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A quantum optical shutter

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Abstract
A time-dependent dielectric cmedium is used to model a time-varying beam-splitter inside a cavity. The time-varying boundary conditions smoothly evolve from a highly transmitting to a highly reflecting beam-splitter. These approximately correspond to a transformation from a single cavity to a pair of cavities. Quantum field-theoretic calculations show that such a smooth change yields non-singular evolution of the field. However, it predicts a production of photons up to frequencies comparable to the rate of change of the transition. We find that a time-varying beam-splitter operating at optical frequencies would produce an observable number of photons.

Keywords: time-varying dielectric, beam-splitter, optical shutter

In this paper we deal with a model problem in a second-quantized field theory: an ideal laser cavity with a variable reflectivity mirror at its centre. Such a partial mirror is called a beam-splitter. Absorption is neglected. We are interested in changing the reflectivity of the central 'partition' in order to study the evolution from a single cavity to a pair of disconnected cavities. The outer mirrors remain perfectly reflecting.

We start by reviewing the quantum theory of static beam-splitters and give a heuristic explanation for why time-varying beam-splitters might be expected to yield photon production. Then we consider a field-theoretic formulation of a time-varying beam-splitter.

Static beam-splitters

Earlier treatments of the quantized time-stationary beam-splitter [1–5] all rely on the behaviour of the classical, time-stationary beam-splitter [6]. In this case, the beam-splitter is a two-input, two-output device that obeys relations like

$$\begin{aligned} \hat{a}^{\rm out}(t) &= \cos\theta\, \hat{a}^{\rm in}(t) + {\rm e}^{{\rm i}\varphi} \sin\theta\, \hat{b}^{\rm in}(t) \\ \hat{b}^{\rm out}(t) &= \cos\theta\, \hat{b}^{\rm in}(t) - {\rm e}^{-{\rm i}\varphi} \sin\theta\, \hat{a}^{\rm in}(t), \end{aligned} \tag{1}$$

where $\hat{a}$ and $\hat{b}$ are the annihilation operators of the incoming and outgoing modes, t denotes time and where the transmission and reflection amplitude coefficients are $\cos\theta$ and ${\rm e}^{{\rm i}\varphi}\sin\theta$, respectively. One of these papers [4] introduces an inner product to confirm that the modes are orthogonal in order to derive the above input–output relations (or so-called Bogoliubov transformations), though their choice of inner product appears to be somewhat ad hoc.

In this paper we seek to incorporate dynamics into the beam-splitter element by allowing its transmission and reflection coefficients to change in time in some prescribed manner [7]. We show that the above equations are no longer sufficient as they stand. One effect of these time-varying boundary conditions is particle creation.

A heuristic way to see that particle creation might result from a time-varying beam-splitter is to consider the effect of a time-varying reflection coefficient on equation (1). The simplest time-dependence we might add is harmonic

$$\theta(t) = \Omega t.$$

Decomposing the output modes into their frequency components, we find two predominant effects: each incoming frequency ω produces a pair of sidebands at $\omega \pm \Omega$; negative frequency contributions appear when $\omega < \Omega$. The former effect is the expected sideband modulation from the time-varying transmission. The latter is a purely quantum effect. By making the replacement

$$\hat{a}(-\omega) \to \hat{a}^{\dagger}(\omega),$$

the creation of quanta is expected for frequencies up to the beam-splitter modulation frequency. We confirm this qualitative behaviour for our model of a time-varying beam-splitter.

Time-varying beam-splitter

There are numerous ways to construct a variable beam-splitter in the laboratory. Our model is an infinitely thin slab of dielectric, for which we assume full control over the dielectric permittivity. Consider the 'electromagnetic' field Lagrangian density in one spatial dimension:

$$\mathcal{L} = \frac{1}{2}\left[\varepsilon(x,t)\left(\frac{\partial\phi}{\partial t}\right)^2 - \left(\frac{\partial\phi}{\partial x}\right)^2\right],$$

with boundary conditions $\phi(x = \pm L, t) = 0$. Here $\varepsilon(x,t)$ is the time-dependent, inhomogeneous dielectric permittivity and $\phi(x,t)$ is the scalar analogue of the vector potential in one spatial dimension. For convenience, units were chosen so that the magnetic permeability and the speed of light are unity.

Several people have studied quantum fields with time-dependent metrics, boundaries or dielectrics [8–26]. In our field-theoretic calculations we follow Law's 'frozen mode' formulation [16]. The exact equations of motion are linear:

$$\frac{\partial}{\partial t}\left(\varepsilon(x,t)\frac{\partial\phi}{\partial t}\right) - \frac{\partial^2\phi}{\partial x^2} = 0, \tag{2}$$

with $\phi(x = \pm L, t) = 0$. Canonical quantization of the field $\hat{\phi}(x,t)$ proceeds by enforcing the standard equal-time commutation relations with conjugate momentum $\hat{\pi}(x,t) \equiv \varepsilon(x,t)\,\partial\hat{\phi}(x,t)/\partial t$. At each instant in time the quantum field $\hat{\phi}(x,t)$ can be decomposed into the 'instantaneous' modes satisfying the eigenvalue equation

$$\left(\frac{\partial^2}{\partial x^2} + \varepsilon(x,t)\omega_k^2(t)\right)U_k^{(t)}(x) = 0,$$

with $U_k^{(t)}(x = \pm L) = 0$ and the frequencies $\omega_k(t)$ chosen to satisfy the instantaneous boundary conditions [16]. The time t parametrizes the set of mode functions. In accordance with the Sturm–Liouville theory, let these modes be represented as a complete orthonormal set for any 'time' t, satisfying

$$\int_{-L}^{L} \mathrm{d}x\; \varepsilon(x,t)\, U_k^{(t)}(x)\, U_{k'}^{(t)}(x) = \delta_{kk'}.$$

Using these instantaneous modes we may define an instantaneous 'annihilation operator'

$$\hat{a}_k \equiv \int_{-L}^{L} \mathrm{d}x\; U_k^{(t)}(x) \times \left(\sqrt{\frac{\omega_k(t)}{2}}\,\varepsilon(x,t)\,\hat{\phi}(x,t) + \frac{\mathrm{i}}{\sqrt{2\omega_k(t)}}\hat{\pi}(x,t)\right),$$

which, by virtue of the equal-time commutation relations for $\hat{\phi}$ and $\hat{\pi}$, satisfies the canonical commutation relations

$$[\hat{a}_k, \hat{a}_{k'}^{\dagger}] = \delta_{kk'}.$$

When $\varepsilon(x,t)$ is constant in time, these annihilation operators and their associated instantaneous modes take on the conventional interpretation.

The equations of motion for these operators may be derived from the field equation (2) and the above decomposition, yielding

$$\frac{\partial\hat{a}_k}{\partial t} = -\mathrm{i}\,\omega_k(t)\,\hat{a}_k + \frac{1}{2}\frac{\partial\ln\omega_k(t)}{\partial t}\hat{a}_k^{\dagger} + \tfrac{1}{2}\sum_{k'}[(\mu_{kk'} + \mu_{k'k})\,\hat{a}_{k'}^{\dagger} + (\mu_{kk'} - \mu_{k'k})\,\hat{a}_{k'}], \tag{3}$$

with

$$\mu_{jk}(t) \equiv -\sqrt{\frac{\omega_j(t)}{\omega_k(t)}}\int_{-L}^{L}\mathrm{d}x\;\varepsilon(x,t)\,U_j^{(t)}(x)\,\frac{\partial}{\partial t}U_k^{(t)}(x),$$

in agreement with Law's effective Hamiltonian [16]. It should be noted that no approximations have been made and that this represents an exact solution to the equations of motion equation (2). Finally, the physical occupation number for each mode is readily accessible whenever $\mathrm{d}\varepsilon(x,t)/\mathrm{d}t = 0$ with

$$\langle\hat{n}_k(t)\rangle = \langle\hat{a}_k^{\dagger}(t)\,\hat{a}_k(t)\rangle, \tag{4}$$

in the Heisenberg picture.

To apply these results to our beam-splitter model let $\varepsilon(x,t) = 1 + \kappa(t)\,\delta(x)$, where $\kappa(t)$ represents the integrated permittivity in the thin dielectric slab. The thickness of the slab is taken to be zero to avoid multiple reflections from its surfaces. This approximation greatly simplifies our calculations and allows us to concentrate on the effects of the time-varying boundary conditions. Because of the symmetry of our cavity, the instantaneous mode functions $U_k(x)$ may be written as

$$u_n(x) = \frac{\sin[\omega_n(L - |x|)]}{\sqrt{L + \kappa\sin^2(\omega_n L)/2}}$$

$$v_m(x) = \sin(m\pi x/L)/\sqrt{L},$$

in terms of the symmetric and antisymmetric modes, respectively, with

$$\tan(\omega_n L) = \frac{2}{\omega_n\kappa},$$

following from satisfying the instantaneous boundary conditions. Since the antisymmetric modes always satisfy the instantaneous boundary conditions they are not affected by changes in κ. We henceforth drop them from our discussions. We shall suppose that the integrated permittivity undergoes a smooth transition between κ_0 and κ_1 chosen as

$$\kappa(t) \equiv \begin{cases} \kappa_0, & t \leqslant 0 \\ \kappa_0 + (\kappa_1 - \kappa_0)\sin^2(\Omega t), & 0 < t < \pi/2\Omega \\ \kappa_1, & \pi/2\Omega \leqslant t, \end{cases}$$

where Ω represents the rate of transition. A very similar model is studied by Crocce *et al* [26], though with a periodically time-varying permittivity instead of our idealized model of a shutter smoothly 'opening' or 'closing.'

Numerical integration of equation (3) and use of equation (4) yields the mean number of quanta created from the vacuum by opening or closing the central partition. Figure 1 shows the mean number of quanta per mode created in this manner for integrated permittivities κ ranging between 0.1 and 10. Five hundred field modes were used for the numerical integration. We take the (half) cavity size as $L = 1$ and calculate $\langle\hat{n}_m(\pi/2\Omega)\rangle$ for two frequencies $\Omega = 1$ and 10. These plots show that a tenfold increase in the opening or

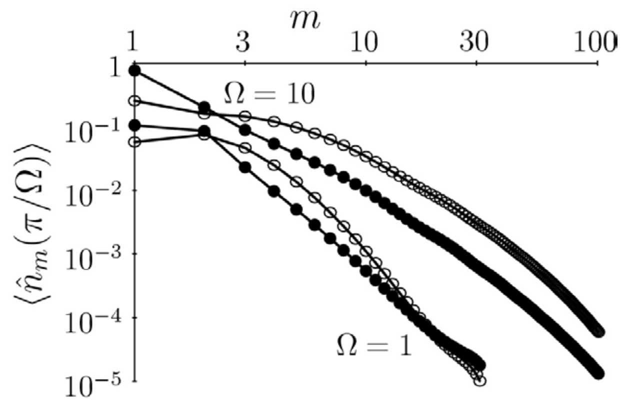

Figure 1. Log–log plots of the mean number of quanta created from vacuum versus mode number (starting arbitrarily with $m = 1$) for transitions in the integrated permittivity κ between 0.1 and 10 for fixed cavity size $L = 1$. The plots with open circles denote 'opening' the central partition with $\kappa_0 = 10$ and $\kappa_1 = 0.1$; those with closed circles denote 'closing' it with $\kappa_0 = 0.1$ and $\kappa_1 = 10$. The lower pair of plots was computed for $\Omega = 1$ and the upper pair for $\Omega = 10$.

closing rate roughly corresponds to a tenfold increase in energy production. It is difficult to extend these numerical calculations to the limit $\kappa \to 0$ (opening) because successively more modes must be included. By contrast, the limit $\kappa \to \infty$ (closing) appears to present no special problems.

Yablonovitch [10], Hizhnyakov [13] and Mendonça *et al* [25] have investigated the quantum emission in an infinite medium with a time-dependent refractive index using Maxwell's equations. They interpreted this infinite-medium model as a non-linear optical equivalent of an accelerating mirror. In their calculations particle creation comes from the properties of the bulk medium rather than from boundary conditions. By contrast, we have studied the opposite limit of an infinitely thin time-varying dielectric. We are tempted to interpret this as a model of the time-varying boundary conditions such as one might expect in a time-varying topology. For these boundary conditions to be satisfied, a smooth change of the dielectric permittivity must be imposed; instantaneous changes in the boundary conditions would result in divergences[1] that are directly analogous to those found in quantum fields propagating on a classical topology change [27–30]. We have found that smooth time-varying boundary conditions can produce a finite number of quanta if the boundary conditions change at the amplitude level.

Fearn *et al* [31] have considered the sudden replacement of a cavity mirror by a detector. Their analysis was semi-classical in that the mirror boundary conditions were treated heuristically, glueing together the dynamics of a closed cavity to that of an open cavity without ensuring self-consistent behaviour of the electromagnetic field in response to such instantaneous changes. They showed that a detector placed outside the cavity could immediately detect photons if the cavity had an 'excited' atom in it. Their heuristic explanation for this was that the excited atoms are actually dressed and that the excitation is shared between atom and cavity mode which immediately begins propagating away once the cavity is opened. By comparison to their completely classical treatment of the field's time-varying boundary conditions, in our model removal of a cavity end-mirror on optical timescales could produce a detectable number of photons even with an initially empty cavity in vacuum. The sudden removal of a mirror (which is not possible in our model) might be approximated if the mirrors are good reflectors only within a finite bandwidth, implying a natural cut-off.

To conclude, we have studied the quantization of a field-theoretic model for a time-varying beam-splitter based upon a time-varying permittivity within a narrow dielectric. We found that quanta may be generated with frequencies up to the inverse timescale required to shift between highly transmitting and highly reflecting limits. An observable number of photons could be produced if such changes occurred on optical timescales.

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[1] The key point is that an instantaneous change of boundary condition will, in general, enforce an infinite gradient on the field which will introduce an infinite energy. For example, suppose we had a smooth field $\phi(x, t)$ at time t. If we were to introduce a perfectly reflecting boundary instantaneously at $x = x_0$ the instantaneous new solution would be $\phi(x, t)|2\Theta(x - x_0) - 1|$, where $\Theta(x)$ is the Heaviside–Lorentz step function. However, the infinite gradient at $x = x_0$ would introduce an infinite energy via the term $(\partial\phi(x, t)/\partial x)^2$ and hence result in divergences.

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