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Quantum Teleportation

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Abstract

Given a single copy of an unknown quantum state, it is impossible in principle to identify it. The mostly inaccessible information carried by the state is termed quantum information. In contrast to classical information, it cannot be copied or cloned. This concept provides a theoretical underpinning for all aspects of quantum communication and quantum computation. Here we use it to consider quantum teleportation.

1. Introduction

During this symposium we have heard many things about Heisenberg's life as a brilliant physicist, including his struggles with both scientific and moral questions. While preparing to give this talk I looked at information about his life and found a very surprising fact: he nearly failed his final Ph.D. orals. The difficulty arose when Wien, who was on his examining committee, moved from mathematical questions to those on experimental physics. Heisenberg was unable to derive the resolving power of such devices as microscopes and telescopes. In the end, Heisenberg received a III, equivalent perhaps to a C, as the overall grade for his doctorate. This was in 1923.

It is perhaps ironic then, that this young man had within 4 years understood the limits to resolution better than anyone else of his time. The Heisenberg microscope gedanken experiment helped lay the foundations of quantum mechanics and the uncertainty principle. This principle has re-emerged as a corner stone of our understanding of the limitations of quantum information processing. Here we will consider its role in our understanding of quantum teleportation.

2. What's in a Name

Before we begin we shall discuss the term 'teleportation' which has only recently appeared within the scientific literature [1].

Having seen sufficient science-fiction movies in my time, I can make an initial stab at defining teleportation as:

`Some kind of instantaneous 'disembodied' transport.`

Now, this is inconsistent with relativity, so let's immediately change our definition to

`Some kind of 'disembodied' transport.`

The term disembodied requires some explanation, however, we shall prefer to be vague about it: The object moves from one place to another without manifestly appearing within the intervening space.

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In the classical day-to-day world we already have many examples which could fit this definition: a fax machine transports an image as electricity; a telephone transporting sound waves; overhead projectors, etc. Should these really count as teleportation? After all they are all really copying processes. They leave the image, the sound, what-have-you behind and send the copy across space in some disembodied way. An object is scanned, the information is transmitted in a different form and a copy is reconstructed.

Let us keep away from philosophical questions at the moment. However, we note that the above procedure has intrinsic limitations. It will not work, in general, for quantum systems. Consider for example a single-photon wavepacket prepared in a state of linear polarization down to a 1° accuracy. This means that the preparation procedure requires almost 8 bits worth of polarization orientation information.

Any attempt at measuring this orientation, will yield at most one bit's worth of information. For example, simply running this photon through a polarization-dependent beamsplitter will cause the photon to be transmitted or reflected. Yielding at most one bit. This demonstrates the principle, that in general, it is impossible to accurately determine an *unknown* quantum state. This is essentially a general statement of the Heisenberg uncertainty principle for quantum systems. A consequence of this is that it is likewise impossible to perfectly copy an unknown quantum state. For if a device were capable of perfect copying, we could apply it many times to our original system, yielding a vast ensemble. Then with this ensemble we could then measure the system's state to any desired accuracy.

This latter form of the quantum limitation is today called the no-cloning theorem [2] and embodies many of the key features of Heisenberg's original principle.

3. Teleportation Protocol

If no-cloning appears to forbid a copying mechanism for performing disembodied transport, what can teleportation of an unknown quantum state be? Clearly, it cannot be a copying procedure. Nonetheless, the aim of quantum teleportation is to:

`move an unknown quantum state across a classical communication channel.`

Let us call the sender Alice and the receiver Bob. There is no in-principle restriction to the eavesdropping on a classical communication channel (classical information may be freely copied). So it would seem that another agent Bob$'$ could receive a copy of Alice's message and follow the same protocol as the real Bob. His actions would produce a copy of Alice's state! However, this cannot be, as it would violate the no-cloning theorem.

To get around this problem quantum teleportation, distinguishes the 'real Bob' as special, since only he shares one-half of a maximally entangled state with Alice, whose half is somehow used in her part of the protocol. Even if another agent were to share such a state with her, it would not participate in her actions. This 'quantum link' between Alice and Bob is sufficient to get around the limitations placed on transport through a classical communication channel.

To see how this works, we shall give the mathematics here only for the teleportation of a state from a two-dimensional Hilbert space. A key feature which allows the protocol to work is that there is, in general, a basis of maximally entangled states

$$\begin{aligned}
|\mathrm{Ent}_1\rangle &\propto |\updownarrow\leftrightarrow\rangle - |\leftrightarrow\updownarrow\rangle \\
|\mathrm{Ent}_2\rangle &\propto |\updownarrow\leftrightarrow\rangle + |\leftrightarrow\updownarrow\rangle \\
|\mathrm{Ent}_3\rangle &\propto |\updownarrow\updownarrow\rangle - |\leftrightarrow\leftrightarrow\rangle \\
|\mathrm{Ent}_4\rangle &\propto |\updownarrow\updownarrow\rangle + |\leftrightarrow\leftrightarrow\rangle\,.
\end{aligned} \tag{1}$$

Let us label the incoming state given to Alice as $|\psi\rangle_{\rm in}$ and suppose that Alice and Bob share between themselves the first of these entangled states. The total state would then be described as

$$|\Phi\rangle_{{\rm in},A,B} = |\psi\rangle_{\rm in} \otimes |{\rm Ent}_1\rangle_{A,B} \,. \tag{2}$$

Since Eqs. (1) describe a complete set for a four-dimensional Hilbert space, we may decompose the states in Alice's hand. Formally we have

$$|\Phi\rangle_{{\rm in},A,B} = \frac{1}{2}\sum_{j=1}^{4} |{\rm Ent}_j\rangle_{{\rm in},A} \otimes U_j\,|\psi\rangle_B \,, \tag{3}$$

where U_j are *fixed* unitary operators on a two-dimensional Hilbert space suitably chosen for this decomposition to hold. It is clear from the linearity of this equation that the U_j cannot depend on the state $|\psi\rangle$ itself.

Let us now suppose that Alice makes a measurement in this basis identifying her result by the associated subscripts $j_0 = 1, 2, 3, 4$. Conditioned on knowing this value the best description of the state remaining in Bob's hand is then given by

$$\to U_{j_0}\,|\psi\rangle_B \,. \tag{4}$$

If Alice now communicates this identification j_0 to Bob (requiring two bits of classical communication) then Bob knows which basis state Alice found. Based on this knowledge, he then operates $U_{j_0}^\dagger$ on his state, yielding

$$\to U_{j_0}^\dagger U_{j_0}\,|\psi\rangle_B = |\psi\rangle_B \,. \tag{5}$$

At this stage the protocol is complete and the state has been transferred between Alice and Bob. This procedure utilizes two channels: one potentially pre-existing channel of shared quantum entanglement; and one of classical communication used for Alice to transmit her measurement result to Bob. Neither by themselves contains any information about the original state. This form of transport would seem qualify as disembodied. Further, at no stage did we produce a copy of the original, nor has either Alice or Bob discovered any details about the state they participated in transporting.

4. Continuous Quantum Variables

Here we shall discuss one particular implementation of quantum teleportation based, not on states from a finite-dimensional Hilbert space, but on coherent and squeezed states of the electromagnetic field. An idealized version was first discussed by Vaidman [3]. This assumed maximally entangled states which in infinite Hilbert space dimensions corresponds to infinite energy states. Here we discuss a realizable scheme as originated by Braunstein and Kimble [4].

Optical entanglement is easily created. It turns out that almost any pure states (excluding coherent states) combined at a beamsplitter will yield entanglement. The simplest choice is of a pair of squeezed states with opposite squeezing, as shown in Fig. 1.

In this case, a pair of so-called twin beams are generated. To detect this kind of entanglement, one simply reverses the procedure: twin beams of this sort recombined at a beamsplitter resolve into unentangled squeezed states. The identity of the specific twin-beam state may be determined by measuring the 'displacement' of the position-squeezed beam and the

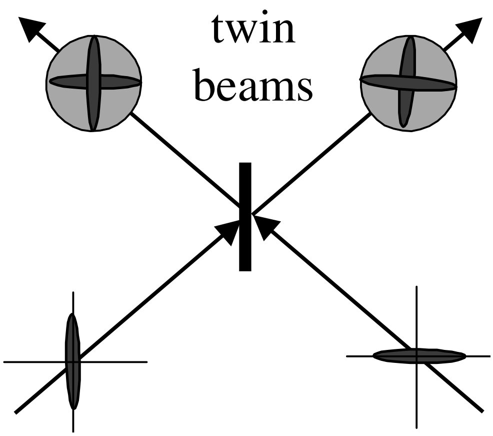

Fig. 1. Twin-beam entanglement created by combining squeezed states on a beamsplitter. Entanglement is 'resolved' into unentangled states by the time-reversed procedure.

'kick' of the momentum squeezed one. As we saw in the previous section the ability to create and detect entanglement are the key requirements in implementing teleportation. This also holds for teleportation in infinite dimensional Hilbert spaces. More details about the scheme for continuous quantum variables may be found in Ref. [4].

5. Criteria and Evidence for Teleporting Unknown Coherent States

In any realistic experiment we will not be able to consider teleporting *completely* unknown states. We will have some partial information about them. For example, we may wish to consider the teleportation of a limited class of easily constructed states, such as coherent states with a limited peak amplitude [5].

What criteria shall we test in such a circumstance? The limited set of states may be described as an alphabet of states $\{|\psi_\alpha\rangle\}$ with some associated probabilities P_α for their selection in any given run. If Alice and Bob follow a teleportation protocol, Bob hopes to have recreated the original state $|\psi_\alpha\rangle$ during each run. However, in a real experiment he will only achieve an approximate (probably mixed) state ρ_α compared to the original.

The average overlap probability

$$F = \int d\alpha\, P_\alpha \langle \psi_\alpha | \rho_\alpha | \psi_\alpha \rangle\,, \tag{6}$$

will be a measure of the performance of the teleportation. It is called the average fidelity of the teleportation.

The criteria we consider requires that this mean fidelity be better than Alice and Bob could achieve using a classical communication channel alone. Such a performance cannot be reproduced by a conventional copy-and-send strategy. Demonstrating a quantum state transfer protocol with mean fidelity better than could be achieved without entanglement guarantees that a quantum channel has been utilized. In teleportation protocols the only quantum channel is due to the entanglement.

For an alphabet of coherent states it can be shown that no more than a 50% mean fidelity is possible in the absence of shared entanglement [5]. This criteria assumes that Alice and Bob know only the *form* of the alphabet, but not the actual state being transmitted during each run. It makes no other assumption about their actions provided only that they communicate via a classical communication channel.

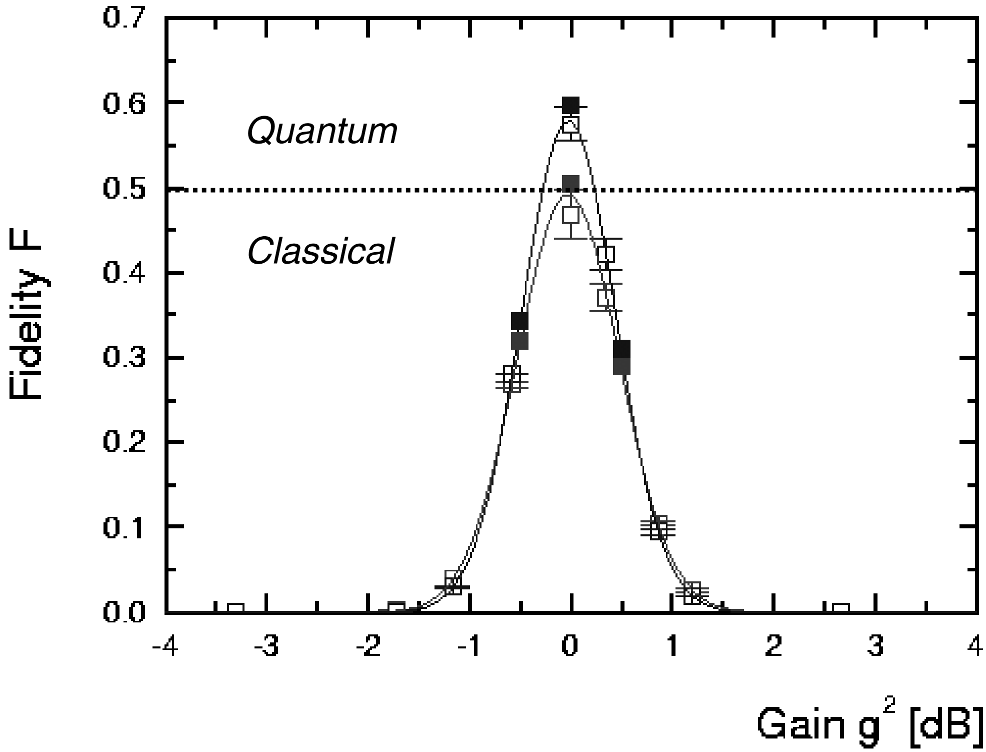

Fig. 2. Data from Ref. [6]. The lower plot shows the mean fidelity achievable in the absence of shared entanglement. Even at optimal feed-forward gain it never exceeds 50%. The upper plot shows the mean fidelity with twin-beam entanglement shared between sending and receiving stations.

Furusawa et al. implemented just such a protocol and criteria in their experimental demonstration of teleportation [6]. Data from that experiment is shown in Fig. 2. The bound of 50% mean fidelity was beaten only when sender and receiver had access to shared entanglement.

6. Conclusion

Quantum teleportation is a wholly new protocol for the transmission of quantum states. No experiments so far have demonstrated all of the features which teleportation has to offer. A preliminary list of these features reads:

- Quantum transport through a classical channel,
- Entanglement assisted transport of a quantum state,
- Many bits can be ‘sent’ via few bits,
- Can teleport to an *unknown* location,
- Can teleport half of an entangled state,
- Can teleport an entire space or filter a subspace.

Ultimately, which features are most important to achieve will depend on the applications to which teleportation is put. After all these years of our studying quantum mechanics, it still has beautiful surprises for us. There is no uncertainty about that.

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