry properties of W_{MQC} in Eq. (1) underline this]. The squeezing responsible for the entanglement corresponds to $|10\log_{10}[(\sqrt{M}-1)/(\sqrt{M}+1)]|$ dB if $r_1 = r_2$ (about 7.7 dB for $M = 2$, 5.7 dB for $M = 3$, 4.8 dB for $M = 4$, and 4.2 dB for $M = 5$). In agreement with $F_{\mathrm{clon},1,M}^{\mathrm{coh\ st},\infty} \to F_{\mathrm{class}}$ for $M \to \infty$, the squeezing and hence the entanglement approach zero as M increases.

Their bipartite character is what W_{MQC} and for example the qubit telecloning state proposed by Murao *et al.* [5] have in common. However, as opposed to W_{MQC} (for $M > 1$), the qubit state contains maximum bipartite entanglement for any M. On the other hand, the qubit states are in some sense more symmetric and even more "multiuser friendly," as they are actually $2M$-partite states containing bipartite entanglement between M parties "on the left side" and M parties "on the right side." Because of this symmetry, *each* particle on *each* side can function as a port enabling the transfer of quantum information to all particles on the other side [5]. We can also construct such an MQC for continuous variables with exactly the same properties as the qubit state, but the price we have to pay is that we need infinite squeezing, i.e., maximum bipartite entanglement for any M. The corresponding $2M$-mode state is generated by first producing an infinite-squeezing EPR state [7] and then sending *both* halves each together with $M-1$ ancilla modes through an M-splitter. This MQC also enables optimal $1 \to M$ telecloning of coherent states, but instead of a fixed port mode, any mode on the left side built from the left EPR half or on the right side built from the right EPR half can now function as a port for sending quantum information to the other side. Let us emphasize the analogy between this particular continuous-variable MQC and Murao *et al.*'s qubit telecloning state [5] by displaying the former for $M = 2$ and $s = 0$ in the Schrödinger representation (position basis):

$$|\psi_{\mathrm{MQC}'}\rangle \propto \int dx\,dy\,dz\,\exp(-y^2 - z^2) \times |x+y\rangle|x-y\rangle|x+z\rangle|x-z\rangle. \tag{10}$$

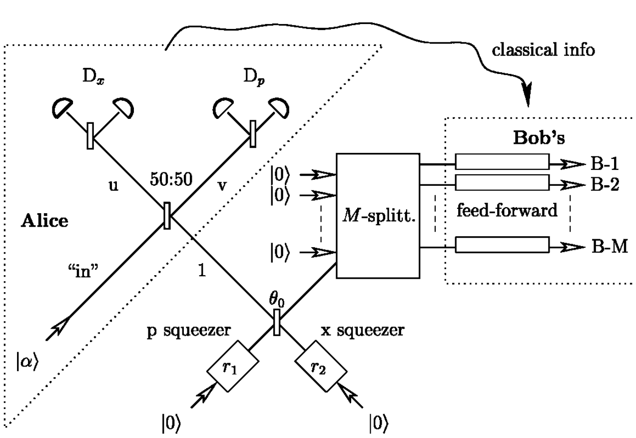

FIG. 1. Optimal telecloning of an arbitrary coherent state from Alice to M spatially separated Bob's. Alice and the Bob's share the entangled state W_{MQC}, here generated with vacuum ancilla modes $|0\rangle$ corresponding to $s = 0$ in W_{MQC}.

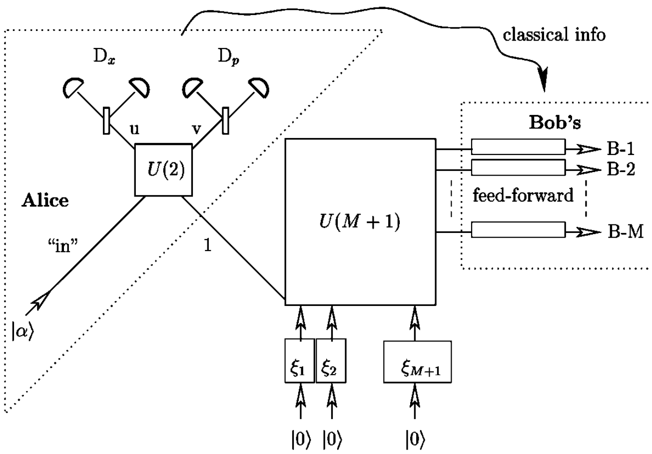

FIG. 2. Telecloning of an arbitrary coherent state from Alice to M spatially separated Bob's. Alice and the Bob's share a multipartite entangled state generated via an arbitrary quadratic interaction Hamiltonian.

Despite its nice symmetry properties, it is an unphysical (unnormalizable) state as opposed to W_{MQC} which does without infinite squeezing. Our results suggest that also for qubits (or generally qu*d*its), a less symmetric but more economical version of an MQC might exist (see also Ref. [10]).

An important question now is whether W_{MQC} is indeed the *most* economical version of an MQC. Does our telecloning protocol rely on minimal squeezing resources? At least the linear optics part, one beam splitter followed by an M-splitter, is certainly the simplest possible choice. Nevertheless, let us consider a much broader class of $(M + 1)$-mode states, namely, all multipartite entangled states that can be generated via quadratic interaction Hamiltonians (i.e., an arbitrary combination of multiport interferometers, squeezers, down-converters, etc.). This arbitrary combination may be decomposed by Bloch-Messiah reduction [25] into a set of $M + 1$ squeezers $\hat{b}'_j = \cosh\xi_j \hat{b}_j + \sinh\xi_j \hat{b}_j^\dagger$ with vacuum inputs $\hat{b}_j$, and a subsequent linear multi-port [unitary transformation $U(M + 1)$], $\vec{a} = U(M+1)\vec{b}'$ with $\vec{a} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{M+1})^T$, etc. Without loss of generality, mode $\hat{a}_1$ can be chosen as a port, and rather than assuming a phase-free symmetric beam splitter before the "Bell detection," we consider now any unitary matrix $U(2)$ acting on the input mode $\hat{a}_{\mathrm{in}}$ and $\hat{a}_1$, $(\hat{a}_{\mathrm{u}}, \hat{a}_{\mathrm{v}})^T = U(2)\,(\hat{a}_{\mathrm{in}}, \hat{a}_1)^T$ (see Fig. 2). An arbitrary unitary matrix acting on $M + 1$ modes can be decomposed into beam splitters and phase shifters as $U(M+1) = (B_{M\,M+1} \times B_{M-1\,M+1} \cdots B_{1\,M+1} B_{M-1\,M} \cdots B_{12} D)^{-1}$ [26]. Each of the $M(M+1)/2$ beam splitter operations depends on a reflectivity/transmittance parameter and a phase, $B_{kl} \equiv B_{kl}(\theta_{kl}, \phi_{kl})$, where B_{kl} is an $(M + 1)$-dimensional identity matrix with the entries I_{kk}, I_{kl}, I_{lk}, and I_{ll} replaced by $e^{i\phi_{kl}}\sin\theta_{kl}$, $e^{i\phi_{kl}}\cos\theta_{kl}$, $\cos\theta_{kl}$, and $-\sin\theta_{kl}$, respectively. Extra phase shifts are included by the diagonal matrix D with elements $e^{i\beta_1}, e^{i\beta_2}, \ldots, e^{i\beta_{M+1}}$. The entire telecloning process based on this generalization depends on $M^2 + 3M + 6$ parameters. With an optimization algorithm based on a genetic code [27], we numerically confirmed for $M = 2$ (16 parameters) and $M = 3$ (24 parameters) that W_{MQC} uses the least total squeezing. In every calculation, the optimization of coherent-state telecloning forces $M - 1$ auxiliary modes to approach vacuum and only a pair of modes to be squeezed, each mode by at least $10\log_{10}[(\sqrt{M} - 1)/(\sqrt{M} + 1)]$ dB (if equally squeezed, otherwise less squeezing in one mode is at the expense of more squeezing in the other mode, exactly as for our proposed state).

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