PHYSICAL REVIEW A VOLUME 43, NUMBER 1 1 JANUARY 1991

Fundamental limits to observations of squeezing via balanced homodyne detection

Samuel L. Braunstein
Research House, 2639 East Fourth Street, Tucson, Arizona 85716-4417

David D. Crouch
Joint Institute for Laboratory Astrophysics, University of Colorado and National Institute of Standards and Technology Boulder, Colorado 80309-0440

(Received 15 June 1990)

A photodetector's electronic bandwidth is the frequency range over which beats between different frequencies in the input can be observed in the output. The output of a balanced homodyne detector, the prototypical device for detecting squeezed light, consists of beats between a signal to be measured and a local oscillator. Beats between the signal and quantum local-oscillator fluctuations lead to noise that degrades the observed squeezing. Through wide-band calculations, we show that this noise contains contributions from the *entire* signal bandwidth, even when this bandwidth is orders of magnitude larger than the detector's electronic bandwidth. This allows us to place an upper limit on the total power that can enter the detector's signal port (both signal and extraneous light) within the detector's optical bandwidth in order for a given level of squeezing to be observable.

I. INTRODUCTION

The balanced homodyne detector[1] is the prototypical device for the measurement of squeezing.[2,3] In this paper, we examine the effect of local-oscillator (LO) fluctuations on measurements of squeezing by such detectors. Even if the LO is devoid of classical fluctuations, quantum fluctuations remain and lead to noise in the output that can limit the observable squeezing. Several authors[1,4,5] have studied this effect for a single signal mode; they miss, however, an important feature that emerges when the signal has a large spectral bandwidth.

There are two relevant bandwidths to consider in our simplified treatment of balanced homodyne detection: the photodetector's electronic bandwidth—the bandwidth over which beats between different frequencies in the incident fields can be seen in the output photocurrent—and the spectral bandwidth of the signal. We show that quantum LO fluctuations produce noise in the output of a balanced homodyne detector, and that the entire signal bandwidth contributes to this noise, even when the signal's spectral bandwidth is orders of magnitude larger than the detector's electronic bandwidth.

We are interested in the broadband nature of the detector's response to a broadband signal. Thus, in Sec. II, we review the formalism required for a description of wide-band squeezed-state light. Balanced homodyne detection is analyzed in Sec. III, where we calculate the spectrum of the output of such a detector, and demonstrate that quantum LO fluctuations add noise to the output that may limit the detectable squeezing. We discuss the limits to the detection of squeezing in Sec. IV in the context of two specific examples, narrow-band squeezed light, as produced by an ideal optical parametric oscillator, and wide-band squeezed light. Our analysis applies equally well to general signals as to the squeezed signals from which we derived our fundamental limits. Our result shows that in order to see a given amount of squeezing an upper limit is placed on the ratio of the total signal power to the local-oscillator power, even when the signal power has contributions from frequencies far outside the electronic bandwidth but still inside the optical bandwidth of the detector.

II. A WIDE-BAND DESCRIPTION OF SQUEEZED-STATE LIGHT

The operators most relevant to a wide-band description of squeezed-state light are the quadrature phases and their Fourier components, the quadrature-phase amplitudes.[6,7] From the second-order noise moments of the quadrature-phase amplitudes arise the elements of the spectral-density matrix,[7] which contains spectral information about the quadrature-phase noise. In this section, we define the quadrature phases, the quadrature-phase amplitudes, and the spectral-density matrix. We also calculate the elements of the spectral-density matrix for squeezed light produced by a generic nonlinear device through a unitary transformation of an input vacuum field.

We approximate the signal by a nearly monochromatic, nearly plane-wave electromagnetic field with a single polarization. Such a signal traveling through free space in the positive-z direction can be described by the positive-frequency part of the electric-field operator

$$\hat{E}^{(+)}(z,t) = [\hat{E}^{(-)}(z,t)]^\dagger = \int_S d\omega \frac{1}{2\pi}\left(\frac{2\pi\hbar\omega}{c\sigma}\right)^{1/2} \hat{a}(\omega)\exp[-i\omega(t-z/c)]\,, \quad (2.1)$$

where the integral runs over a band S of relevant signal frequencies, and σ is the field's effective cross-sectional area, which crudely accounts for its transverse structure. The operators $\hat{a}(\omega)$ and $\hat{a}^\dagger(\omega)$ are annihilation and creation operators for the field; they satisfy the commutation relations

$$[\hat{a}(\omega),\hat{a}(\omega')]=0\ ,\quad [\hat{a}(\omega),\hat{a}^\dagger(\omega')]=2\pi\delta(\omega-\omega')\ . \tag{2.2}$$

We assume that the band S is centered on some optical frequency Ω, and we let $\omega=\Omega\pm\epsilon$, where ϵ is positive by definition and $\epsilon<\Omega$. The electric-field operator $\hat{E}=\hat{E}^{(+)}+\hat{E}^{(-)}$ can then be expressed as

$$\hat{E}(z,t)=\hat{E}_1(\phi;z,t)\cos[\Omega(t-z/c)-\phi]+\hat{E}_2(\phi;z,t)\sin[\Omega(t-z/c)-\phi]\ , \tag{2.3}$$

where $\hat{E}_1(\phi;z,t)$ and $\hat{E}_2(\phi;z,t)$ are the quadrature phases of the electric field relative to the frequency Ω. The quadrature phases, in turn, are given by

$$\begin{aligned}\hat{E}_1(\phi;z,t)&=\left(\frac{8\pi\hbar\Omega}{c\sigma}\right)^{1/2}\int_S d\epsilon\frac{1}{2\pi}\left[\hat{\alpha}_1(\phi;\epsilon)e^{-i\epsilon(t-z/c)}+\hat{\alpha}_1^\dagger(\phi;\epsilon)e^{i\epsilon(t-z/c)}\right]\ ,\\ \hat{E}_2(\phi;z,t)&=\left(\frac{8\pi\hbar\Omega}{c\sigma}\right)^{1/2}\int_S d\epsilon\frac{1}{2\pi}\left[\hat{\alpha}_2(\phi;\epsilon)e^{-i\epsilon(t-z/c)}+\hat{\alpha}_2^\dagger(\phi;\epsilon)e^{i\epsilon(t-z/c)}\right]\ ,\end{aligned} \tag{2.4}$$

where the Fourier components $\hat{\alpha}_1(\phi;\epsilon)$ of $\hat{E}_1(\phi;z,t)$ and $\hat{\alpha}_2(\phi;\epsilon)$ of $\hat{E}_2(\phi;z,t)$ are the quadrature-phase amplitudes:

$$\begin{aligned}\hat{\alpha}_1(\phi;\epsilon)&=\frac{1}{2}\left[\left(1+\frac{\epsilon}{\Omega}\right)^{1/2}\hat{a}(\Omega+\epsilon)e^{-i\phi}+\left(1-\frac{\epsilon}{\Omega}\right)^{1/2}\hat{a}^\dagger(\Omega-\epsilon)e^{i\phi}\right]\ ,\\ \hat{\alpha}_2(\phi;\epsilon)&=-\frac{i}{2}\left[\left(1+\frac{\epsilon}{\Omega}\right)^{1/2}\hat{a}(\Omega+\epsilon)e^{-i\phi}-\left(1-\frac{\epsilon}{\Omega}\right)^{1/2}\hat{a}^\dagger(\Omega-\epsilon)e^{i\phi}\right]\ .\end{aligned} \tag{2.5}$$

Here ϕ is an arbitrary reference phase. The quadrature-phase amplitudes obey the following commutation relations:

$$\begin{aligned}&[\hat{\alpha}_1(\phi;\epsilon),\hat{\alpha}_1(\phi;\epsilon')]=[\hat{\alpha}_2(\phi;\epsilon),\hat{\alpha}_2(\phi;\epsilon')]=[\hat{\alpha}_1(\phi;\epsilon),\hat{\alpha}_2(\phi;\epsilon')]=0,\\ &[\hat{\alpha}_1(\phi;\epsilon),\hat{\alpha}_1^\dagger(\phi;\epsilon')]=[\hat{\alpha}_2(\phi;\epsilon),\hat{\alpha}_2^\dagger(\phi;\epsilon')]=\frac{\epsilon}{2\Omega}2\pi\delta(\epsilon-\epsilon')\ ,\\ &[\hat{\alpha}_1(\phi;\epsilon),\hat{\alpha}_2^\dagger(\phi;\epsilon')]=[\hat{\alpha}_1^\dagger(\phi;\epsilon),\hat{\alpha}_2(\phi;\epsilon')]=\frac{i}{2}2\pi\delta(\epsilon-\epsilon')\ .\end{aligned} \tag{2.6}$$

These commutation relations constrain the second-order noise moments of the quadrature-phase amplitudes, and thus the elements of the spectral-density matrix, which will now be defined.

The autocorrelation function for the quadrature phases is given by

$$\langle\Delta\hat{E}_m(\phi;z,t)\Delta\hat{E}_n(\phi;z,t+\tau)\rangle_{\rm sym}=\frac{4\hbar\Omega}{c\sigma}\int_S d\epsilon\,{\rm Re}[S_{mn}(\phi;\epsilon)e^{-i\epsilon\tau}],\quad m,n=1,2 \tag{2.7}$$

where we have assumed

$$\begin{aligned}&\langle\Delta\hat{\alpha}_m(\phi;\epsilon)\Delta\hat{\alpha}_n(\phi;\epsilon')\rangle_{\rm sym}=0\ ,\\ &\langle\Delta\hat{\alpha}_m^\dagger(\phi;\epsilon)\Delta\hat{\alpha}_n(\phi;\epsilon')\rangle_{\rm sym}=\pi S_{mn}(\phi;\epsilon)\delta(\epsilon-\epsilon')\ .\end{aligned} \tag{2.8}$$

Here, for any operators $\hat{A}$ and $\hat{B}$, $\Delta\hat{A}\equiv\hat{A}-\langle\hat{A}\rangle$ and $\langle\hat{A}\hat{B}\rangle_{\rm sym}\equiv\frac{1}{2}\langle\hat{A}\hat{B}+\hat{B}\hat{A}\rangle$. Equations (2.8) characterize *time-stationary quadrature-phase noise*,[7] i.e., they guarantee that Eq. (2.7) is independent of the time t. Equations (2.8) define the elements $S_{mn}(\phi;\epsilon)$ of the spectral-density matrix, which contains the spectral information about the quadrature-phase noise. For vacuum fluctuations,

$$S_{11}(\phi;\epsilon)=S_{22}(\phi;\epsilon)=\tfrac{1}{2}\ ;$$

the signal is squeezed when

$$\min[S_{11}(\phi;\epsilon),S_{22}(\phi;\epsilon)]<\tfrac{1}{2}\ , \tag{2.9}$$

for some ϕ. The commutation relations in Eq. (2.6) place the following constraints on the elements of the spectral-density matrix:[7,8]

$$S_{11}(\phi;\epsilon), S_{22}(\phi;\epsilon) \geq \frac{\epsilon}{2\Omega}, \qquad (2.10)$$

$$S_{11}(\phi;\epsilon) S_{22}(\phi;\epsilon) \geq \tfrac{1}{4}. \qquad (2.11)$$

Equation (2.10) imposes minimum values on the diagonal elements of the spectral-density matrix, limiting the degree of squeezing possible at any frequency ϵ. Equation (2.11) is an uncertainty relation for the spectral densities of the two quadratures, and states that the price one pays for squeezed fluctuations in one quadrature is increased fluctuation in the orthogonal quadrature.

The source of the signal is a nonlinear optical device (an optical parametric oscillator, for example) that performs a unitary transformation on the annihilation and creation operators $\hat{b}(\omega)$ and $\hat{b}^\dagger(\omega)$ of the input field, yielding the output operators

$$\hat{a}(\Omega+\epsilon) = M(\Omega+\epsilon)\hat{b}(\Omega+\epsilon) + N(\Omega+\epsilon)\hat{b}^\dagger(\Omega-\epsilon),$$

$$\hat{a}^\dagger(\Omega-\epsilon) = M^*(\Omega-\epsilon)\hat{b}^\dagger(\Omega-\epsilon) + N^*(\Omega-\epsilon)\hat{b}(\Omega+\epsilon). \qquad (2.12)$$

Both $\hat{a}(\omega)$ and $\hat{b}(\omega)$ must satisfy Eq. (2.2); as a result, M and N are constrained in such a way that

$$|M(\Omega\pm\epsilon)|^2 - |N(\Omega\pm\epsilon)|^2 = 1, \qquad (2.13)$$

$$M(\Omega+\epsilon)N(\Omega-\epsilon) = M(\Omega-\epsilon)N(\Omega+\epsilon). \qquad (2.14)$$

By taking

$$M(\Omega\pm\epsilon) = |M(\Omega\pm\epsilon)|\exp[i\theta_M(\Omega\pm\epsilon)]$$

and

$$N(\Omega\pm\epsilon) = |N(\Omega\pm\epsilon)|\exp[i\theta_N(\Omega\pm\epsilon)],$$

we find upon combining Eqs. (2.13) and (2.14) that

$$|M(\Omega+\epsilon)| = |M(\Omega-\epsilon)| \equiv |M(\epsilon)|,$$

$$|N(\Omega+\epsilon)| = |N(\Omega-\epsilon)| \equiv |N(\epsilon)|, \qquad (2.15)$$

and

$$\theta_M(\Omega+\epsilon)+\theta_N(\Omega-\epsilon) = \theta_M(\Omega-\epsilon)+\theta_N(\Omega+\epsilon) \equiv \theta_{MN}(\epsilon). \qquad (2.16)$$

It is then straightforward to show that the elements of the spectral-density matrix resulting from the transformation in Eq. (2.12) are[6]

$$S_{11}(\phi;\epsilon) = \frac{1}{2}\left[|M(\epsilon)|^2 + |N(\epsilon)|^2 + 2\left(1-\frac{\epsilon^2}{\Omega^2}\right)^{1/2}|M(\epsilon)||N(\epsilon)|\cos[\theta_{MN}(\epsilon)-2\phi]\right],$$

$$S_{22}(\phi;\epsilon) = \frac{1}{2}\left[|M(\epsilon)|^2 + |N(\epsilon)|^2 - 2\left(1-\frac{\epsilon^2}{\Omega^2}\right)^{1/2}|M(\epsilon)||N(\epsilon)|\cos[\theta_{MN}(\epsilon)-2\phi]\right], \qquad (2.17)$$

$$\begin{aligned} S_{12}(\phi;\epsilon) &= S_{21}^*(\phi;\epsilon) \\ &= \left(1-\frac{\epsilon^2}{\Omega^2}\right)^{1/2}|M(\epsilon)|\,|N(\epsilon)|\sin[\theta_{MN}(\epsilon)-2\phi] - i\frac{\epsilon}{2\Omega}[|M(\epsilon)|^2+|N(\epsilon)|^2]. \end{aligned}$$

We can express the power in the signal described by Eq. (2.12) in terms of the diagonal elements of the spectral-density matrix, Eq. (2.17):

$$\begin{aligned} P_S = \frac{c\sigma}{2\pi}\langle \hat{E}^{(-)}(z,t)\hat{E}^{(+)}(z,t)\rangle &= \hbar\Omega\int_S d\epsilon\frac{1}{2\pi}[S_{11}(\phi;\epsilon)+S_{22}(\phi;\epsilon)-1] \\ &= \hbar\Omega\int_S d\epsilon\frac{1}{2\pi}[|M(\epsilon)|^2+|N(\epsilon)|^2-1]. \end{aligned} \qquad (2.18)$$

We will find this result to be useful in our subsequent discussion of balanced homodyne detection.

III. BALANCED HOMODYNE DETECTION

A phase-sensitive detection scheme is needed to measure the phase dependence of the fluctuations in a beam of squeezed light; a balanced homodyne detector,[1] shown in Fig. 1, is preferred for this purpose. The signal beam $\hat{E}_S(z,t)$ is combined with a local-oscillator beam $\hat{E}_{\rm LO}(z,t)$ by a 50-50 beam splitter, whose outputs fall on the identical photodetectors D_1 and D_2; the output current $\hat{I}_D(t)$ is obtained by subtracting the output $\hat{I}_2(t)$ of D_2 from the output $\hat{I}_1(t)$ of D_1.

The outputs of the 50-50 beam splitter are

$$\hat{E}_h(z,t) = \frac{1}{\sqrt{2}}[\hat{E}_{\rm LO}(z,t)+\hat{E}_S(z,t)],$$

$$\hat{E}_v(z,t) = \frac{1}{\sqrt{2}}[\hat{E}_{\rm LO}(z,t)-\hat{E}_S(z,t)]. \qquad (3.1)$$

The fields $\hat{E}_h(z,t)$ and $\hat{E}_v(z,t)$ fall on the photodetectors D_1 and D_2, respectively, which we assume are located at $z=0$.

There is an unresolved question[6,8–12] as to whether a photodetector responds to the energy flux or to the number flux of photons. If the former assumption holds then our analysis of Sec. II applies directly. If the latter assumption holds, however, we could simply drop all the $\sqrt{\Omega}$ corrections (i.e., the terms proportional to ϵ) throughout Sec. II and the remaining broadband analysis would apply. In fact, we shall see that our results are reasonably insensitive to this assumption.

Here we follow the treatment of Glauber[13] by assuming that the photodetectors are bolometers, i.e., they respond to energy flux so that $\hat{I}(t) \propto \hat{E}^{(-)}(t)\hat{E}^{(+)}(t)$. With this assumption, we find that the photocurrent operators are

$$\begin{aligned}\hat{I}_1(t) &= \frac{ec\sigma_D}{2\pi\hbar\Omega}\hat{E}_h^{(-)}(t)\hat{E}_h^{(+)}(t)\\ &= \frac{ec\sigma_D}{4\pi\hbar\Omega}[\hat{E}_{\rm LO}^{(-)}(t)\hat{E}_{\rm LO}^{(+)}(t)+\hat{E}_{\rm LO}^{(-)}(t)\hat{E}_S^{(+)}(t)+\hat{E}_S^{(-)}(t)\hat{E}_{\rm LO}^{(+)}(t)+\hat{E}_S^{(-)}(t)\hat{E}_S^{(+)}(t)]\ ,\end{aligned} \tag{3.2}$$

$$\begin{aligned}\hat{I}_2(t) &= \frac{ec\sigma_D}{2\pi\hbar\Omega}\hat{E}_v^{(-)}(t)\hat{E}_v^{(+)}(t)\\ &= \frac{ec\sigma_D}{4\pi\hbar\Omega}[\hat{E}_{\rm LO}^{(-)}(t)\hat{E}_{\rm LO}^{(+)}(t)-\hat{E}_{\rm LO}^{(-)}(t)\hat{E}_S^{(+)}(t)-\hat{E}_S^{(-)}(t)\hat{E}_{\rm LO}^{(+)}(t)+\hat{E}_S^{(-)}(t)\hat{E}_S^{(+)}(t)]\ ,\end{aligned}$$

where the z-dependence of the field operators has been suppressed, and we have neglected mode mismatching between the photodetectors and the fields. The differenced photocurrent operator $\hat{I}_D(t)$ is then

$$\begin{aligned}\hat{I}_D(t) &\equiv \hat{I}_1(t)-\hat{I}_2(t)\\ &= \frac{ec\sigma_D}{2\pi\hbar\Omega}[\hat{E}_{\rm LO}^{(-)}(t)\hat{E}_S^{(+)}(t)+\hat{E}_S^{(-)}(t)\hat{E}_{\rm LO}^{(+)}(t)]\ .\end{aligned} \tag{3.3}$$

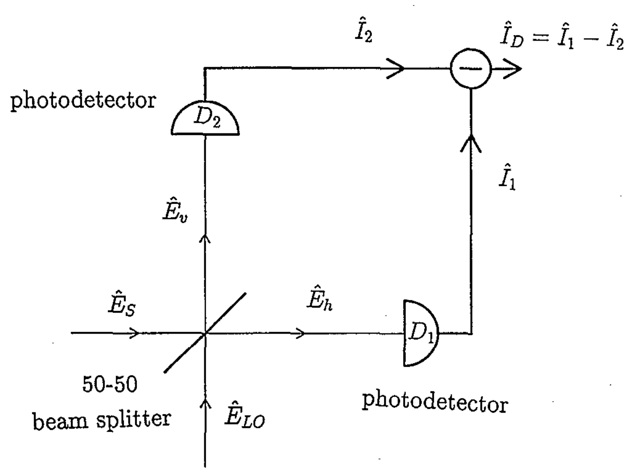

FIG. 1. Schematic diagram of an ideal balanced homodyne detector. The signal field $\hat{E}_S(t)$ combines with the local oscillator $\hat{E}_{\rm LO}(t)$ at a 50-50 beam splitter to produce sum and difference fields at photodetectors D_1 and D_2, respectively. In any counting period, these ideal photodetectors produce photocurrents proportional to the number of incident quanta. The output of the device is the difference photocurrent $\hat{I}_D(t)=\hat{I}_1(t)-\hat{I}_2(t)$, which is then fed into a spectrum analyzer.

Here e is the electronic charge, σ_D is the photodetector's effective cross-sectional area, and Ω is the LO frequency.

We assume that all classical noise has been removed from the LO, leaving only the quantum fluctuations. The positive-frequency part of the LO electric-field operator is modeled by

$$\begin{aligned}\hat{E}_{\rm LO}^{(+)}(z,t) = \int d\omega \frac{1}{2\pi}\left(\frac{2\pi\hbar\Omega}{c\sigma}\right)^{1/2}\hat{b}(\omega)\\ \times \exp[-i\omega(t-z/c)]\ ,\end{aligned} \tag{3.4}$$

where

$$\hat{b}(\omega) = \langle\hat{b}(\omega)\rangle + \Delta\hat{b}(\omega) \tag{3.5}$$

and

$$\langle\hat{b}(\omega)\rangle = A\exp(i\phi_{\rm LO})2\pi\delta(\omega-\Omega)\ , \tag{3.6}$$

which represents an ideal narrow-band laser. We can then define quadrature phases and quadrature-phase amplitudes for the LO using Eqs. (2.4) and (2.5) by replacing $\hat{a}(\omega)$ by $\Delta\hat{b}(\omega)$ and ϕ by $\phi_{\rm LO}$. The LO electric-field operator can then be expressed in terms of the LO quadrature phases as

$$\hat{E}_{\rm LO}(z,t)=\left[\left(\frac{8\pi P_{\rm LO}}{c\sigma}\right)^{1/2}+\Delta\hat{E}_{{\rm LO},1}(z,t)\right]\cos[\Omega(t-z/c)-\phi_{\rm LO}]+\Delta\hat{E}_{{\rm LO},2}(z,t)\sin[\Omega(t-z/c)-\phi_{\rm LO}]\;; \tag{3.7}$$

the LO quadrature phases, $\Delta\hat{E}_{{\rm LO},1}(z,t)$ and $\Delta\hat{E}_{{\rm LO},2}(z,t)$, represent quantum LO amplitude and phase fluctuations, respectively, and $P_{\rm LO}=\hbar\Omega A^2$ is the LO power.

We find it useful to express the differenced photocurrent operator $\hat{I}_D(t)$ in terms of the quadrature phases of the signal and the LO. From Eqs. (2.1), (2.3), (2.4), and (2.5) can be derived the useful relations

$$\hat{E}^{(+)}(z,t)=\tfrac{1}{2}[\hat{E}_1(\phi;z,t)+i\hat{E}_2(\phi;z,t)]\exp\{-i[\Omega(t-z/c)-\phi]\}\,,$$
$$\hat{E}^{(-)}(z,t)=\tfrac{1}{2}[\hat{E}_1(\phi;z,t)-i\hat{E}_2(\phi;z,t)]\exp\{i[\Omega(t-z/c)-\phi]\}\,. \tag{3.8}$$

Equations (3.8) hold for both the signal and the LO; using Eqs. (3.8), we can express the differenced photocurrent $\hat{I}_D(t)$ in terms of the signal and the LO, obtaining

$$\hat{I}_D(t)=\frac{ec\sigma_D}{4\hbar\Omega}\left[\left(\frac{8\pi P_{\rm LO}}{c\sigma}\right)^{1/2}\hat{E}_1(\phi_{\rm LO};t)+\Delta\hat{E}_{{\rm LO},1}(t)\hat{E}_1(\phi_{\rm LO};t)+\Delta\hat{E}_{{\rm LO},2}(t)\hat{E}_2(\phi_{\rm LO};t)\right]\,. \tag{3.9}$$

We see from this result that a balanced homodyne detector measures the quadrature of the signal that is in phase with the local oscillator in the absence of LO fluctuations.[8] Quantum fluctuations are always present in the LO, however; the second and third terms in Eq. (3.9), which result from LO fluctuations, will be shown to limit the degree of squeezing that can be measured by an ideal balanced homodyne detector.

Experimentally, we are interested in the *spectrum* of the differenced photocurrent. The relevant quantity here is the spectral density of $\hat{I}_D(t)$, which is the Fourier transform of the autocorrelation function $\langle\Delta\hat{I}_D(t)\Delta\hat{I}_D(t+\tau)\rangle_{\rm sym}$. There are *two* contributions to the autocorrelation function, and thus to the spectral density. The first contribution, which we call the *classical* contribution, arises from the first term in Eq. (3.9); it is the spectral density that would be obtained with a noiseless *classical* LO. The second contribution, which we call the *quantum* contribution, arises from the second and third terms in Eq. (3.9) and is due to *quantum* LO fluctuations. As there are no correlations between the signal and the LO, we can calculate the two contributions separately. Actually, this split between a classical versus quantum contribution is somewhat artificial since any classical noise in the LO would contribute to what we call the quantum contribution, increasing it above the theoretical minimum which we investigate in this paper.

Beats between the classical part of the LO at frequency Ω and signal frequencies $\Omega+\epsilon$ and $\Omega-\epsilon$ yield a Fourier component of the classical part of $\hat{I}_D(t)$, $[\hat{I}_D(t)]_C$, at frequency ϵ. Such beats, however, can be seen only for frequencies within the photodetector's electronic bandwidth $\Delta_{\rm el}$. Here we assume that each photodetector has the same electronic bandwidth, and that each responds with unit efficiency for $\epsilon\le\Delta_{\rm el}$ and with zero efficiency otherwise. Using Eqs. (2.4) and (2.5) in the first term of Eq. (3.9), we obtain

$$[\hat{I}_D(t)]_C=\left(\frac{4e^2P_{\rm LO}}{\hbar\Omega}\right)^{1/2}\int_S d\epsilon\frac{1}{2\pi}f(\epsilon)[\hat{a}_1(\phi_{\rm LO};\epsilon)e^{-i\epsilon t}+\hat{a}_1^\dagger(\phi_{\rm LO};\epsilon)e^{i\epsilon t}]\,, \tag{3.10}$$

where $f(\epsilon)=1$ for $\epsilon\le\Delta_{\rm el}$ and is zero otherwise, and we have set $\sigma_D=\sigma$. The corresponding contribution to the autocorrelation function is

$$[\langle\Delta\hat{I}_D(t)\Delta\hat{I}_D(t+\tau)\rangle_{\rm sym}]_C=\frac{4e^2P_{\rm LO}}{\hbar\Omega}\int_S d\epsilon\frac{1}{2\pi}f(\epsilon)S_{11}(\phi_{\rm LO};\epsilon)\cos\epsilon\tau\,. \tag{3.11}$$

By taking the Fourier transform of this equation we obtain the classical contribution to the spectral density of the differenced photocurrent:

$$S_C(\phi_{\rm LO};\epsilon)=\int_{-\infty}^{\infty}[\langle\Delta\hat{I}_D(t)\Delta\hat{I}_D(t+\tau)\rangle_{\rm sym}]_C e^{i\epsilon\tau}\,d\tau$$
$$=\frac{2e^2P_{\rm LO}}{\hbar\Omega}f(\epsilon)\,S_{11}(\phi_{\rm LO};\epsilon)\,. \tag{3.12}$$

There are also beats between the signal and the quantum LO fluctuations. Such beats are responsible for the quantum part of $\hat{I}_D(t)$, $[\hat{I}_D(t)]_Q$. By substituting Eqs. (2.1) and (3.4) into the second and third terms of Eq. (3.9), we find

$$[\hat{I}_D(t)]_Q = \frac{e}{\Omega}\int_S d\omega \frac{1}{2\pi}\int_0^\infty d\omega' \frac{1}{2\pi}(\omega\omega')^{1/2} f(|\omega-\omega'|)\exp[i(\omega-\omega')t][\hat{a}^\dagger(\omega)\Delta\hat{b}(\omega') + \hat{a}(\omega')\Delta\hat{b}^\dagger(\omega)] \,. \tag{3.13}$$

For a LO with vacuum fluctuations at all frequencies, the resulting quantum contribution to the spectral density is

$$\begin{aligned} S_Q(\epsilon) &= \int_{-\infty}^{\infty} [\langle \Delta\hat{I}_D(t)\Delta\hat{I}_D(t+\tau)\rangle_{\text{sym}}]_Q e^{i\epsilon\tau}\,d\tau \\ &= e^2 f(\epsilon)\int_S d\epsilon' \frac{1}{2\pi}\left[S_{11}(\phi_{\text{LO}};\epsilon') + S_{22}(\phi_{\text{LO}};\epsilon') - 1 - \frac{\epsilon'}{\Omega}\left(\frac{\epsilon'}{\Omega} + 2\,\text{Im}[S_{12}(\phi_{\text{LO}};\epsilon')]\right)\right] . \end{aligned} \tag{3.14}$$

Here the signal assumes the role previously played by the LO by acting as a local oscillator for quantum LO fluctuations. For *any* frequency ω within S, one obtains detectable beats with LO fluctuations at all frequencies ω' satisfying $|\omega-\omega'| < \Delta_{\text{el}}$. As this occurs whether the signal frequency ω itself lies inside or outside the electronic bandwidth, the *entire* signal band S contributes to $S_Q(\epsilon)$.

Adding Eqs. (3.12) and (3.14), we find that the total spectral density $S_T(\phi_{\text{LO}};\epsilon)$ is

$$S_T(\phi_{\text{LO}};\epsilon) = \frac{e^2 P_{\text{LO}}}{\hbar\Omega} f(\epsilon)\left[2S_{11}(\phi_{\text{LO}};\epsilon) + \frac{P_S}{P_{\text{LO}}} - \frac{\hbar\Omega}{P_{\text{LO}}}\int_S d\epsilon' \frac{1}{2\pi}\frac{\epsilon'}{\Omega}\left(\frac{\epsilon'}{\Omega} + 2\,\text{Im}[S_{12}(\phi_{\text{LO}};\epsilon')]\right)\right] , \tag{3.15}$$

where the signal power P_S is given by Eq. (2.18). We now define a normalized spectral density $S_N(\phi_{\text{LO}};\epsilon)$ by normalizing Eq. (3.15) to the vacuum level, i.e., so that $S_N(\phi_{\text{LO}};\epsilon) = 1$ for a signal consisting only of vacuum fluctuations. For such a signal we find that

$$S_{11}(\phi_{\text{LO}};\epsilon) = S_{22}(\phi_{\text{LO}};\epsilon) = \tfrac{1}{2} \,,$$

and

$$S_{12}(\phi_{\text{LO}};\epsilon) = -\frac{i\epsilon}{2\Omega} \,,$$

and it is obvious that $P_S = 0$; from Eq. (3.15), the spectral density for such a signal is

$$[S_T(\phi_{\text{LO}};\epsilon)]_{\text{vac}} = \frac{e^2 P_{\text{LO}}}{\hbar\Omega} f(\epsilon) \,. \tag{3.16}$$

The normalized spectral density for frequencies $\epsilon \le \Delta_{\text{el}}$ is then defined by

$$\begin{aligned} S_N(\phi_{\text{LO}};\epsilon) &\equiv \frac{S_T(\phi_{\text{LO}};\epsilon)}{[S_T(\phi_{\text{LO}};\epsilon)]_{\text{vac}}} \\ &= 2S_{11}(\phi_{\text{LO}};\epsilon) + \frac{P_S}{P_{\text{LO}}} - \frac{\hbar\Omega}{P_{\text{LO}}}\int_S d\epsilon' \frac{1}{2\pi}\frac{\epsilon'}{\Omega}\left(\frac{\epsilon'}{\Omega} + 2\,\text{Im}[S_{12}(\phi_{\text{LO}};\epsilon')]\right) . \end{aligned} \tag{3.17}$$

For a signal generated by the unitary transformation in Eq. (2.12), Eq. (3.17) becomes

$$S_N(\phi_{\text{LO}};\epsilon) = 2S_{11}(\phi_{\text{LO}};\epsilon) + \frac{\hbar\Omega}{P_{\text{LO}}}\int_S d\epsilon' \frac{1}{2\pi}\left(1 + \frac{\epsilon'^2}{\Omega^2}\right)[S_{11}(\phi_{\text{LO}};\epsilon') + S_{22}(\phi_{\text{LO}};\epsilon') - 1] \,, \tag{3.18}$$

where $S_{11}(\phi_{\text{LO}};\epsilon)$ and $S_{22}(\phi_{\text{LO}};\epsilon)$ are given by $S_{11}(\phi;\epsilon)$ and $S_{22}(\phi;\epsilon)$, respectively, from Eq. (2.17). We can see that this result is relatively insensitive to the assumption of the bolometric nature of the photodetectors, the correction coming in only as $O(\epsilon'^2/\Omega^2)$. Since the optical bandwidth S of the detector is likely to be somewhat less than Ω, we can drop the term proportional to ϵ'^2/Ω^2 in the integral in Eq. (3.18) and obtain

$$S_N(\phi_{\text{LO}};\epsilon) = 2S_{11}(\phi_{\text{LO}};\epsilon) + \frac{P_S}{P_{\text{LO}}} \,, \tag{3.19}$$

where we have used Eq. (2.18).

The result in Eq. (3.19) is very similar to the one describing the amount of squeezing obtained from a parametric amplifier.[14–16] There the pump plays the role played here by the local oscillator. There are, however, some important differences. With a parametric amplifier, the pump phase fluctuations mix the amplified quadrature of the field with its squeezed quadrature. Only a tiny portion of the signal bandwidth[15,16] is mixed in, however, corresponding to the phase-matched bandwidth of the pump itself. Here, all quadratures of the signal field act as local oscillators for fluctuations in the physical local oscillator; furthermore, the entire signal bandwidth contributes in this way, not just a tiny portion.

We will use Eq. (3.19) in Sec. IV when we consider the effects of quantum LO fluctuations on the detection of squeezing by a balanced homodyne detector.

IV. FUNDAMENTAL LIMITS ON THE DETECTION OF SQUEEZING

Quantum mechanics limits the squeezing detectable by a balanced homodyne detector in two ways. The first way is by imposing a limit on the squeezing in the signal itself through Eq. (2.10); this limit depends only on the detection process being bolometric. Since it is likely that $\epsilon \ll \Omega$ over the signal bandwidth S, however, the limit imposed by Eq. (2.10) is not the relevant one for squeezing at optical frequencies, and will not be discussed further. The second way is by means of the quantum noise that must accompany the LO, which makes a contribution to the normalized spectral density in Eq. (3.19) that may mask the desired effect produced by a squeezed signal. In this section, we discuss this limit in the context of two specific examples, narrow-band squeezed light as produced by an optical parametric oscillator (OPO), and wide-band squeezed light.

Let us first consider narrow-band squeezed light as produced by an OPO. For such a signal, we find that[17]

$$M(\Omega \pm \epsilon) = \frac{(\gamma/2)^2 + \epsilon^2 + g^2}{(\gamma/2 \mp i\epsilon)^2 - g^2}\,, \qquad N(\Omega \pm \epsilon) = \frac{\gamma g}{(\gamma/2 \mp i\epsilon)^2 - g^2}\,. \tag{4.1}$$

From these equations we see that $\theta_{MN}(\epsilon) = 0$, so by choosing $\phi = 0$ in Eqs. (2.17) and by dropping terms of order ϵ/Ω, we obtain

$$S_{11}(\epsilon) \equiv S_{11}(0;\epsilon) = \frac{1}{2}\frac{(\gamma/2 - g)^2 + \epsilon^2}{(\gamma/2 + g)^2 + \epsilon^2}\,, \qquad S_{22}(\epsilon) \equiv S_{22}(0;\epsilon) = \frac{1}{2}\frac{(\gamma/2 + g)^2 + \epsilon^2}{(\gamma/2 - g)^2 + \epsilon^2}\,. \tag{4.2}$$

Here γ is a cavity damping rate, and g is a parametric gain rate. In our approximation of neglecting terms of $O(\epsilon/\Omega)$, the off-diagonal elements of the spectral-density matrix are zero. From Eq. (2.18) and Eqs. (4.2), the signal power is

$$P_S = \frac{\hbar\Omega(\gamma/2)g^2}{(\gamma/2)^2 - g^2}\,. \tag{4.3}$$

If we then set $g = s(\gamma/2)$, where $0 \le s < 1$, we find

$$S_N(\epsilon) \equiv S_N(0;\epsilon) = \frac{(1-s)^2 + (2\epsilon/\gamma)^2}{(1+s)^2 + (2\epsilon/\gamma)^2} + \frac{s^2}{1-s^2}\frac{\hbar\Omega\gamma}{2P_{\rm LO}}\,. \tag{4.4}$$

We now want to calculate the value of s at which maximum squeezing is *detected*. We find that, for $\epsilon \ll \gamma/2$, $S_N(\epsilon)$ is a minimum when

$$s \approx 1 - \left(\frac{\hbar\Omega\gamma}{2P_{\rm LO}}\right)^{1/3}, \tag{4.5}$$

and that the corresponding minimum value of $S_N(\epsilon)$ is

$$[S_N(\epsilon)]_{\rm min} \approx \frac{3}{4}\left(\frac{\hbar\Omega\gamma}{2P_{\rm LO}}\right)^{2/3}, \quad \epsilon \ll \gamma/2\,. \tag{4.6}$$

For $\Omega/2\pi = 5\times10^{14}$ Hz, $\gamma = 10^7\,{\rm s}^{-1}$, and $P_{\rm LO} = 1.0$ mW, we find $[S_N(\epsilon)]_{\rm min} \approx 10^{-6}$; for the given parameter values, $\epsilon/\Omega \ll \gamma/2\Omega = 1.6\times10^{-9}$, so that the minimum imposed by LO fluctuations is three orders of magnitude larger than the limit imposed by Eq. (2.10) on the squeezing in the signal itself. The approximation we made by dropping terms of order ϵ/Ω is then justified in this case. The output statistics used in Eq. (4.1) assumes an ideal OPO, i.e., with a classical pump. Including OPO pump fluctuations[18] would result in a practical limit to squeezing; here we discuss fundamental limits.

Now let us consider detection of wide-band squeezed light. Specifically, we will consider a signal that is uniformly squeezed over a bandwidth $\Delta/2\pi$. For such a signal, one has

$$M(\epsilon) = |M(\epsilon)| = \begin{cases} \cosh r & \text{if } 0 \le \epsilon < \Delta \\ 1 & \text{otherwise} \end{cases} \qquad N(\epsilon) = |N(\epsilon)| = \begin{cases} \sinh r & \text{if } 0 \le \epsilon < \Delta \\ 0 & \text{otherwise} \end{cases} \tag{4.7}$$

where r is the squeezing parameter. Using Eqs. (2.17), (2.18), and (3.19), we find with $\phi_{\rm LO} = 0$ that

$$\begin{aligned} S_N(\epsilon) &= \cosh^2 r - 2\left(1 - \frac{\epsilon^2}{\Omega^2}\right)^{1/2}\cosh r \sinh r \\ &\quad + \sinh^2 r + \frac{\hbar\Omega\Delta \sinh^2 r}{\pi P_{\rm LO}} \\ &\approx \left(1 + \frac{\hbar\Omega\Delta}{4\pi P_{\rm LO}}\right)e^{-2r} + \left(\frac{\hbar\Omega\Delta}{4\pi P_{\rm LO}}\right)e^{2r} - \frac{\hbar\Omega\Delta}{2\pi P_{\rm LO}} \end{aligned} \tag{4.8}$$

for frequencies ϵ within the photodetector's electronic bandwidth, i.e., $\epsilon < \Delta_{\rm el}$.

From Eq. (4.8) we can calculate the maximum squeezing possible at a frequency $\epsilon < \Delta_{\rm el}$. By differentiating $S_N(\epsilon)$ with respect to r and assuming that $\hbar\Omega\Delta/4\pi P_{\rm LO}$ is much less than 1, we find that the minimum occurs when

$$e^{2r} = \left(\frac{\hbar\Omega\Delta}{4\pi P_{\rm LO}}\right)^{-1/2}. \tag{4.9}$$

The corresponding minimum value of $S_N(\epsilon)$ is

$$[S_N(\epsilon)]_{\rm min} \approx \left(\frac{\hbar\Omega\Delta}{\pi P_{\rm LO}}\right)^{1/2}. \tag{4.10}$$

For $\Omega/2\pi = 5 \times 10^{14}$ Hz, $P_{\rm LO} = 1.0$ mW, and $\Delta/2\pi = 100$ GHz, a squeezing bandwidth that might be produced by an inherently wide-band device such as a parametric amplifier[19] or a forward four-wave mixer,[20] we find $[S_N(\epsilon)]_{\rm min} \approx 0.008$, which is a much more severe limitation than that found in the narrow-band case. One finds $P_S/P_{\rm LO} \approx 7 \times 10^{-7}$ for the previously considered OPO, and $P_S/P_{\rm LO} \approx 4 \times 10^{-3}$ in the present case, which accounts for the difference.

The result in Eq. (3.19) applies equally well to general signal fields as to the squeezed fields from which we obtained our fundamental limits. Thus any extraneous noise entering the signal port will act as a local oscillator for the fluctuations of the local oscillator and will limit the amount of observable squeezing. For example, with $P_{\rm LO} = 1.0$ mW, we would need to limit the total signal power (signal plus noise) to $P_S \ll 10\ \mu$W in order to see squeezing down to a value of $S_N(\epsilon) = 0.01$.

V. CONCLUSION

We have presented a wide-band analysis of balanced homodyne detection in which we have examined the effect of quantum local-oscillator fluctuations on the normalized spectral density $S_N(\phi_{\rm LO};\epsilon)$ of the detector's output. We found that such fluctuations produce noise in the output that can mask the desired effect produced by a squeezed signal, limiting the observable squeezing. The contribution to the normalized spectral density of the output $S_N(\phi_{\rm LO};\epsilon)$ is approximately $P_S/P_{\rm LO}$, where P_S is the total signal power and $P_{\rm LO}$ is the local-oscillator power; thus *every* Fourier component of the signal contributes to this excess noise, even components at frequencies lying outside the detector's electronic bandwidth. These results put a limit on the total power entering the signal port, an effect which would be important for experiments sooner than the fundamental limit we derive.

Two specific examples were used to illustrate the effects of local-oscillator fluctuations on the spectrum: narrow-band squeezed light as produced by an ideal optical parametric oscillator, and wide-band squeezed light as might be produced by a wide-band device such as a parametric amplifier or a forward four-wave mixer. For signals optimized to minimize $S_N(\phi_{\rm LO};\epsilon)$ at a specific frequency, the limit imposed on wide-band squeezed light is much more severe than that imposed on narrow-band squeezed light. This is due to the much greater power in the wide-band signal than in the narrow-band signal.

Although the model we have used for photodetection is greatly simplified, we believe that the limits on balanced homodyne detection discussed here are fundamental in the sense that such limits cannot be overcome by building photodetectors that more closely approximate the ideal; these limits result from the quantum nature of the signal and the local oscillator. These limits are not of immediate concern to those involved in current efforts to produce highly squeezed light, but they may be in the future, especially if it should become possible to produce highly squeezed light over a large bandwidth.

ACKNOWLEDGMENTS

The authors appreciated discussions with Carlton Caves. S. L. B. was funded in part by the Office of Naval Research Fund No. N00014-88-K-0294. The work of D. D. C. was supported by the National Science Foundation through Grant No. PHY-8604504.

[1]H. P. Yuen and V. W. S. Chan, Opt. Lett. **8**, 177 (1983).
[2]D. F. Walls, Nature (London) **306**, 141 (1983).
[3]For accounts of current research on squeezed states, see J. Opt. Soc. Am. B 4 (1987); and J. Mod. Opt. **34** (1987).
[4]B. L. Schumaker, Opt. Lett. **9**, 189 (1984).
[5]B. Huttner and Y. Ben-Aryeh, Phys. Rev. A **40**, 2479 (1989).
[6]B. Yurke, Phys. Rev. A **32**, 311 (1985).
[7]C. M. Caves and B. L. Schumaker, Phys. Rev. A **31**, 3068 (1985).
[8]C. M. Caves and B. L. Schumaker, in *Quantum Optics IV*, edited by J. D. Harvey and D. F. Walls (Springer, Berlin, 1986), p.21.
[9]R. S. Bondurant and J. H. Shapiro, Phys. Rev. D **30**, 2548 (1984).
[10]J. H. Shapiro and S. S. Wagner, IEEE J. Quantum Electron. **QE–20**, 803 (1984).
[11]J. H. Shapiro, IEEE J. Quantum Electron. **QE–21**, 237 (1985).
[12]R. S. Bondurant, Phys. Rev. A **32**, 2797 (1985).
[13]R. J. Glauber, Phys. Rev. **130**, 2529 (1963).
[14]M. Hillery and M. S. Zubairy, Phys. Rev. A **29**, 1275 (1984).
[15]C. M. Caves and D. D. Crouch, J. Opt. Soc. Am. B 4, 1535 (1987).
[16]D. D. Crouch and S. L. Braunstein, Phys. Rev. A **38**, 4696 (1988).
[17]M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[18]J. Gea-Banacloche and M. S. Zubairy, Phys. Rev. A **42**, 1742 (1990).
[19]D. D. Crouch, Phys. Rev. A **38**, 508 (1988).
[20]M. J. Potasek and B. Yurke, Phys. Rev. A **35**, 3974 (1987).