NUCLEAR ELECTRIC DIPOLE MOMENTS

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ABSTRACT

The existence or non-existence of electric-dipole moments (EDM) on elementary particles has profound implications for the symmetry structure of basic physical theory. This thesis poses the question whether it is possible to find a more sensitive method of determining the EDM of elementary particles by looking at great ensembles of them simultaneously, as opposed to sensitive methods which look at them one at a time. The Schiff screening theorem indicates in a limited way that this may be difficult. In this thesis an attempt to circumvent the implications of the Schiff theorem was undertaken, in the course of which regretably the theorem was strengthened and generalized, making the realization of the circumvention by experimental means beyond reasonable possibility.

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PREFACE

This thesis may be divided into three main areas. The first is a general review of the theoretical and experimental status of fundamental electric dipole moments. The next deals with the discussion of feasibility of our suggested experiment. The third investigates the problem of shielding, and includes a review of much of the work that has already been done on this topic.

References are collected at the end of the thesis and arranged alphabetically. Throughout the thesis, references are noted by the name of their author and the year they were published. The numbering of equations and figures is recommenced at the beginning of each chapter.

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CHAPTER 1

Section 1.1

OUTLINE OF THESIS.

Interest in the electric dipole moment (EDM) of the neutron dates back to the nineteen-fifties. Even in the absence of a demonstration of the existence of an EDM for the neutron, or indeed for any other fundamental particle, this interest has continued to grow due to its significance for the space-time symmetries of modern physical theories. If EDMs were found to exist then we would have to conclude that the interactions involving the neutron and/or other particles would need to have a somewhat lower symmetry than was once thought possible. This will be discussed in section 1.2 and 1.3.

So far, the most precise experiments have used the spin resonance method. These experimental techniques (review chapter 3) have been refined over many years, and further significant gains in sensitivity seem unlikely. Therefore new techniques will probably be necessary if EDMs below the limits established by this method are to be measured. This thesis develops a feasibility study of a novel experimental approach originated by G. I. Opat.

The sensitivity of spin resonance techniques relies on the accuracy of modern frequency measurements. A drastic limitation of this method in the measurement of the EDM of the neutron is the low intensity of presently available neutron beams. Clearly a method measuring the EDM of atoms (instead of neutrons) would avoid this short-coming, since it would enable us to perform experiments on a larger number particles simultaneously. It is then of interest to extract the EDM of the different particles comprising the atoms investigated. An outline of such an experiment is presented in section 1.4, and a full discussion of its feasibility in chapter 3. Some of the difficulties involved in the extraction are discussed in chapter 2.

Unfortunately, due to atomic shielding effects which were more severe than was initially anticipated, the sensitivity of this method, even at its best, does not exceed that of spin resonance techniques. This problem is reviewed in chapter 2 and is considered more completely in chapter 4.

Section 1.2

SYMMETRIES IN PHYSICS

Symmetries are a central part of the study of physics. By reviewing the connection between EDMs and two fundamental symmetries of physics (parity and time-reversal), we will see that the study of EDMs can provide information on the exactness of these symmetries.

As it is conveyed in everyday language the concept of symmetry is somewhat intuitive, somehow describing the simplicity, pattern or perfection exhibited by an object.

In physics, we necessarily want to give the term a precise meaning. A system which is unchanged after it has been acted upon by some operation, is then said to be "symmetric" with respect to that operation. For instance, a planar square is unaffected by rotations through ninety degrees about the central axis perpendicular to its plane. In physics we say a physical law is symmetric (or invariant) under a given transformation if the law applies to all systems derivable from the original one using the transformation. At a higher level of abstraction we would need to apply the mathematical theory of groups, however this formalism is not necessary here and will not be further discussed.

The classical approach of physics was generally to derive the symmetries from the physical laws, which were themselves based on experimental evidence. More recently, modern physics has reversed this approach in seeking to derive physical laws from fundamental symmetries. The outstanding examples of this latter technique are special relativity and quantum electrodynamics. Special relativity is based on a symmetry principle which holds that our physical laws are the same independent of the inertial frame we are in. Included in this symmetry is the postulate that the speed of light is the same in any inertial frame, thus requiring our laws to be invariant under Lorentz transformations. Quantum electrodynamics is not derived completely from symmetry principles, however the way in which electron fields interact with photons is dependent upon the assumption that this interaction is invariant under any (local) change of phase of the electron fields.

THREE FUNDAMENTAL SYMMETRY OPERATIONS

We introduce three fundamental symmetries: these being parity, time-reversal, and charge-conjugation. It is conventional in physics to use these same terms to describe the operations associated with the symmetries involved, we shall follow this convention throughout this thesis.

Parity

The operation of inversion of all space coordinates is called the parity operation (P). It may be thought of as the physical system obtained from the original by observing it through three mirrors set orthogonally to each other. Thus the parity operation on a right-handed screw gives a left-handed screw. For parity to be a perfect symmetry of nature both the original and parity-inverted systems must occur with equal incidence.

Many molecules can have either right or left-handed forms and both forms may be produced with equal ease by chemical means. Yet it is well known that living creatures can often metabolise only one variety of these organic molecules. Might this mean that parity is broken somehow in the chemistry of living matter? Considering the equal probability of production using chemical processes it is usually concluded that this is not the case. The preference is seen as an historical accident - at least locally on our planet. At the organic level if these two forms were competing for the same nourishment we see from probability theory that even though each has an equal chance to survive one must eventually vanish, since it only has a finite population to repopulate from. However, this explanation leaves many more questions open than it attempts to answer. In spite of this observed asymmetry of molecules on the macroscopic scale, there is no suggestion that on the microscopic level the Hamiltonian is not symmetric with respect to parity.

Time-Reversal

Closely related to parity is the time-reversal operation (T); it is the operation whereby all the particles in a system have their momenta reversed, with their spatial coordinates unchanged. The standard illustration of time-reversal is that of watching a system recorded on a movie film which is subsequently run backwards. Since we see a ball flying through the air or two colliding balls as equally intuitively sensible when the film is run backwards as forward Newton's laws are "proved" invariant under time-reversal.

However, this conclusion seemingly contradicts the general experience of supposedly irreversible events; such as shattering a glass. On a microscopic level, of atoms and molecules in collision, time-reversal holds, yet for the splinters to fly off from the ground and to make a whole glass again is not believable. The way out of this dilemma is to realise that the macroscopic event would require the orchestrating of many microscopic ones with the initial conditions set up exactly to produce the desired effect. Obviously the tolerances necessary to do this are not possible, even in the laboratory.

Charge-Conjugation

The final fundamental symmetry considered here is the operation of charge-conjugation (C). This operation exchanges every particle for its antiparticle, which has the internal quantum numbers of the original particle reversed in sign. As particles and antiparticles annihilate when they come into contact, there can be no question that the locally observed asymmetry between these two types of matter indicates any breaking of charge conjugation.

As there are no macroscopic objects made of antiparticles in our common experiences, it is worth considering some consequences of individual particles being invariant under charge-conjugation. This would imply that a particle and an antiparticle would have identical masses and lifetimes. This also follows as a consequence of the PCT theorem (Streater and Wightman, 1964), which states that any local theory satisfying Lorentz invariance is symmetric under the combined symmetry operations of P, C, and T taken in any order.

SYMMETRY VIOLATION AND THE EDM OF THE NEUTRON

If a particle is symmetric under the P or T transformations then its EDM must identically vanish. Thus the existence of an EDM of a particle would imply a violation of both symmetries of P and T. We give a simple version of this argument, and follow it with a brief review of the development of the ideas involved.

All symmetry arguments rely on the simplicity inherent in the definition of the structure of elementary particles, so we must naturally be careful if we wish to apply them to more complicated systems. They rely on the assumption of quantum theory which holds that the particle be completely specified by a few simple quantum numbers, such as charge, mass, and spin. In particular, the simplicity of the structure of the particles requires that they have only one possible axis, that of their spin, and magnetic-dipole moment, with which the EDM must be aligned or anti-aligned. The EDM is considered to be a "polar vector" as it would be if it were caused by displaced charges, while the magnetic-dipole moment is an "axial vector" as if it were due to circulating currents.

With these simplifications we may form a mental picture of a particle, as is shown in figure 1, and its transformations under the symmetries of P, T and C. Under the action of either the P or T operations the orientation of the magnetic-dipole moment to the EDM is reversed, while under the C operation it is left unchanged since one is an axial and the other a polar vector.

For particles to be symmetric under P or T they would have to exist in two configurations; one with the EDM and magnetic-dipole moment aligned, and one with them antialigned. So either the EDM is zero, or the neutron is a doublet. This latter possibility would contradict our understanding of nuclear spectroscopy, since now the Pauli principle would allow us to fit two spin-up neutrons in the same energy and momentum states with their EDMs

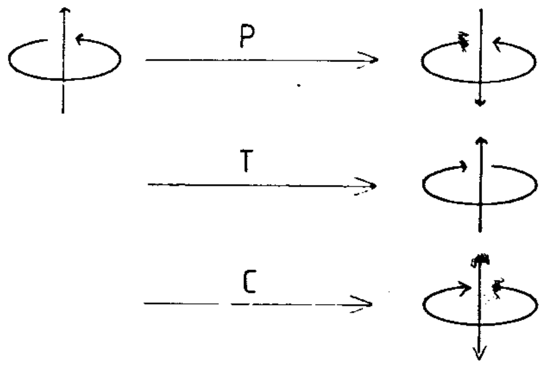

FIGURE 1. The EDM and magnetic-dipole moment are represented as polar (arrow) and axial (loop) vectors respectively. Under the parity or time-reversal operations the relative orientation of the EDM and magnetic-dipole moment is reversed, while under charge-conjugation it is left unchanged.

oppositely directed, thus giving us twice as many neutrons in each orbital as can actually be placed there. This follows for protons and electrons as well. It is interesting to note that some molecules do exhibit EDMs. However these states are not true eigenstates of the Hamiltonian of the isolated molecule, but are due to two almost degenerate states of opposite parity (eg. ammonia).

HISTORICAL PERSPECTIVE

Before 1957 P, C, and T were all believed to be perfect symmetries of physics, hence by the the above arguments, the EDM of any fundamental particle would have to be zero. The work of Lee and Yang (1956) and Wu et al (1957) demonstrated that P and C could no longer be considered to be exact symmetries. However, the T-symmetry was still believed to be exact, and to still rule out the possibility of an EDM for fundamental particles. The experiment involving the decay of the neutral kaon (Christenson et al, 1964) demonstrated that CP was not an exact symmetry so by the PCT theorem neither was T. So symmetry arguments of the kind given above no longer forbade the possibility of EDMs at the fundamental level.

As early as 1950 Purcell and Ramsey realised that Dirac's magnetic charge (1948), if it existed, was an example of a theory which violated parity conservation and hence permited EDMs. Therefore, they concluded that these symmetries do not have "a priori" validity and must be tested experimentally. They went on to argue that the matter-antimatter asymmetry in our region of the universe suggested that C might not be a symmetry at all, so why should any other symmetry, such as P or T, hold true? They concluded that the assumption of parity symmetry and the existence of an EDM must therefore be empirically based. They suggested that a search of the EDM of a neutron might be practicable, arguing that the EDM of atoms and molecules would be too small.

Lee and Yang (1956) surveyed the experimental information concerning parity and concluded that there was insufficient evidence at that time to decide whether parity was an exact symmetry of weak interactions. They also concluded that there was evidence which supported the conservation of parity in both the strong and electromagnetic interactions to a high degree of accuracy. Their investigation was motivated by a problem which is now called the tau-theta puzzle:

The so called θ^+ ($\rightarrow 2\pi$) and the τ^+ ($\rightarrow 3\pi$) mesons were found to have identical masses and lifetimes to within experimental errors, suggesting that they were the same particle. On the other hand, the decay distributions of these particles strongly suggested, on the basis of angular momentum conservation, that they had opposite parity. Lee and Yang saw that if parity was not strictly conserved, the θ^+ and τ^+ would simply represent two different decay modes of the same particle (subsequently called the K^+). They suggested that this analysis be confirmed by seeking further examples of parity violation in the weak decay of nuclei, hyperons, and mesons.

Parity violation was indeed confirmed in the famous 1957 experiment of Wu et al on the angular distribution of the beta particles from the decay of polarized Cobalt-60 nuclei.

Landau (1957) considered that the isotropy of space and momentum conservation precluded parity violation being an isolated asymmetry. He pointed out that, whereas strong interactions were invariant under space-inversion and charge-conjugation, it could be that weak interactions were invariant only under their product (CP). Landau's scheme, together with the PCT theorem, implies that T-invariance is a symmetry of strong and weak interactions, hence EDMs would not exist.

Ioffe (1957) extended Landau's argument to the case in which P and T were violated separately but PT was conserved. By applying these symmetries to the S-matrix and using the principle of detailed balance he was able to distinguish which observable quantities in decay distributions would be non-zero if the symmetries of CP and PT were violated. His argument shows from the description of the beta-decay of Cobolt-60 that charge-conjugation as well as parity is violated, which is the result Lee and Yang recognised from the data and is the conclusion Landau came to in order to preserve the isotropy of space.

We have come a full circle at this point with the symmetry argument again being used to deny the possibility of EDMs.

Shortly after Landau's paper, Smith et al (1957) published the results of an experimental measurement of the EDM of the neutron, using a neutron-beam magnetic resonance method. They established that, if it existed, the EDM of the neutron was less than 5×10^{-20}e.cm ('e' is the charge on the proton). Although the negative results were fully consistent with the theoretical beliefs of the day, their motivation was to put T-invariance to experimental test, much as P-invariance had been earlier, by using the search for an EDM as a sensitive probe.

This trend against the purely theoretic restriction of EDMs to zero was supported by Ramsey (1958). He pointed out that however appealing symmetry arguments were, they were not necessarily valid. He reconsidered the possibility of magnetic monopoles as a mechanism for producing EDMs, and argued that if magnetic-monopoles existed (and Dirac had already given reasons for expecting their existence) then the PCT theorem would have to be replaced by the PCTM theorem, where M is magnetic pole conjugation. In Landau's argument PC is conserved hence instead of T alone being conserved, the combined product of T and M - TM would be conserved. This, Ramsey argued, would allow the T symmetry to be violated without affecting CP symmetry or Landau's cosmological argument. With T symmetry being put in question, the old argument against EDMs once again came to be re-examined. Ramsey concluded from this example that ultimately the validity of all symmetry arguments must be empirically testable.

Let us now consider a question raised by Zel'dovich (1958). Cobalt-60 beta-decay showed an asymmetry of the distribution of emitted electrons; this demonstrated a violation of the parity symmetry. By the Heisenberg uncertainty principle, the nucleus can undergo a virtual decay, emitting an electron and capturing it again. If the asymmetry in real decays gives rise to a similar asymmetry in virtual decay, this would lead to an asymmetry in the virtual electron cloud, and therefore an EDM. But then we would conclude that T-invariance was automatically broken, simply because P-invariance was broken. Zel'dovich solved this problem by showing that the decay asymmetry due to parity violation is not necessarily related to the asymmetry of the virtual-particle cloud. He did this by showing that the real decays and virtual decays depend on the real and imaginary parts of the S-matrix elements respectively, and so are independent.

Another possible failure of simple theoretical arguments was analysed by Sandars (1968), who showed that the usual proof for the non-existence of an EDM in the presence of P and T symmetry is not strictly valid in a uniform external magnetic field. This raised the possibility that, under these circumstances, the existence of an EDM is forbidden by parity but not by time-reversal invariance. This would be a critical result if true, since then the detection of an EDM would no longer be evidence for the violation of T-invariance. He went on, however, to rule out this possibility by showing that, even in an arbitrarily strong magnetic field, the existence of an EDM is forbidden by T-invariance as long as the state is non-degenerate and the magnetic field is uniform.

Many of the objections to the theoretical arguments were based on models of particles involving magnetic monopoles. The work of Opat (1976) and Jackson (1977) shows that all known intrinsic magnetic moments are caused, to a high degree of precision, by circulating electric currents and not by magnetic charges. This was done by studying the effect of a nucleus made up of magnetic charge on the hyperfine spitting of Hydrogen and also by considering the scattering due to matter made up of magnetic charges.

Although this tends to discredit the examples raised as objections to the theoretical symmetry arguments, it does not necessarily refute the underlying point. All physical theories must rely on experimental data if they are to be ultimately credible.

CP-Violation

In 1964 Christenson et al discovered the CP-violating decay of the neutral kaon (see Kabir (1968) and Rowe and Squires (1969) for detailed discussions). Indeed this remains the only known manifestation of CP-violation, and assuming the validity of the PCT theorem, this violation implies a violation of T symmetry. Further experiments on K^0 decays suggest that this is an accurate conclusion (Schubert et al, 1970).

We do not as yet understand the exact source of this CP-violation. The various CP-violating theories, which are consistent with the facts of kaon decay, differ in their predictions; in amoung other things, respect to the size of the EDM of the neutron. A measurement of the EDM of the neutron would distinguish between them. This precisely determinable static property would doubtless be more constraining of the theories than a dynamical one, eg. a decay, which is incapable of such precision.

Section 1.3

THEORIES FOR EDMS

In this section we assume the validity of the PCT theorem (Streater and Wightman, 1964). Hence we shall consider the terms CP and T violation to be interchangeable.

It is known that the CP symmetry is violated in neutral kaon decays. The question is whether the effects of CP violation are manifested in any other way. As explained in the last section the existence of an EDM of a particle would provide an additional display of CP violation. For this reason the neutron is and has been the subject of intensive experimental studies. We now look at the mechanisms by which a neutron may acquire an EDM.

Mechanisms for the Occurence of a Neutron EDM

In the literature there are many ways in which the EDM of the neutron has been evaluated, and it is impossible to review all of them here; Ramsey (1982) and Wolfenstein (1974) however offer such reviews.

Those calculations which make use of the seminal paper of Koboyashi and Maskawa (1973) are, we feel, the most significant. This paper is concerned with the extension of the Weinberg - Salam theory of weak and electromagnetic interactions with extra generations of quarks.

Brief Survey of Sources of CP Violation

Many new interactions have been postulated which could explain CP violation; they may be divided into four types: superweak theories; milliweak theories; electromagnetic theories; and millistrong theories.

It is easy enough to estimate the order of magnitude of the EDM of a particle caused by the interaction responsible for the violation of the symmetries. The magnitude of a particle's EDM can be characterised by a length representing the separation of opposite charges within the particle. On dimensional grounds this length can be estimated to be some characteristic length of the particle multiplied by the strength of the interactions causing the parity and time-reversal violations necessary for an EDM to exist. The characteristic length is taken to be the Compton wavelength of the particle.

The electromagnetic interaction of hadrons was suggested (Bernstein et al, 1965) to be intrinsically T violating but parity conserving. Thus an EDM could be caused by radiative corrections to the weak amplitudes giving a neutron EDM (d_n) of magnitude:

$$d_n \simeq e \frac{G m_n^2 . c^2}{4\pi} \frac{\hbar c}{m_n} \simeq 10^{-20} e.cm$$

where G is the Fermi constant and m_n is the mass of the neutron. This coincides with more detailed calculations (Ramsey, 1982) which range from 10^{-20} to 10^{-23}e.cm. As the neutron EDM has been experimentally shown (see the references and discussion in chapter 3) to be less than 6×10^{-25}e.cm such a putative explanation for T violation has been discarded.

For milliweak theories the effective coupling is three orders of magnitude smaller than the weak interaction giving a neutron EDM of:

$$d_n \simeq 10^{-23} \text{e.cm.}$$

Calculations from various models for the milliweak interaction give an EDM for the neutron typically between 10^{-21} and 10^{-24}e.cm, though there are models as low as 10^{-29}e.cm (Ramsey, 1982). Clearly these latter models cannot be discarded on present experimental limits.

The superweak interactions have an effective coupling of the order 10^{-9}G, thus they should give an EDM of about 10^{-29}e.cm. More accurate calculations for differing superweak models give 10^{-28} to 10^{-43}e.cm (Ramsey, 1982).

Finally, the millistrong interaction violates T but leaves P invariant. Thus, weak force corrections would be required to produce an EDM. It would seem that only one calculation (Clark and Randa, 1975) has been made for this correction which yields an EDM of 10^{-26}e.cm.

Although the failure to observe an EDM at the present level of sensitivity leaves many of these theories unchallengeable, we see that an improvement of even a few orders of magnitude to the present experimental limit would be very interesting. The absence of an EDM at this level would strongly favour the superweak interaction, though this would not entirely rule out milliweak theories as an explanation of T violation. If it were possible to determine the size of the EDM, this extra parameter would be an aid to the refinement of present theories, thus hopefully extending our understanding of symmetry violations.

The Koboyashi - Maskawa Scheme

The discovery of the τ lepton and the Υ meson (which is thought to be the bound state of a pair of the new b quarks) means that the SU(2)xU(1) theory includes at least three doublets of left-handed leptons:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L,\ \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$$

and three doublets of left-handed quarks:

$$\begin{pmatrix}u\\ d\end{pmatrix}_L,\ \begin{pmatrix}c\\ s\end{pmatrix}_L,\ \begin{pmatrix}t\\ b\end{pmatrix}_L$$

Koboyashi and Maskawa (1973) have shown that this number of doublets is sufficient for the introduction into the theory of CP violation.

To see this we consider the standard model of Weinberg and Salam for N generations (or doublets) of quarks. In this model the charge-changing currents are described by an NxN unitary matrix U:

$$J_\mu^- = 1/2\cdot\bar{u}_i\gamma_\mu(1-\gamma_5)U_{ij}d_j \qquad i,j=1,2,\ldots,N$$

where $\begin{pmatrix}u_i\\ d_i\end{pmatrix}$ are the quark doublets,

and γ_μ, γ_5 are the Dirac gamma matrices. This matrix U describes the fact that the weak eigenstates and mass eigenstates are not the same; this is often called the mixing matrix.

In the case N equal to 2, the SU(4) flavour model of Glashow et al (1970) has

$$U = \begin{pmatrix}\cos\theta_c & \sin\theta_c\\ -\sin\theta_c & \cos\theta_c\end{pmatrix}$$

where θ_c is the Cabibo angle. It turns out that this gives a charged current which is invariant under CP and hence T.

However, for N = 3 Koboyashi and Maskawa parameterized U as:

$$U = \begin{pmatrix} c_1 & -s_1 & -s_1s_3\\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta}\\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2s_3e^{i\delta}\end{pmatrix}$$

where $s_i = \sin\theta_i$, $c_i = \cos\theta_i$ $\qquad i = 1,2,3$

If we ignore the real mixing angles we can extract from $J_\mu^+W^{\mu-} + J_\mu^-W^{\mu+}$

an interaction of the form:

$$e^{i\delta}.\bar{u}_L\gamma_\mu d_L W^{\mu +} + e^{-i\delta}.\bar{d}_L\gamma_\mu u_L W^{\mu -} \qquad (1)$$

under the operation of CP this becomes:

$$e^{i\delta}.\bar{d}_L\gamma_\mu u_L W^{\mu -} + e^{-i\delta}.\bar{u}_L\gamma_\mu d_L W^{\mu +}$$

which is the same as equation (1) except for the exchange of phase angles. Thus for N = 3 the SU(2)xU(1) model encompasses T violation if this phase parameter δ is non-zero.

Neutron EDM in the Koboyashi - Maskawa Scheme

There have been many calculations of the neutron EDM over the past seven years in the Koboyashi - Maskawa scheme. As there is essentially only the one parameter involved in T violation, the data from neutral kaon CP violation is sufficient to fix the theory. Even so, there was much debate in the literature as to just how large a neutron EDM is predicted by this scheme.

Since it is known that the static properties of baryons can be obtained by simply adding up the properties of the individual quarks, the problem of the value of the quark EDM was studied first using the Koboyashi - Maskawa scheme. By looking at a few Feynman diagrams Maiani (1976), Ellis et al (1976) and Lee (1977) estimated the order of magnitude of the quark EDM as being about 10^{-30}e.cm. A more careful investigation by Shabalin (1979) has shown that there are cancellations in all purely weak interactions up to fourth order in the semi-weak coupling constant "g".

This problem of cancellation of diagrams occured again in attempts to calculate the contribution from the strong interaction to the quark EDM, and again when exchange terms between the different quarks in the neutron were included (see Shabalin, 1980, and references therein). So by 1980 the value of the neutron EDM attributed to the Koboyashi - Maskawa mechanism was of the order of 10^{-34}e.cm, making it ten orders of magnitude lower than the best experimental limit of the time.

More recently it has been found that some higher order Feynman diagrams can actually give a larger contribution to the neutron EDM. The largest value was obtained by Gavela et al (1982), who calculated a dipole of size

10^{-30}e.cm due to the so-called penguin diagrams. It would seem that the cancellations in these sort of diagrams are not as severe as those encountered in earlier contributions. The model independence of this estimate has been verified by Eeg and Picek (1982).

Thus the presently accepted value of the neutron EDM as predicted from the Koboyashi - Maskawa mechanism is about 10^{-30}e.cm. This value is unlikely to be directly testable in the near future unless new experimental techniques are employed (see the review on the present experimental status in chapter 3).

Section 1.4

GENERAL CONSIDERATION FOR MEASURING EDMS

We start with some general considerations about measuring EDMs, and then briefly outline the proposed experiment; the feasibility study is left to chapter 3.

Type of Particles to be Studied

The question of which particles should be considered for measurement will greatly depend upon the method that is to be used. The basic idea is to see how well one can align the particles in an electric field, this being proportional to the size of the EDM on the particle.

There are two points that provide guidance in this matter. The fact that the EDM is a static property suggests the use of a stable particle; and the fact that charged particles wildly accelerate under any applied electric field suggests the use of a neutral particle. That these guidelines are sensible is borne out by the best available data; the neutron, which most closely fits the above guidelines, presently has the lowest value for the upper limit to its EDM.

Parenthetically, these guidelines are not meant to rule out any measurement in other situations. For instance the EDM of the muon has been measured directly in the CERN Muon Storage Ring by Bailey et al (1978). The technique used follows closely the "g-2" precession experiments. As a result of the Lorentz transformation of the laboratory magnetic field the muon experiences an electric field in its rest frame proportional to the particle velocity. If the muon has an EDM, this electric field will cause an extra precession over that predicted by quantum electrodynamics from the muon anomalous magnetic moment.

Motivation Behind Bulk Measurements

In chapter 3 the neutron-spin resonance experiments are reviewed. The major disadvantages that become apparent from this review, are low densities of neutrons and short intervals during which the neutrons are in the apparatus.

Nonetheless, the great success with these neutral particles leads one to consider measuring the EDM of neutral atoms - which may be obtained in almost any quantity, in almost any variety. Ideally, we choose an atom whose nucleus most resembles the neutron. This is done because theoretical interest is primarily in determining the EDM of the neutron rather than that of other particles.

The question of using Helium-3 for such a bulk experiment then arises. Helium-3 consists of a pair of protons and a neutron, all assumed in S-states. The protons are in a spin singlet state; so the angular momentum, magnetic moment and EDM of Helium-3 are presumably caused solely by the neutron. The magnetic-dipole moment of the Helium-3 nucleus is $-2.1275\mu_n$, where μ_n is the nuclear magneton, which differs from the neutron's magnetic moment of $-1.91315\mu_n$ by only about 11%, so one might similarly expect the EDM of the Helium-3 nucleus to be good approximation to the EDM of the neutron. Even so, it is necessary to make a less tenuous connection between the EDM of the neutron, and the result our experiment would yield the EDM of the Helium-3 nucleus. This problem is investigated by considering the case of the deuteron in chapter 2.

Of far greater importance is the fact that as an atom, the Helium-3 nucleus is surrounded by electrons which may seriously shield the nuclear EDM. Much work has already been done in this area; in chapter 2 some interesting aspects of this shielding are uncovered which throw light on the extent of nuclear shielding due to the protons. We also give this subject of shielding considerable attention in chapter 4.

General Overview of a Bulk Experiment

As was pointed out in section 1.2, the EDM of a system must be directed along the spin axis of that system. The magnetic-dipole moment is also directed along this axis. Thus these two properties are tied together so that if we can align one of them, we necessarily align the other. In most experiments, including the suggestion we are investigating, this co-alignment is the basis of the EDM measurement.

If an isolated neutral system with an EDM is placed in a uniform electric field, the spin of the system will precess about the electric field vector, just as a magnetic-dipole moment will precess in a magnetic field. This precession will not produce any net alignment. If, however, this system is not isolated, for instance an atom in a fluid, then the spin-spin interactions between the atoms will allow the spin orientations to change. If the fluid is given sufficient time to reach equilibrium there will be a net alignment of both the electric and magnetic-dipole moments.

Since the fluid will contain many magnetic-dipole moments aligned with the electric field, the fluid will have a net magnetization which could be measured very accurately with superconducting SQUID magnetometers (see figure 2 in chapter 3, and the related discussion for the particular case of interest, that of liquid Helium-3 as the fluid). This alignment will then give a measure of the size of the EDM on each particle in the fluid.

CHAPTER 2

Section 2.1

OUTLINE OF CHAPTER

As suggested in chapter 1 the shielding effects of the electrons surrounding the nucleus may be severe. In fact there are classical arguments which conclude that this shielding of the nuclear EDM should be complete. This is important if true, because it says that the experiment we propose, or indeed any variation of it, whether with atoms or molecules, is doomed to be completely insensitive to the EDM which it seeks to find. We discuss these points briefly and point out some difficulties associated with them.

The first attempts we made at calculating the shielding involved calculations for atomic Hydrogen and Helium. These calculations showed that there should be only relatively mild screening of the nuclear EDM. In each case the screening is about 20%. These calculations, however, were soon shown to be inadequate by a very general quantum-mechanical proof that the shielding is in fact complete.

Because of the surprising generality of this theorem we reconsider our calculations to determine the source of the discrepancy. In doing so we discover where the maximum contribution to this screening comes from. This discovery has important consequences for the size of the nuclear shielding and is discussed further in section 2.3.

Section 2.2

Classical Argument for Atomic Shielding of the EDM

As early as 1950, Purcell and Ramsey argued that most experiments that might be used to search for nuclear EDM's on atoms and molecules would in fact be very insensitive, owing to the smallness of the electrical field at the position of a charged nucleus, that is owing to the shielding of the electrons.

This argument may be explained a little more clearly. Consider a charged particle with a non-zero EDM on it. One must apply an electrical field at the particle in order to generate an interaction energy, which is observed in some parameter. Now, a charged particle cannot exist unaccelerated in an electric field. However, charged particles in neutral matter (atoms, molecules, or solids) have time-averaged motions which are either at rest or uniform (drift) velocity with respect to the applied electric field. Such arrangements have essentially no sensitivity to EDMs. All the forces acting on the particle in matter are electrical; the charged particles will move at constant velocity in zero-average field.

Thus proposals to observe a shift in the resonant frequency in nuclear magnetic resonance carried out in the presence of a strong electrical field, fail because the nucleus simply shifts to the new minimum in the total electric potential which it experiences. This same objection would apply to the experiment we proposed in chapter 1.

This argument was in fact put forward by Garwin and Lederman in 1959, but it glosses over two points. Firstly, the way the electric-dipoles and charges interact with an electric field differs, thus the statement that all forces acting on a particle in matter are electrical is no longer correct. If we had an EDM of large magnitude in neutral matter where the charges were of vanishingly small size, it is hard to believe that there would be any shielding of significance, since the charges being so weak could not be accelerated to their new screening positions fast enough.

Secondly, we must consider atoms in terms of quantum mechanics, which describes particles as wavefunctions spread out in space. Thus, even if the particle were at the minimum of the electric potential, its wavefunction could sample the region away from this position, hence the EDM on the particle could have some interaction with the external electric field.

These two objections put the classical argument in question. In fact the first point shows that even if the classical theorem is true, it is only true for small EDM's. Now the argument is based on the experimental fact that all charged particles in neutral matter have time-averaged motions of uniform velocity, or at least observably so. Thus we must conclude that the EDMs involved are of negligible size. If we take them as being small compared to the product of characteristic length involved times the characteristic charge, this immediately puts a limit on the EDM of electrons and all nuclei of familiar matter as having a value below 10^{-10}e.cm. We would expect an even smaller limit from the same experimental fact if the classical argument were not true for even small values of the EDM.

SHIELDING IN SIMPLE CASES

Since the classical shielding argument of the last section may be questioned, it is worth investigating two simple cases: shielding in Hydrogen and Helium.

Hydrogen

We consider the shielding of a nuclear EDM in atomic Hydrogen. To calculate this shielding we consider a non-relativistic, time-independent perturbation calculation for Hydrogen, neglecting the magnetic moments of the particles; we write the Hamiltonian as: $H = H^o + H^1$

where
$$H^o = \frac{-\hbar^2}{2\mu_e}\nabla^2 - \frac{e^2}{4\pi\varepsilon_o}\frac{1}{r}$$

and
$$H^1 = \frac{-ed_n}{4\pi\varepsilon_o}\frac{\cos\theta}{r^2}$$

(μ_e is the reduced electron mass, and d_n is the nuclear EDM taken along the positive z-axis). As d_n is very small, first order terms in it will suffice; as we want the induced EDM to first order, we must have the wavefunction to first order:

$$|\psi^{n(1)}\rangle = |\psi_n^{(0)}\rangle - \sum_{k\neq n} |\psi_k^{(0)}\rangle \frac{\langle\psi_k^{(0)}|H^1|\psi_n^{(0)}\rangle}{E_k^{(0)} - E_n^{(0)}}$$

where $|\psi_n^{(1)}\rangle$ is the unnormalised eigenstate of H (normalisation changes $|\psi_n(0)\rangle$ to only second order in the EDM), where $|\psi_n^{(0)}\rangle$ are the exact eigenstates of H^o, with eigenenergies $E_n^{(0)}$. We see from this expression that the eigenstate of H is a mixture of the eigenstates of H^o; this mixing of electron states may be thought of as a distortion of the electron's charge cloud, which sets up an EDM.

Taking the dipole operator in the z-direction, $D = -e.r.\cos\theta$

then $$\langle D\rangle = \sum_{k\neq n} \langle\psi_n^{(0)}|D|\psi_k^{(0)}\rangle \frac{\langle\psi_k^{(0)}|H^1|\psi_n^{(0)}\rangle}{E_k^{(0)} - E_n^{(0)}} + c.c. \qquad (1)$$

If we now take the contribution from only the first opposite parity state 2p, mixing with the 1s ground state gives:

$$\langle D\rangle = \frac{-4d_n}{E_{2p} - E_{1s}} \langle 1s|r\cos\theta|2p\rangle\langle 1s|\frac{\cos\theta}{r^2}|2p\rangle$$

which gives $\langle D\rangle = -0.2081d_n$, so the atomic wavefunction of Hydrogen sets up an EDM which shields the nuclear EDM by 20.8%. It is reasonable not to expect much from the other states as in equation (1) the overlap integrals successively reduce, and the energy denominator becomes larger.

If this analysis is correct, we may conclude that the EDM of the deuterium atom would be 80% of the neutron's EDM; taking the proton and electron as having zero EDM.

Helium

Having gained confidence from the calculation of this screening, we next calculate it for Helium. The Hamiltonian is taken as: $H = H^o + H^1$

where $$H^o = -\hbar^2\left(\frac{\nabla_1^2}{\mu_1} + \frac{\nabla_2^2}{\mu_2}\right) - \frac{2.e^2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + \frac{e^2}{4\pi\varepsilon_0}.\frac{1}{r_{12}}$$

As the perturbation H^1 is independent of spin, only excited states which are spin singlets can mix with the ground state $|1s^2{:}^1S\rangle$. Again, we argue that the first spin-singlet excited state $|1s2p{:}^1P\rangle$ will give the greatest contribution. Unlike the case of Hydrogen, we do not have access to the exact wavefuntions for the Helium atom described by the Hamiltonian H^o. Instead, we shall use the wavefunctions determined by Eckart (1930) using the variational principle.

We start with a simplified ground state consisting of two S-wave electrons in identical orbits, and the simplest spin-singlet opposite parity state as two electrons occupying an S and P wave orbit. As the wavefuntion must be antisymmetric with respect to the interchange of particle labels and they are already antisymmetric in the spin-component of their wavefunctions, the spatial part of these wavefunctions is written in terms of one body wavefunctions as:

$$|1s^2\!:\!{}^1S\rangle = |1s\rangle_1\,|1s\rangle_2$$

and $$|1s2p\!:\!{}^1P\rangle = \frac{1}{\sqrt{2}}\left(|1s'\rangle_1\,|2p\rangle_2 + |2p\rangle_1\,|1s'\rangle_2\right)$$

the prime on the 1s orbit in the excited state signifies the "natural" 1s orbit according to the variational procedure, which is different, depending on the places other electrons take in a state.

Eckart takes the one body wavefunctions (neglecting spin parameters) as:

$$\begin{aligned} 1s\rangle &= \sqrt{(4\alpha^3)}.e^{-\alpha r}.Y_{00} \quad , \quad \alpha \simeq 2\text{-}5/16 \\ 1s'\rangle &= \sqrt{(4\beta^3)}.e^{-\beta r}.Y_{00} \quad , \quad \beta \simeq 2 \\ 2p\rangle &= (4/3.\gamma^5).r.e^{-\delta r}.Y_{10} \quad , \quad \gamma \simeq 0.5 \end{aligned}$$

The induced EDM, calculated in the same manner as for Hydrogen is:

$$\langle D\rangle = \frac{-2^{16}}{3\Delta E}\,\frac{\alpha^6\,\beta^3\,\gamma^5}{(\alpha+\beta)^6(\alpha+\gamma)^7}\,d_n \quad ,\text{where } \Delta E = 1.514\text{Ryd.}$$

thus $\langle D\rangle = -0.1381 d_n$, which represents a shielding of the nuclear EDM by 13.8%.

Using Eckart's more sophisticated ground state:

$$|1s''1s'''\rangle = \frac{1}{\sqrt{[2(1+c^2)]}}\left(|1s''\rangle_1|1s'''\rangle_2 + |1s'''\rangle_1|1s'''\rangle_2\right)$$

where 1s'' and 1s''' are of the form 1s only with the parameters τ and σ replacing the parameter α of 1s, then:

$$c = \frac{8\,\tau^{3/2}\,\sigma^{3/2}}{(\tau+\sigma)^3}$$

In this case, the induced atomic EDM is:

$$\langle D \rangle = \frac{-d_n 2^{15} \tau^3 \sigma^3 \beta^3 \gamma^5}{3(1 + c^2)\Delta E}$$

$$\times \left\{ \frac{1}{(\tau+\gamma)(\sigma+\beta)^6} + \frac{1}{(\tau+\gamma)^5(\sigma+\gamma)^2(\tau+\beta)^3(\sigma+\beta)^3} + (\tau \leftrightarrow \sigma) \right\}$$

and from Eckart τ = 2.14, σ = 1.19, and ΔE = 1.5106, this gives $\langle D \rangle = -0.1986 d_n$, or 20% screening.

Given the above and the picture of the Helium-3 nucleus as closely resembling a neutron, it would seem that, in contrast with the classical screening arguments, measurement of the EDM of liquid Helium-3 in bulk involves no intrinsic difficulties.

General Quantum Screening Argument

Although these above calculations may look reasonable, the work of Schiff (1963) shows that they are seriously inadequate.

Schiff has shown under very general conditions that there is complete shielding of nuclear EDMs. He showed that for a quantum system of point-charged particles in a net-neutral system that there is no term in the interaction energy that is of first order in the EDM, regardless of the magnitude of the external potential.

It is interesting to note at this point that our objections to the classical argument are in agreement with Schiff's work. Firstly, there are terms of second order in the EDM in the expression for the interaction energy according to Schiff, thus, if the magnitude of the EDM were large, these terms could no longer be neglected. Secondly, Schiff shows for a particle whose wavefunction describes a charge distribution and EDM distribution that differ, that the shielding is not complete, although it is still excessive. Schiff does not, however, say what mechanism could cause this difference in distributions; we discuss this further in chapter 4.

Including magnetic effects due to the magnetic-dipole moments, Schiff concludes that the nuclear EDM is shielded to one part in 10^7. We leave further detailed discussion of Schiff's theorem and the exceptions to it for chapter 4.

Further Investigation of Hydrogenic Shielding

As Schiff's argument uses only operator algebra, and does not consider a mixture of individual states, it does not directly tell us where our assumptions were wrong, at least for the case of Hydrogen.

Apart from the method of calculation our assumptions differ from Schiff's in only one respect; we include only the contribution from the first excited state. To see if the inclusion of higher excited states makes a difference, we calculate that contribution which is due solely to the complete series of odd parity bound states. In doing this, we need only restrict ourselves to P-states, as the perturbing Hamiltonian changes angular momentum by a single unit when applied to first order, due to the $\cos\theta$ term. Calculating this gives:

$$\langle D\rangle = \frac{-2^7}{3} d_n . \sum_{n=2}^{\infty} \frac{n^4 (n-2)!}{(n+1)} \times \sum_{l=0}^{n-2} \left\{ \frac{(-2)^l}{l!(n-l-2)!(l+2)(l+3)(n+1)^2} \right\} \times \sum_{k=0}^{n-2} \left(\frac{(-2)^k (k+4)}{k!(n-k-2)!(n+1)^k} \right)$$

that is, about 28% shielding.

Thus we have concluded correctly that the higher bound-states do not give a large contribution. This result is different from the earlier hydrogenic result by about 7%, in terms of total shielding. Further, we must conclude that it is the mixing of the continuum states with the ground state which gives the dominant contribution to shielding; for Hydrogen this contribution is around 72%.

Preliminary Discussion of Nuclear Screening

Just as the electron charge clouds can redistribute themselves so as to almost completely screen the nuclear EDM, so the proton charge-clouds may shield an internal EDM on an unpaired particle in the nucleus. Neither the Schiff theorem nor the classical argument apply to this situation since not only is the Coulomb force not the only force involved, but it is not even the most significant force in the nucleus, which is the strong nuclear force.

Thus, the exact screening should not occur.

Now, the nuclear system can be considered a much "stiffer" system than the atomic system. This is because the typical energy differences are 10^6 times larger for the nuclear system. Thus, one might expect that the mixing of nuclear states that is necessary to make up an opposing EDM would be negligibly small. Looking at equation (1), however, for the expectation value of this induced EDM, we see that matrix elements of the dipole operator and perturbing Hamiltonian are also involved, so taking into account the fact that the nuclear system is five orders of magnitude smaller than the atomic system, the screening due to the nucleus may be about 10%.

We note here that if we only included mixing with bound states there would be no shielding in the deuteron as it has only one bound state, its ground state. Hence trivially, in this case, the mixing of the ground state with continuum states must dominate. This domination by the continuum states in mixing occurs then for Hydrogen, Helium, the deuteron and probably most other cases. We consider this in more detail in section 2.3.

<u>Plane Wave Calculations</u>

Clearly a perturbation sum over discrete states is inadequate. To include the sum over the free states entails considerable work, especially if we wish to investigate anything more complicated than a two-body system. Going back to the perturbation expansion of equation (1), all we really need to do is sum over any complete set of states excluding only the ground state as this term has an infinite denominator. Even if we keep this term in explicitly, it will automatically vanish because the operators involved change parity. Thus we can choose any complete set (we should of course point out that the bound states of Hydrogen do not form a complete set).

For simplicity we choose a basis of plane waves. This choice will make it easy to "sum the free states", though it may not give the contribution from the bound states very well, as the expansion of a localised bound state into plane waves requires infinitely many terms up to infinite frequency, the tail of this series then converges only very slowly. This is not critically important since the greatest contribution comes from the free states.

The beauty of this method of calculation is that we are only required to have a good ground state. The induced EDM is then:

$$\langle D\rangle \simeq -\int_k \frac{\langle g.s.|D|k\rangle\langle k|H^1|g.s.\rangle}{E_k - E_{g.s.}} + c.c.$$

where $|g.s.\rangle$ is the ground state wavefunction for Hydrogen, and $|k\rangle$ is the plane wave state, $e^{i\vec{k}.\vec{r}}$, for a one particle system, and D is the dipole operator $e.r.\cos\theta_r$, using the decomposition:

$$e^{i\vec{k}.\vec{r}} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{+l} i^l j_l(kr)Y_{lm}^*(\Omega_k)Y_{lm}(\Omega_r)$$

we find that the general matrix element is:

$$\langle k|O(r)\cos\theta|g.s.\rangle = 4\pi i.\cos(\theta_k)\int_0^k r^2 O(r)j_1(kr)\phi(r)$$

where $j_1(x)$ is the first order spherical Bessel function. Using this expression, the induced EDM may be shown to be:

$$\langle D\rangle = \frac{-2^{12}}{3^2} a_B^{2}\,2.E_B.d_n\int_0^{\infty}\frac{z^4.dz}{(z^2+1)[a(z^2+1)^2+b-a]}$$

where $a = \hbar^2/2\mu$, μ is the reduced electron mass, and $b = a_B^2E_B$, a_B is the Bohr radius and E_B is the ionisation energy of the Hydrogen ground state. This integral was calculated out to z = 50, which gave $\langle D\rangle \simeq -0.91636d_n$, that is 92% shielding. Again we know the answer should be 100% shielding; the discrepancy is attributed to the problems with the plane wave basis, as already described. Even though imperfect, this method promises to give an answer good to within about 10%; or even better when fewer bound states occur as, perhaps, in nuclear calculations. However, it must be pointed out that this accuracy is dependent on the accuracy of the ground state employed in the calculation.

We have not used this method in any other situation, though it promises to be very useful for many-body calculations especially when applied to the nucleus. However, as we have shown that the complete sum of states, both bound and free, gives almost total shielding (92% being only 8% different from the 100% expected) and the sum of all bound states gives 28% shielding, all for the case of Hydrogen, we have clearly demonstrated the cause of the failure of our earliest calculations; the free state mixing makes the dominant contribution to the screening.

As already mentioned, Schiff has shown for the case of Helium that the shielding is complete to one part in 10^7; this will decrease the sensitivity of our proposed experiment by seven orders of magnitude.

Section 2.3

EDM of a Nucleus

Although any observation of an EDM on an atom or nucleus would yield new information about the mechanism of T violation, it is important to relate any observed parameter back to theory. At present, any theory predicting T violation is used to calculate essentially one value, the EDM of the neutron. As the experiment we propose does not measure this value directly, it is necessary to make the connection between the neutron's EDM and the EDM of a nucleus if we wish to use any results we might obtain to distinguish between the large variety of theories.

We start by considering the deuteron nucleus.

Shielding of the Neutron's EDM by the Proton in the Deuteron

To calculate the EDM of the deuteron due to an EDM on the neutron, we take a simple model Hamiltonian:

$$H^0 = \frac{-\hbar^2}{2\mu}\nabla^2 + V(r)$$

where μ is the reduced mass of the nucleon. With the ground state taken as:

$$|\psi\rangle = \frac{1}{\sqrt{(2a^3)}}.e^{-(r/2a)}.Y_{00} \qquad (2)$$

with a = 1.5fm to give the correct deuteron root-mean-square charge radius. The induced EDM is then given by

$$\langle D\rangle = \frac{e^2 d_n}{4\pi\epsilon_0} \sum_{i\neq\psi} \frac{1}{W_\psi - W_i}(\langle\psi|\frac{\cos\theta}{r^2}|i\rangle\langle i|r\cos\theta|\psi\rangle + \text{c.c.}) \qquad (3)$$

where W_i is the energy of state $|i\rangle$.

The potential $V(r)$ of H^0 is assumed to have no velocity dependence; otherwise it has no restrictions on the form it might take. This approximation is good to better than 10%, and it allows us to use the method of Dalgarno and Lewis (1955). This method requires that we find an operator F which satisfies:

$$[F,H^0]|\psi\rangle = r.\cos\theta|\psi\rangle \tag{4}$$

If this can be done then, since:

$$\langle i|r.\cos\theta|\psi\rangle = \langle i|[F,H^0]|\psi\rangle = (W_\psi - W_i)\langle i|F|\psi\rangle$$

we may remove the energy denominator in equation (3) and use closure to obtain

$$\langle D\rangle = \frac{e^2}{4\pi\varepsilon_0} d_n(\langle\psi|\frac{\cos\theta}{r^2}.F|\psi\rangle + c.c) \tag{5}$$

Now F can be any operator, so if we further suppose that it is solely spatial, then it will automatically commute with the potential V(r), since we are neglecting its velocity dependence. Under this ansatz, (4) becomes a simple differential equation with a solution:

$$F = \frac{-\mu}{\hbar^2} r.a(4a + r).\cos\theta$$

Substituting this into equation (5) gives:

$$\langle D\rangle = \frac{-2e^2}{4\pi\varepsilon_0}\frac{\mu a}{\hbar^2} d_n = -0.0512.d_n \tag{6}$$

Thus, taking the ground state of the form in equation (2) yields 5% shielding. This calculation is essentially independent of the potential we use; the dependence actually enters the calculation in the exact shape of the ground state (which we only approximate).

If we consider a more compact wavefunction than that given by equation (2), which is after all a hydrogenic wavefunction and so not entirely suitable for a nuclear calculation, we might try:

$$|\psi\rangle = \frac{2}{\pi^{1/4}.\alpha^{3/2}}.e^{-(r^2/2\alpha^2)}$$

To get the same root-mean-square charge radius as before we choose $\alpha = 2\sqrt{2}a$. In this case we find:

$$F = \frac{-\mu}{\hbar^2}.\alpha^2 r.\cos\Theta$$

so

$$\langle D \rangle = \frac{-e^2}{4\pi\varepsilon_0}\frac{\mu}{\hbar^2}\frac{4\alpha}{3\sqrt{\pi}}d_n = -0.109d_n \qquad (7)$$

that is 11% shielding (we have again neglected the velocity dependence).

These calculations demonstrate that the EDM of the nucleus is about 90% to 95% of the EDM of the neutron. We can expect similar results for many nuclei, noting that the results shown in equations (6) and (7) display the proportionality of the induced EDM to the size of the system, so the shielding is likely to increase as $Z^{1/3}$.

To conclude this discussion, we point out that these results do not modify the predictions of any theory as all these theories only estimate the neutron EDM to within an order of magnitude or so.

CHAPTER 3

Section 3.1

Outline of Chapter

The aim of this chapter is two-fold. Firstly, we briefly review the magnetic beam resonance method used in searching for the EDM of the neutron - presently the most sensitive method. We then consider a novel method employing an idea using liquid Helium-3. The advantages of a cryogenic experiment are: working with large numbers of particles (three grams of Helium-3 constitutes one mole - $6x10^{23}$ particles),and the fact that at these low temperatures it is possible to use very sensitive superconducting magnetometers capable of measuring 10^{-4} fluxoids, at low noise levels.

Unfortunately, the analysis shows this novel technique, even at its best, to be no more sensitive than the present beam method employing neutrons.

Section 3.2

Review of Neutron Measurements

The most sensitive experiments seeking to discover the EDM of any particle are those which employ magnetic beam resonance methods on neutrons. We shall now review how these neutron experiments are performed, what their limitations have been and are at present, and what are the prospects for their future improvement.

The Magnetic Resonance Technique

Consider figure 1. The neutrons enter the apparatus and are polarized by total reflection from a polished magnetized iron mirror at 1. The beam then passes into the region of uniform magnetic field produced by the magnet 2, and uniform electric field applied parallel to the magnetic field produced by the parallel plates 3. The polarization of the beam is analysed at 4 before the neutrons are passed into the detector at 5 and counted. When a radio-frequency magnetic field is applied to the separated coils 6 and 7, at the

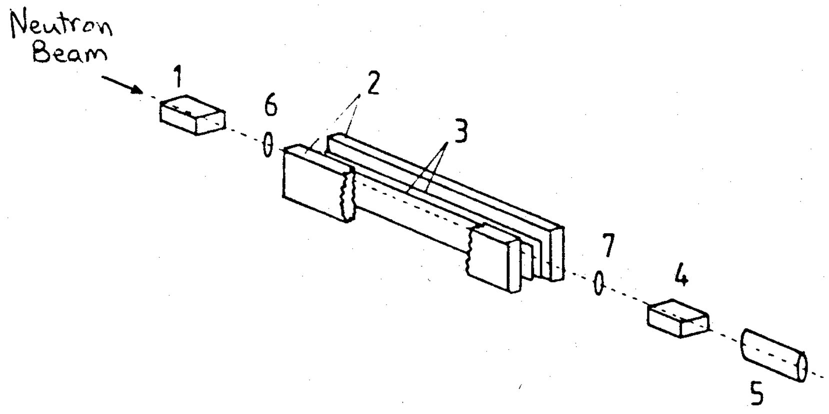

Figure 1. Schematic diagram of magnetic resonance apparatus for measuring the EDM of the neutron. 1. Spin polarizer, iron magnetic mirror; 2. Pole pieces for the uniform magnetic field; 3. Electrostatic plates for the uniform electric field; 4. Spin analyzer, magnetized iron transmission analyzer; 5. Neutron detector; 6. and 7. Coils for the radio-frequency magnetic field.

average Lamour frequency:

$$f = \frac{\mu B}{\hbar} \qquad (1)$$

where μ is the magnetic moment of the neutron, spin-flip transitions can occur. The intensity of the beam passing through the analyzer will consequently decrease. If the neutrons had an EDM oriented along the spin, the homogeneous electric field at 3 would alter the torque on the neutron and would cause a change in the precession frequency. This, in turn, would result in a shift in the frequency at which a maximum number of spin-flip transitions was induced. The experiment consists of measuring the resonance curves. As the radio-frequency of the magnetic field is changed, the intensity varies since the neutrons have their spins flipped by varying amounts. These curves are then fitted to theoretical curves with the neutron EDM as the free parameter.

Discussion of the Limitations of the Beam Methods

This procedure was first used by Smith et al (1957) who obtained a limit on the neutron EDM of $d_n < 5\times10^{-20}$e.cm (all limits are quoted at the 90% confidence level). Since then, many improved measurements have been made; the most accurate result obtained with this method is $d_n < 3\times10^{-24}$e.cm by Dress et al (1977, this paper also contains a survey of these improvements).

By about 1977, the possibilities for using neutron beams had been exhausted for two major reasons. The first is that as the phase angle to be measured is proportional to the duration of the precession, and so is proportional to the time the neutron spends in the apparatus, then the sensitivity of the system to EDM's could be increased by lengthening the region of uniform fields, and lowering the speed of the neutrons. The 1977 experiment used cold neutrons with speeds of 200m/s from the refrigerated moderator at the high-flux reactor in Grenoble. The use of still slower neutrons involves losses of statistical accuracy due to low fluxes at these speeds, and lengthening the apparatus encounters considerable technical difficulties.

The second limitation is that in the rest frame of the neutron, the electric field looks like a magnetic field, thus causing an additional precession due to the magnetic-dipole moment μ_n. This leads to a spurious effect which could be interpreted as an EDM of size:

$$d_n = \mu_n \frac{v_n}{c} \sin\theta \qquad (2)$$

where θ is the angle between the directions of the magnetic and electric fields. We can now see the technical difficulties associated with increasing the size of the operational part of the apparatus; the magnetic and electric fields must be accurately parallel over the entire length.

The way to circumvent these problems was well understood - the use of ultra-cold neutrons in a closed cavity, though for a long time there were no sources which could give adequate quantities of ultra-cold neutrons.

Using such a container with ultra-cold neutrons, instead of a neutron beam, one can increase the time neutrons spend in the fields by several orders of magnitude and so raise the sensitivity. Also the low velocities sharply reduce the spurious effect (see (2)). In fact the apparatus can now have smaller dimensions, and still have large transit times, the alignment achievable between the magnetic and electric fields has in some instances rendered the spurious effect negligible.

The last result using this technique, and the best so far, $d_n < 6\times10^{-25}$e.cm was obtained in 1981 by Altarev et al (for a more complete review see Ramsey, 1982).

It must be pointed out that this increase of accuracy for bottled neutrons over beam methods was obtained with a reactor with a neutron flux an order of magnitude smaller than that of the extremely "bright", high luminosity, I. L. L. Grenoble reactor. This means that a further advance in accuracy can be expected in the not too distant future.

Section 3.3

Outline of the Proposed Method for Measuring EDMs in Bulk

We next consider a very different type of experiment for measurement of the EDM of the neutron. Consider figure 2, where the material under study is placed between two capacitor plates, 1, which are used to produce a large electric field. If the atoms in the material have an EDM, they will be aligned by this field. Since this alignment is an alignment of the spins of the atoms, the material will possess a net magnetization which is measured by the SQUID magnetometer, 2.

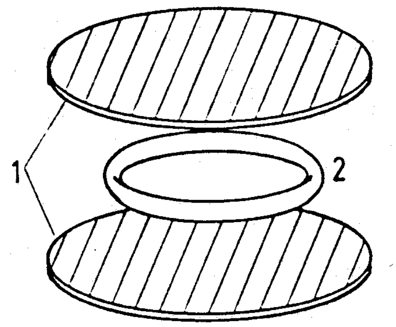

Figure 2. Schematic diagram of our method for measuring the EDM of Helium-3. 1. The capacitor produces a uniform electric field across the liquid Helium-3. 2. The SQUID magnetometer measures the net magnetization of the liquid.

The advantages that this method has over the spin-resonance method lie mainly in modern cyrogenic apparatus. Liquid Helium-3 refrigerators already exist which can supply large quantities of Helium-3 at temperatures as low as 1mK. At these temperatures or even as high as a few degrees Kelvin, it is possible to use sensitive SQUID magnetometers to measure the magnetic field induced. Again this technology already exists. In addition, the low temperatures ensure low noise in the measuring devices, although there is still considerable Brownian paramagnetic noise this can be significantly decreased by synchronous detection techniques.

Previous Methods for Measuring EDM's in Bulk

In fact this method is not as novel as might be supposed; Schiff (1963) mentions a private communication from Fairbank, in which the latter suggests measurement using a dilute solution of Helium-3 in Helium-4, since great sensitivity can be achieved by looking for a precession in a time of the order of hours or days. This is not possible for neutrons, since their mean life of 15 minutes sets an upper limit to their storage time.

Another paper by Vasil'ev and Kolycheva (1978) outlines a very similar method to the one considered here. In this paper, they describe an experiment they performed seeking the EDM of the electron. Their experiment used a SQUID to detect the change in flux through a sample of a Nickel-Zinc ferrite when an electric field was applied. To overcome the problem of thermal noise, they used a digital synchronous detector with a time constant increased to several hours, and they flipped the field at 30Hz. They obtained a limit of $3x10^{-22}$e.cm for the electron's EDM. They concluded that this method was unlikely to be improved in sensitivity because of many sources of noise with large time constants. Their result may be compared with results using a metastable xenon beam (Player and Sandars, 1970) where a limit to the electron's EDM of $2x10^{-24}$e.cm was set.

Flux Quantization

The term SQUID refers to a loop of small inductance containing at least one Josephson junction (a detailed account of these devices may be found in many places, eg. Van Duzer and Turner, 1981). For quantum mechanical reasons a superconducting loop can only contain certain quantized values of magnetic flux given by:

$$\Phi_B = \frac{nh}{2e} \tag{3}$$

where n is an integer. Thus we may talk of a fundamental unit of magnetic flux as a "fluxoid":

$$\Phi_B = \frac{h}{2e} = 2.068\text{x}10^{-15}\text{Wb}$$

This would seem at first glance to be the limit of measurement, but in fact it is possible to use a SQUID to interpolate "between fluxoids". With the aid of a flux transformer an accuracy of around 10^{-4} fluxoids is obtainable.

Thermal Fluctuation in the Net Magnetization

In order to calculate the thermal fluctuation in the net magnetization of the liquid Helium-3, we suppose our system of spin-half particles suitably approximates a paramagnetic system; this could be achieved in practice by using a dilute solution of Helium-3 in Helium-4, much as was suggested by Fairbank (Schiff, 1963).

The single particle energies of this system in an external field are given by:

$$E_1 = \mp x/\beta, \quad x = d.E.\beta$$

for the spin aligned/anti-aligned with the field, where d is the EDM, E is the external electric field, and $\beta = 1/kT$ (k is the Boltzmann constant, T is the temperature). The total partition function is given by:

$$Z = (e^x + e^{-x})^N$$

The probability of having N_+ spins aligned with the field, and N_- anti-aligned ($N = N_+ + N_-$) is then:

$$Pr(N_+) = \frac{(e^x)^{N_+}(e^{-x})^{N_-}}{Z} \frac{N!}{N_+!N_-!}$$

and so:

$$\langle N_+^m \rangle = \sum_{N_+=0}^{N} N_+^m.Pr(N_+)$$

$$\langle N_+^m\rangle = \sum_{N_+=0}^{N} N_+^m \frac{e^{-xN}}{Z} e^{2xN_+} \frac{N!}{N_+!N_-!}$$

Although this is difficult to calculate directly, by noting that:

$$\frac{d^m}{dx^m} e^{2xN} = (2N_+)^m e^{2xN_+}$$

and using $\langle 1\rangle = \langle (N_+)^0\rangle = 1$, we find that:

$$\langle N_+^m\rangle = \frac{1}{2^m} \frac{e^{-xN}}{(e^x+e^{-x})^N} \frac{d^m}{dx^m}[e^{xN}(e^x+e^{-x})^N]$$

and so $\langle N_{net}\rangle = \langle N_+ - N_-\rangle = \langle 2N_+ - N\rangle = N.\tanh x$ (4.1)

further, $(\Delta N_{net})^2 = \langle N_{net}^2\rangle - \langle N_{net}\rangle^2 = 4(\langle N_+^2\rangle - \langle N_+\rangle^2)$

so $\Delta N_{net} = \sqrt{(N - N.\tanh^2 x)}$ (4.2)

or for $x \ll 1$: $\langle N_{net}\rangle = N.x$ (5.1)

and $\Delta N_{net} = \sqrt{N}$ (5.2)

Net Magnetization in a Static Electric Field

To be able to detect a net magnetization of our material under a small applied electric field, we would like to have the net magnetization at least ten times larger than the thermal fluctuations, ie. $\langle N_{net}\rangle > 10.\Delta N_{net}$ (as $x \ll 1$) which from equation (5) gives $Nx > 10\sqrt{N}$. Thus, for any net magnetization to persist above the thermal fluctuations, we need: $x > 10^{-10}kT \simeq 10^{-10}$, that is we need $d.E > 10^{-10}kT$. Taking the electric field as approximately $1kV.cm^{-1}$, to find an EDM of at least 10^{-25}e.cm, we would require $T < 1.2\times10^{-8}$ °K. At present the most ambitious experiment could be done at 10^{-3} °K, even at this low temperature a static electric field arrangement will be restricted to a sensitivity down to a value of only 10^{-20}e.cm.

PARASITIC EFFECTS

It turns out that there are at least two effects that could echo the EDM: (1) the interaction of the magnetic-dipole moment with the electric field; and (2) magnetoelectric effects.

Magnetic-Dipole Effects

This effect constituted the major limiting factor in the neutron beam resonance experiments, and so merits first consideration. In the rest frame of the Helium-3 atom, the electric field will look like a magnetic field, thus producing a spurious dipole of a size given by equation (2), although now **r** is the angle between the capacitor plates and the face of the SQUIDs pickup coil.

Even with excellent alignment this effect could still be troublesome, since both the sample and the pickup coil are likely to distort the electric field, even if the capacitor plates supplying the electric field are large enough so that the fringing field causes no trouble. As the sample is a fluid, the average velocity is zero, and so the net effect will average to zero. However, this will contribute to the thermal noise of the system. To estimate the size of this affect, we take the average speed of a Helium-3 atom between collisions to be given by equating the kinetic energy of the atom to the equipartition energy: $1/_2.mv^2 = 3/_2.kT$, where m is the mass of the Helium-3 atom. This gives: $v/c \simeq 3x10^{-7}\sqrt{T}$, hence this spurious EDM will fluctuate with a size:

$$\Delta d \simeq \frac{1}{\sqrt{N}}\mu \frac{v}{c} \sin\theta \simeq 6x10^{-21}\sqrt{\frac{T}{N}} \sin\theta \text{ e.cm}$$

where N is the number of Helium-3 atoms involved. Even with concentrations as low as 0.1% we get: $\Delta d \simeq 6x10^{-31}$ T.sinθ.e.cm.

This calculation will begin to be inaccurate at low temperatures because of the zero-point motion of the particles. We may estimate this zero-point velocity from the Heisenberg uncertainty principle, giving:

$$\frac{v}{c} = \frac{\Delta pc}{mc^2} \simeq \frac{hc}{mc^2\Delta x} \simeq 6x10^{-7}$$

where Δx, the uncertainty in the position of the particle, is taken to be about one angstrom. It follows that this fluctuation causes no limitation to either the working temperatures or to alignments for any measurement on a system of dipoles, down to about 10^{-30}e.cm.

Magnetoelectric Effects

Landau and Lifshitz (1960) predicted that there may be peculiar phenomena in some antiferromagnetic crystals. If a crystal is placed in a constant magnetic (electric) field, an electric (magnetic) moment proportional to the field is produced in it. This effect is called the magnetoelectric effect.

Dzyaloshinskii (1960) showed that since the thermodynamic potential of such a solid must contain terms proportional to E.B, this effect could not occur in a paramagnetic crystal, as its thermodynamic potential is invariant with respect to time-reversal.

We can see that this effect echoes exactly the effect we are searching for, so we have to be certain it does not occur in our case of Helium-3 in Helium-4. Since we also know that this affect does not occur in paramagnetic substances, we must limit the concentration of Helium-3 to ensure that the solution is sufficiently paramagnetic. Unfortunately, Dzyaloshinskii's argument is based purely on symmetry considerations, and gives no mechanism which could be used to calculate the onset of this affect and give us an idea of its size. The question of how low a concentration is low enough has been answered by Legget (1977).

Although a system characterized by a single angular momentum forbids an EDM by T invariance, if a system for some reason has two independently conserved angular momenta, say an orbital angular momentum and a spin, then it can have an EDM which though P nonconserving, preserves T invariance. Legget showed, by investigating the neutral currents of the Weinberg-Salam theory (see chapter 2, and references therein), that the Cooper pairs in superfluid Helium-3 have the property of a spontaneously broken spin-orbit symmetry, hence the relative orientation of spin and orbital coordinates is the same for all pairs, and remains constant in time. Accordingly this EDM is permenant but only occurs in the superfluid phase. Legget predicted the size of this EDM to be about 10^{-29}e.cm. Although this is small, when Schiff screening is taken into account it becomes larger than any T-violating EDM would be, we must therefore ensure that our apparatus would not be operating in the region of superfluidity.

The important thing to note about this affect is that because the EDM occurs with ~~a violation of~~ T invariance it is not of interest in terms of our original motivation, which was to test the various T-nonconserving theories. Khriplovich (1982), states that any EDM must herald broken T symmetry and goes on to suggest measuring this parity nonconserving spontaneous EDM of Legget in the superfluid phase of Helium-3, as a means of testing T-nonconserving theories. It would seem that Khriplovich is in disagreement with Legget on this point.

In order to keep our sample of Helium-3 in Helium-4 both away from the superfluid phase and out of the two-phase region, where the two liquids separate, it would seem (Wilks, 1967) that we are restricted to relatively high concentrations (a molar concentration of Helium-3 of about 0.6 or more) and relatively high temperatures (of around $0.5^{o}K$). This restriction to high concentrations is not what we originally envisaged to keep the system paramagnetic, but avoiding parasitic affects is more important. Furthermore, this restriction to high temperatures could lead to problems of thermal noise.

SENSITIVITY

The overall sensitivity of the experiment depends on a number of factors: (1) limitations arising from the geometry of the experimental setup; (2) the screening of the nuclear EDM; and (3) the limitations associated with noise. We shall consider each of these in turn.

Flux Measured in the SQUID

Using the net alignment of dipoles in a paramagnetic sample, as given by equation (4.1), we find the net magnetization to be:

$$M = m_n N.\tanh x, \qquad (6)$$

along the axis of the external electric field, where m_n is the magnetic moment of Helium-3 and N is the number of Helium-3 atoms per unit volume. As there are no free currents in the sample $\vec{\nabla}\times\vec{B} = \mu_0\vec{\nabla}\times\vec{M}$ and since $\vec{B} = \vec{0}$ when $\vec{M} = \vec{0}$

(B is the magnetic field from the sample), then $B = \mu_0 M$ and does not depend on the susceptibility of Helium-3.

Since the EDM is very small, less than 10^{-25}e.cm, $x = d.E\beta \ll 1$, then $\tanh.x \simeq x$, and we find the measured flux to be approximately:

$$\Phi_B \simeq B.A \simeq \mu_0 m_n N \beta d.E.A$$

where A is the area of the sample. So from equation (3.2), the flux in fluxoids will be:

$$\Phi_B = \frac{2e}{h} \mu_0 m_n NA \frac{d.E}{kT} \quad \text{(fluxoids)} \qquad (7)$$

<u>Flux from a Uniformly Magnetized Disc</u>

In calculating the flux for equation (7) we assumed that there was no fringing of the field, and more importantly, that no lines of flux returned into the sample. Either of these will decrease the flux measured in the SQUID, and may be considered to be a geometric effect, limiting the sensitivity of our experiment.

Although it would seem from equation (7) that we could increase the observed flux simply by increasing the area of the sample and pickup coil, we will see in what follows that such is not the case.

From Lorrain and Corson (1970), the vector potential from a single current loop is given by:

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} x \vec{r}}{r2}$$

$$A = \frac{\mu_0}{4\pi} \int_S \frac{\vec{M} x \vec{n}}{r} ds + \frac{\mu_0}{4\pi} \int_\tau \frac{xM}{r} d\tau$$

if we consider a uniformly magnetized disc $\vec{\nabla} x \vec{M} = \vec{0}$, and we may define an equivalent surface current density: λ_e = Mxn, where n is the unit normal vector to the surface. If the magnetization is directed along the axis of the disc then: λ_e = M. Hence the magnetic field B of a cylinder uniformly magnetized in the direction of its axis of symmetry is identical to that of a solenoid of the same dimensions that carries a current density N'I = M; N' is the number of turns per unit length.

Thus $B_{magnetization} = B_{solenoid}$, with I replacing M. Bartberger (1950) has calculated the magnetic field from a circular loop of wire at z = 0 (z is the distance along the axis from the loop) with radius ρ = a, and a current I to be:

$$B_z = A(I_1 - \rho . I_2/a)$$

where

$$I_1 = \frac{1}{\pi}\int_0^{\pi} \frac{1}{(1-b\cos\theta)^{3/2}} d\theta$$

$$I_2 = \frac{1}{\pi}\int_0^{\pi} \frac{\cos\theta . d\theta}{(1-b\cos\theta)^{3/2}}$$

and

$$A = \mu_0 I . a^2 \frac{1}{2(a^2+z^2+\rho^2)^{3/2}}$$

$$b = \frac{2a\rho}{(a^2+z^2+\rho^2)}$$

He also tabulated I_1 and I_2 for various values of 'b' over its entire range. Thus the flux through a loop (the pickup coil) distance 'z' away, and at a radial distance of 'a' is: $\Phi = \int_{ds} B_s . ds = 2\pi\int_0^a B_z \rho d\rho$

Thus, for our "solenoid":

$$\Phi_{tot} = \int_{z=0}^{h} \Phi(z) dz$$

$$= \mu_0 M . \pi a^2 \left\{ \int_{z=0}^{h/a} \int_{\rho=0}^{1} \frac{I_1(b) - I_2(b)}{(1+z^2+\rho^2)^{3/2}} \rho d\rho dz \right\}$$

where now

$$b = \frac{2\rho}{1+z^2+\rho^2}$$

This expression was numerically integrated for many values of the ratio h/a, where 'h' is the height of the disc, and 'a' is its radius. The results are graphed, see figures 3 and 4, as a function F(h/a), computed so that $\Phi = \mu_0 M . A . F(h/a)$. Thus F(h/a) gives the geometric correction for the flux calculated in equation (7).

Atomic Shielding of the Nuclear EDM for Helium-3

Schiff (1963) has calculated the size of the screening effect of the electron charge-clouds in the case of Helium-3. He has shown that the net EDM on the Helium-3 atom is 1.5×10^{-7} of the nuclear EDM. This is discussed in

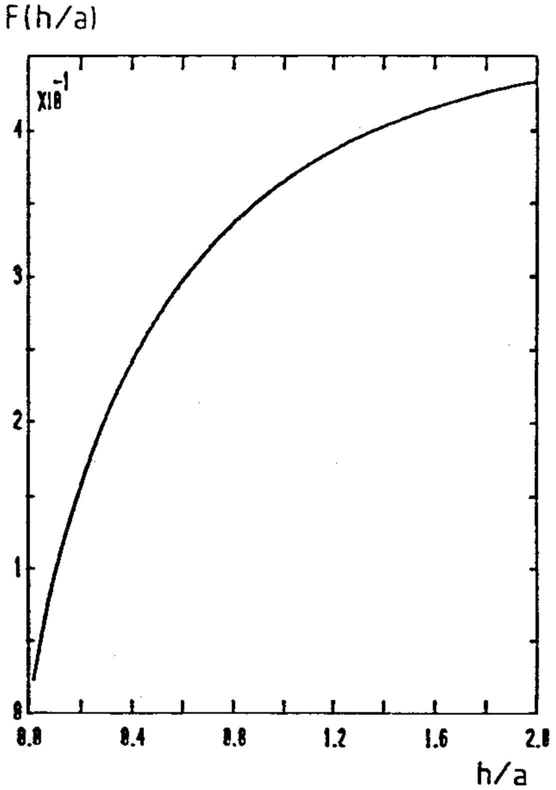

Figure 3. Graph of the geometric correction F(h/a), for the magnetic flux coming from a uniformly magnetized disc of height h and radius a, for h/a over the range of 0 to 2.

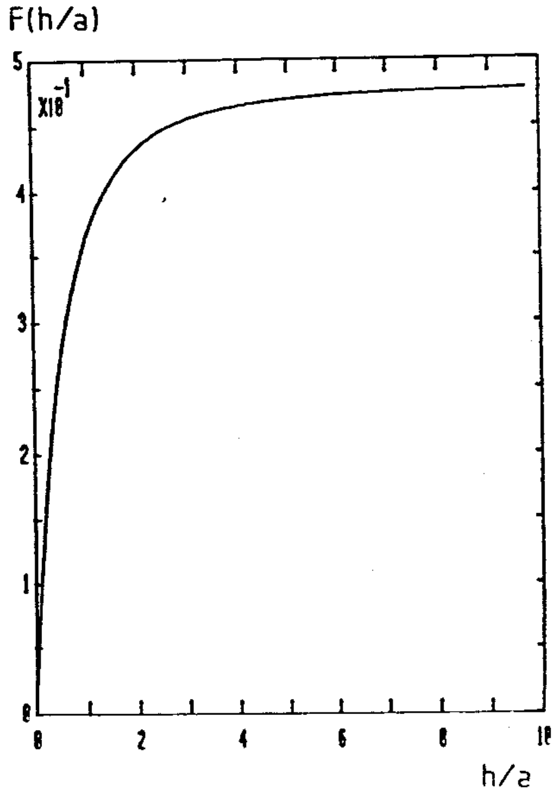

Figure 4. Graph of the geometric correction F(h/a), for h/a over the range 0 to 10.

more detail in chapter 4. Thus, to improve on the present value for the neutron EDM we would need to be able to measure an atomic EDM, in the case of Helium-3, of better than 9×10^{-32}e.cm.

Although the ground state of the Helium atom is a spin-singlet, a contribution to the atomic EDM is also made by the EDM of the electron. Khriplovich (1982) calculated the atomic EDM for Helium-3 to be 3.5×10^{-7} times the electron's EDM. As the current limit on the electron's EDM is 2×10^{-24}e.cm. To improve this limit we would have to measure the atomic EDM of Helium-3 down to at least 7×10^{-31}e.cm.

NOISE

Noise is of various kinds; we will concern ourselves with just two: (1) thermal noise; and (2) low frequency noise.

Thermal Noise

This is caused by the random fluctuations in alignment of the dipole moments. As we have already seen this noise rules out the possibility of having a static electric field and measuring the resulting magnetization, simply because the largest attainable fields are still dwarfed by this form of noise at all but unobtainably low temperatures, viz. 10^{-8} ^{o}K.

However, thermal noise has a spectrum which is flat (that is it is called "white noise"), so if we were to flip our signal at a frequency f_B, and synchronously filter out all information outside this frequency to within a range Δf, we could filter out a great portion of it. The available noise power P_N is given by (Motchenbacher and Fitchen, 1973) as: $P_N \propto kT.\Delta f$, where f is the noise bandwidth of the measuring system. If we require that the true signal gives at least ten times the power than that delivered by the noise, then:

$$d.E > 3 \times 10^{-15} kT.a.\Delta f/f_B \qquad (8)$$

where 'a' is chosen so that the fluctuation over the total frequency range is given by equation (5) (that is summing the right-hand side of equation (8) over this range); as the spectrum for thermal noise is flat there must be a frequency cuttoff, otherwise the available noise power would be infinite;

'a' characterizes this cutoff, and we sef it at unity though it could be many times smaller.

In order to maximize the signal we should choose the frequency at which we flip the electric field to correspond to the spin-relaxation time of diluted Helium-3. This time will be equal to the time required for spins to align under an imposed external magnetic field. Such must be the case, since the system relaxes into an aligned state through the interactions of the magnetic dipoles and is independent of the size of the external field. Thus, if alignment is caused by an electric field it is still a process which occurs solely through the interactions of magnetic moments on the atoms; since the EDMs are many orders of magnitude smaller than the magnetic moments, they contribute only negligibly to this relaxation reaction.

From Wilks (1967), this relaxation time T_r can be of the order of 100 seconds and is roughly given by:

$$T_r = \left\{ \frac{20\pi r}{\gamma^4 \hbar^2 N} \right\} D$$

where 'r' is the radius of the atom, 'γ' is the gyromagnetic ratio, N is the number of spins per unit volume and D is the selfdiffusion coefficient, this last parameter depending heavily on temperature.

Further, Wilks points out that any paramagnetic impurity even in very small amounts, can cause an appreciable reduction in the relaxation times. This should be avoided so as to give us a relaxation time of around 300 seconds, which would allow us to choose $1/f_B \simeq 300$ seconds. Accepting data within a few hertz of this would restrict the working temperatures from equation (8) to $T < 3\times10^{19} d.E/k$, that is $T < 0.4\ ^oK$, for $d \simeq 10^{-25}$e.cm.

Unfortunately, if we use the shielded value for the EDM d as being about 10^{-31}e.cm we would need to operate our system at temperatures below $10^{-6}\ ^oK$; even if this were possible we would have our sample of Helium-3 in the unsuitable superfluid phase.

Low Frequency Noise

Low frequency or 1/f noise has several unique properties; its spectral density increases without limit as frequency decreases. Despite this, the noise does not become infinite at D.C., because there is a lower limit to the frequency response set by the length of time the apparatus is turned on. This low-frequency cutoff attenuates frequency components with periods longer than the "on time" of the equipment.

An important point to realise about a 1/f noise limited experiment is that measurement accuracy cannot be improved by increasing the length of measuring time. By contrast, when measuring with thermal noise, the accuracy increases as the square-root of the measuring time.

Without the knowledge of the details of the sources of this noise no reasonable estimate can be made of how large a problem it will prove to be; possible sources could be external electric or magnetic fields, fluctuations in temperature, etc.

Overcoming the Noise Problem

From our preceeding discussions we can recognize the source of the problem, which is simply: too much noise. Because the coupling between the EDM and the electric field is so weak, only a very meager alignment is possible.

An alternative way of performing this experiment is to use a strong magnetic field to align the magnetic dipoles in the sample, which in turn produces a net polarization. By placing a pair of capacitor plates at the ends of the sample, we may extract charges from this polarization. Again we want: $\langle N_{net} \rangle > 10(\Delta N_{net})$. This gives $x \simeq \tanh .x > 10/\sqrt{N}$, where now we have $x = m_n.B/kT \simeq 3.5 \times 10^{-3}/T$, for a magnetic field B of 5 Tesla. The net charge accumulated on a plate of area A is given by:

$$q \simeq Nd.x.F(h/a).A$$

where F(h/a) is the same geometric factor as the one derived in the initial experimental conception.

For an unshielded EDM of 10^{-25}e.cm and plates around 1m in area, this gives a static charge of about $10^{-3}/T$ electrons on each plate. Although this is not very much, it certainly is an improvement in sensitivity in comparison with the work of Vasil'ev and Kolycheva (1978) in that there are no noise limitations. Hence using a configuation of a flipping field and a charge integrator, with a rectification device, we could in principle achieve any sensitivity in the apparatus, perhaps even as as low as 10^{-28}e.cm for an unshielded EDM.

As already noted, the case of Helium-3 has atomic shielding which decreases this sensitivity by seven orders of magnitude, rendering the method unfeasible.

CHAPTER 4

Section 4.1

Outline of Chapter

The aim of this chapter is to investigate Schiff's total screening theorem (1963). We repeat Schiff's argument in detail to see where loopholes, if any, may arise. Relativistic effects and finite size effects are known to lead to incomplete shielding. Normally, the incomplete shielding is small except where special atomic or molecular configurations occur which can produce a large antishielding.

Next, we investigate a relativistic effect in the nucleus, which has been neglected until now. We show that this effect is one cause of incomplete shielding, known as the "volume effect", and shed some light on its mechanism.

Section 4.2

Schiff's Screening Theorem

We look at Schiff's argument in detail in order to pinpoint exceptions that might exist to its general statement.

Schiff considers the non-relativistic Hamiltonian for a system of N particles of finite size, with mass m_i, charge e_i, EDM d_i and spatial coordinate r_i with respect to some origin, in an external electric potential V(r). He writes it as :

$$H = H^0 + H^1 \qquad (1.1)$$

$$H^0 = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i<j}^{N} \frac{e_i e_j}{4\pi\varepsilon_0} \frac{1}{r_{ij}} + \sum_{i}^{N} e_i V(r_i) \qquad (1.2)$$

$$H^1 = \sum_{i}^{N} \vec{d}_i \cdot \left\{ \sum_{j}^{N} \frac{e_j}{4\pi\varepsilon_0} \vec{\nabla}_{ij} \frac{1}{r_{ij}} + \vec{\nabla}_i V(r_i) \right\} \qquad (1.3)$$

Since the size of the dipole d_i is so small, it makes sense to treat the problem as a perturbation calculation working only to first order in d_i, and similarly neglect dipole-dipole terms.

Equation (1), also implies that all the particles are ideal pointlike quantum particles, and neglects relativistic effects and perturbations possessing a magnetic-dipole moment. All these exceptions are discussed in later sections.

Following Schiff we define a Hermitian operator:

$$Q = \sum_{i=1}^{N} \frac{1}{\hbar e_i} \vec{d}_i \cdot \vec{p}_i$$

We note that this definition fails if the system contains neutral particles. In the case of the atom, for Schiff's argument to apply, neutrons must be lumped together in the nucleus and not considered individually. Noting that:

$$\frac{\partial}{\partial(x_i - x_j)} \frac{1}{r_{ij}} = \frac{\partial}{\partial x_i} \frac{1}{r_{ij}}$$

it is easily shown that:

$$H^1 = i[Q,H^o] \qquad (2)$$

As Q is Hermitian, the operator $U \equiv e^{(iQ/\hbar)}$ is unitary, and we may write equation (1) as:

$$H = H^o + i[Q,H^o] \qquad (3.1)$$

$$= UH^oU^\dagger + {}^1/_2[Q,[Q,H^o]] + \ldots \qquad (3.2)$$

All terms in equation (3.2) except the first are second order or higher in the dipoles d_i and may be neglected. Thus, to first order in the d_i's the eigenstates of H^o which satisfy the Schroedinger equation $H^o|n\rangle = E_n|n\rangle$ determine the eigenstates $U|n\rangle$ of H, to first order, which satisfy $H(U|n\rangle) = E_n(U|n\rangle)$ with the same energy eigenvalues E_n . Schiff concludes from this that since the energy eigenvalues of the new states do not depend on d_i, there is no interaction energy of first order in the dipole moments. Accordingly the net dipole moment can only depend on d_i to at best second order, thus screening any internal EDM.

This argument shows that there must be screening of any EDM in any neutral Coulombic system. Since we are considering systems only in pure eigenstates, Schiff points out that the whole system must be neutral. The reason for this is that a non-neutral system in an external electric field is in a potential with an incomplete boundary. The system would continue to accelerate in this field forever, and therefore have no stationary eigenstate.

Most real systems do not exist in energy eigenstates, but in fact exist in prepared states. For example, the laevo-rotatory sugar molecule, which is in a mixture of opposite parity states, may not shift into an energy eigenstate in a time greater than the age of the universe. The simplest prepared state that may be considered, would consist of a superposed pair of degenerate states. By removing an appropriate degeneracy-lifting perturbation, we could prepare the linear combination of states desired. This simplest situation does not lead to an interesting result, because one result that we obtain from Schiff's theorem is that degenerate states are not split by H^1, and so, however the system is prepared, there is still complete screening of any internal EDM, to first order in the dipole moments d_i.

Section 4.3

Discussion of the Exceptions to Schiff's Screening Theorem

Schiff's result then shows that there can be no average electric field at any particle in an atom or molecule, unless there is some non-electric force which keeps it from accelerating in this field. On noting this, we might speculate about the connection between Schiff's theorem and the classical Earnshaw's theorem, that all maxima of the electric field occur inside charges for a static system. A dipole in a zero averaged field, will have zero average energy.

There would seem to be four exceptions to this result: (1) neutral particles; (2) volume effects; (3) magnetic effects; and (4) relativistic effects.

Neutral Particles

Neutral particles are not accelerated by an external field, and so Schiff's result cannot apply to them. After all, the neutron experiments do not suffer from any shielding problems; this is one of their advantages.

Volume and Magnetic Effects

As the nucleus of an atom has a finite size, part of it must lie in the electric field which does not average to zero. However, as further shown by Schiff, if the charge and EDM are distributed unequally over the nuclear volume the net interaction will not necessarily vanish to first order. This is called the "volume effect".

The nuclear magnetic moment can interact with the electrons distorting their charge clouds, which then pulls the nucleus slightly away from a position of average zero electric field.

Schiff calculated the size of both of these effects for the case of Helium-3. The magnetic moment effect may be calculated as a perturbation, using the form of the interactions as given by the Breit equation (1957). To estimate the size of the volume effect, Schiff assumed that the EDM on the nucleus would be distributed in the same manner as the magnetic moment of that nucleus and he used the then recent data about the root-mean-square radius of the charge and magnetic moment density to approximate these distributions. Schiff found the net EDM on the Helium-3 atom would be 1.5×10^{-7} and 1.4×10^{-9} times that of its nucleus for the magnetic moment and volume effects respectively.

In 1967 Sandars considered the volume effect, and realised that although it would vanish for any atom or molecule with a definite parity, when placed in an external field the parity symmetry can be destroyed, and so the volume effect need not be zero. He also pointed out that this effect can be large in suitable polar molecules, since a polar molecule can be polarized to a far greater extent in laboratory-attainable fields than atoms can be. He proposed the use of Thallium-Fluoride in a magnetic resonance beam experiment as a sensitive measure of the proton EDM, the Thallium nucleus having an unpaired proton.

Hinds and Sandars (1979) recalculated this volume effect for Thallium-Fluoride more accurately and they also determined the size of the magnetic moment. They found that these effects contributed to the atomic EDM being

0.136 and 0.813 of the Thallium nuclear EDM respectively. They coupled these results with those in an experimental paper (1979), in which they used magnetic resonance techniques on beams of Thallium-Fluoride, yielding the upper limit to the Thallium nuclear EDM as 1.4 > $6x10^{-21}$e.cm. Since they had no reason to expect nuclear shielding to occur, they ascribed this limit to the unpaired proton in the Thallium nucleus.

Khriplovich (1976) used the results of atomic Caesium experiments to get a bound on the proton EDM. Apart from finite size effects, he included a new magnetic effect. After evavuating the nuclear magnetic quadrupole moment produced by the EDM of a circulating valence nucleon, he showed that this quadrupole moment in turn induces an EDM on the atom. He calculated the size of the effect, which induces an atomic EDM of $7.8x10^{-4}$ of the Caesium nuclear EDM. He also showed for this case that the volume effect is 0.17 times smaller again. He concluded from this that the proton EDM is less than $5.5x10^{-19}$e.cm. He also pointed out that earlier results invoking the volume effect, particularly in molecular systems, would be very unreliable, since both the dipole distribution on the nucleus used in assessing the volume effect and also the molecular wavefunctions employed were poorly known. He then claims that although his bound is 35 times less stringent than earlier limits, it does not suffer from the above problems and so is likely to be more reliable.

Relativistic Effects

The final violation of Schiff's shielding result, reviewed in the literature, comes from relativistic effects. Rather than giving a fully relativistic treatment of interacting particles - an as yet unsolved problem - Schiff considered the case of a nucleus carrying the EDM and the electrons as Dirac particles. He replaced the terms H^0 of equation (1.2) with their relativistic counterparts, and found that all relativistic terms were small. However, Sandars (1965 and 1968) repeated Schiff's calculations using a relativistically covariant form of H^1, in equation (1.3), which Schiff himself had not done. Sandars considered the case of an EDM on the electrons.

He found essentially what Schiff had found for his calculations with the EDM on the nucleus, namely, that only small effects of the order $Z^2\alpha^2$ existed (Z is the proton number, and α is the fine structure constant).

However, Sandars also considers the case of Hydrogen in the 2S state which is nearly degenerate with the 2P state. If an unperturbed system is in an eigenstate $|\gamma\rangle$ then its measured dipole moment will be given by:

$$\langle D\rangle = \langle\gamma|D|\gamma\rangle - \sum_{j\neq\gamma} \frac{\langle\gamma|D|j\rangle\langle j|H'|\gamma\rangle}{E_j - E_\gamma} + c.c \qquad (4)$$

where D is the dipole operator and H^1 is the relativistically covariant form of H^1 in equation (1.3). Because of the term with the 2S - 2P energy denominator, the overall dipole moment can be very large. This holds solely for the relativistic part of H^1 (the non-relativistic part leads to no net dipole as before). For the 2S state of Hydrogen, this term is so large that it actually produces a multiplication of the electron EDM of order $1/\alpha$. Originally, this result did not lead to an experiment, as the 2S state of Hydrogen, although metastable, becomes very short-lived in the presence of an electric field.

It turns out that the heavier alkaline atoms all have near degeneracies in their ground states, so that the electron EDM is enhanced in each case. Sandars (1966) showed that the atomic EDMs for potassium, rubidium and Caesium were about 2.4, 24 and 120 times the electron EDM respectively. These calculations have been re-done a number of times (Ignatovich, 1969 and Flambaum, 1976) with results closely approximating those of Sandars; they also extended his work to find the electron EDM enhancement factors for other atoms with nearly degenerate ground states (Uranium, Lutetium, Europium, Cerium, Iron and Chromium).

Many experiments depending on this relativistic enhancement of the electron EDM have been performed. The best is with a beam of metastable Xenon in a magnetic resonance experiment. In this way, Player and Sandars (1970) set a limit on the EDM of the electron at 2×10^{-24}e.cm.

Section 4.4

Nuclear EDMs

Since Schiff did not use the proper form of H^1 in his investigation of relativistic effects, he missed out on these quite remarkable enhancements. Surprisingly, no-one has tried to repeat Schiff's calculation with the EDM on the nucleus. In this section we consider this case more thoroughly and try to obtain an order of magnitude estimate of the relativistic effects for this case.

We begin by studying the relativistic enhancement of the electron EDM in an atom. The relativistic form of equation (1) is:

$$H^o = \sum_i [\beta_i mc^2 + c\vec{\alpha}_i.\vec{p}_i - eV_i] + \sum_{j\neq k} 1/2[e^2/r_{jk} + B_{jk}] \qquad (5.1)$$

$$H^1 = - d_e \sum_i \beta_i \vec{\sigma}_i.\vec{E}_i \qquad (5.2)$$

where d_e is the EDM of the free electron, the matrices β_i, α_i, and σ_i are the usual 4x4 Dirac matrices, E_i is the electric field at the i^{th} electron, and B_{jk} is the Breit interaction.

Schiff's theorem implies that the non-relativistic part of the expectation of H^1 must vanish. This is easily seen if we rewrite H^1 in the form:

$$H^1 = d_e/e.[H^o, \sum_i \vec{\sigma}_i.\vec{\nabla}_i] + d_e/e.[\sum_i \vec{\sigma}_i.\vec{\nabla}_i, \sum_{j\neq k} 1/2 B_{jk}] + d_e \sum_i (1 - \beta_i)\vec{\sigma}_i.\vec{E}_i \qquad (6)$$

The first term on the right hand side of equation (6) does not contribute to the expectation of H^1 since it consists of the commutator of an operator with the unperturbed Hamiltonian, the form required by Schiff's theorem. The second term does not occur in a non-relativistic theory because it involves the relativistic Breit operator. The final term is also clearly relativistic since "$1-\beta$" is the operator which picks out the small component of the single particle wavefunction. So, in the non-relativistic limit the expectation of H^1 is zero. However, it will not vanish in general as there will be contributions from the second and third terms.

Sandars (1965) showed that in this case (that is with the EDM sited on the electrons), the second term was smaller than the third. Neglecting it, the net dipole of the system becomes:

$$\langle D\rangle = \langle\gamma| d_e \sum_i (\beta_i - 1)\sigma_{zi}|\gamma\rangle - \sum_{j\neq\gamma} \frac{\langle\gamma|\sum_i ez_i|j\rangle\langle j| d_e \sum_i (\beta_i - 1)\vec{\sigma}_i . \vec{E}_i|\gamma\rangle}{E_j - E_\gamma} + c.c \qquad (7)$$

Here $|\gamma\rangle$ represents the unperturbed state of the atom. When there is an opposite parity state, say $|j\rangle = |\gamma'\rangle$, very close to $|\gamma\rangle$ in energy, the term with this state dominates all others in value in equation (7), and we get a large mixing between these states, giving:

$$\langle D\rangle = \frac{-\langle\gamma|\sum_i ez_i|\gamma'\rangle\langle\gamma'| d_e \sum_i (1-\beta_i)\vec{\sigma}_i . \vec{E}_i|\gamma\rangle}{E_{\gamma'} - E_\gamma} + c.c. \qquad (8)$$

This is just what Sandars (1968) showed and we see that if the denominator is small enough there will be a large enhancement on the electron's EDM.

<u>Relativistic Effects for a Nuclear EDM</u>

In exactly the same way, we may determine the size of the atomic dipole when the EDM is sited on the nucleus. In this case:

$$H^1 = -d_n \sum_i \beta_n \vec{\sigma}_n . \vec{E}_n \qquad (9.1)$$

$$= \frac{d_n}{Ze} [\sum_i \vec{\sigma}_n . \vec{\nabla}_i , H^0] + \frac{d_n}{Ze} [\sum_i \vec{\sigma}_i . \vec{\nabla}_i , \sum_{j\neq k} 1/_2 B_{jk}] + d_n \sum_i (1-\beta_i)\vec{\sigma}_n . \vec{E}_i \qquad (9.2)$$

Here β_n and σ_n are the Dirac matrices for the nucleon with intrinsic EDM d_n, and E_i is now the field at the nucleon, due to each electron in the system.

Once again, the first term of equation (9.2) leads to Schiff's screening result. The second is the relativistic term Schiff (1963) considered and showed to be small. This only leaves the contribution from the third term.

It is interesting to note that if we were to make a perturbative expansion of the EDM, as in equation (7), and only include the first dominant term, we would erroneously conclude that this effect produces an enormous enhancement to the nuclear EDM (of the order of 20000 times for the 2S state of Hydrogen).

The reason that this conclusion would be invalid follows by writing this third term as:

$$-d_n.(1-\beta_n)\vec{\sigma}_n.[\sum_i \vec{\nabla}_i, H^e]$$

where H^e is the purely electronic part of H^o. To the extent that the electronic and nuclear systems are independent, this expression becomes:

$$-d_n(1-\beta_n)\vec{\sigma}_n.[\sum_i \vec{\nabla}_i, H^o]$$

which, as in Schiff's theorem, leads to exact shielding when the Breit terms are neglected. Thus, the above mentioned large enhancement from the first term must be cancelled by the contribution of all the other terms.

The only non-zero contribution that will come from this term will arise from the interactions between the electrons and nucleons, such as the hyperfine interaction of the electrons in the volume effect of the nucleus. It follows that this term of the perturbing Hamiltonian is responsible for the "volume effect" discussed earlier.

Schiff has shown that the volume effect can only occur if there is a difference between the charge and EDM distributions. We can see how this occurs since the nucleon charge density is given by $e\Psi_n^\dagger \Psi_n$ while the EDM distribution is given by $d_n \Psi_n^\dagger \beta_n \Psi_n$, where Ψ_n is the nucleon wavefunction.

We shall only attempt to obtain the contribution from this term correct to the right order of magnitude. For kinetic energies small by comparison with $m_n c^2$ (here m_n is the nucleon mass), we have:

$$\Psi_n \simeq \begin{pmatrix} \phi \\ \dfrac{\vec{\sigma}.\vec{p}}{2m_n} \phi \end{pmatrix}$$

where ϕ is the two-component ultra non-relativistic wavefunction (Bjorken, 1964).

The difference between these two distributions is given by:

$$\Psi_n^\dagger(1-\beta_n)\Psi_n \simeq \phi^\dagger(\vec{\sigma}.\vec{p})^\dagger(\vec{\sigma}.\vec{p})\phi/2m^2$$
$$= \phi^\dagger p^2 \phi/2m^2$$

$$\Psi_n^{\dagger}(1-\beta_n)\Psi_n = \frac{-\hbar^2}{2m^2}\phi^{\dagger}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)-l(l+1)\frac{1}{r^2}\right]\phi \qquad (10)$$

where l is the angular momentum of ϕ.

Thus, given the nucleon wavefunction, we may readily calculate the size of this affect for Hydrogen or Helium. Following Schiff, if we write:

$$\Psi_n^{\dagger}(1-\beta_n)\Psi_n = \sum_l f_l(r)P_l(\cos\theta) \qquad (11)$$

the size of this affect is then given by:

$$\frac{16\pi}{3a_0^2}d_n\int_0^{\infty} r^4[f_0(r) + \frac{4}{25}f_2(r)]dr \qquad (12)$$

(where a_0 is the Bohr radius), for the Hydrogen ground state, and approximately $(27/16)^2$ times this value for Helium (Schiff, 1963).

If we take:

$$\Psi_n = \frac{2(2a)^{3/4}}{\pi^{1/4}}e^{-r^2a}.Y_{00}.\chi_n \qquad (13)$$

where χ_n is the two component nuclear spinor, equation (11) gives

$$f_0(r) = \frac{-\hbar^2}{m^2}\left(\frac{2a}{\pi}\right)^{3/2}a(2r^2a-3).e^{-2r^2a} \quad \text{and} \quad f_2(r) = 0$$

Substituting these into equation (12) we obtain

$$\frac{1}{2}d_n\left(\frac{\hbar c}{m_nc^2}\right)^2\frac{1}{a_0^2} \simeq 1.8\text{x}10^{-11}d_n \qquad (14)$$

and approximately $4.3\text{x}10^{-11}.d_n$ for Helium. Schiff's estimate of this effect is almost two orders of magnitude larger than ours, even though he relied on experimental values of the root-mean-square radius of the charge and dipole distributions. Those values would would be expected to give an order of magnitude uncertainty to his calculation even if as little as 5% uncertainty was attached to them.

We note that this result, given by equation (14), depends neither upon the spin orientation of the nucleon, nor on the radial distribution parameter 'a' of the nucleon wavefunction (see equation (13)).Thus, we expect this effect to increase with the number of nucleons in the nucleus, or equivalently with the volume of the nucleus.

Although the effect is generally very small, we can see it would be at a maximum when the gradient of the electric field at the nucleus is also at a maximum. This situation should hold to a great extent in highly polarized molecules. As already mentioned, Hinds and Sandars (1979) calculated this effect for Thallium-Fluoride and found it to be surprisingly large. They relied, however, on semi-empirical methods to obtain the root-mean-square charge dipole distributions. It would be worth while calculating the size of these parameters from first principles, as we have done for Hydrogen and Helium, though with a nuclear model more sophisticated than ours, since this experiment on Thallium-Fluoride presently sets the best limit on the EDM of the proton.

CHAPTER 5

Summary and Future Prospects

We have shown that although our proposed experiment looked promising, it lacks sufficient sensitivity to improve on the upper limit already available for the EDM of the neutron. We have also seen that a number of forms of this experiment have been considered previously.

There are three points which we can conclude as deserving further attention:

(1) It would be possible to use our proposed experimental set-up to investigate unshielded parasitic effects, such as the effect of Legget (1977) which requires a sensitivity of around 10^{-28}e.cm.

(2) The experiment of Vasil'ev and Kolycheva (1978) could be re-done with an improvement of up to six orders of magnitude in sensitivity, by making only minor modifications to their basic approach.

and (3) An improved calculation for the Thallium nucleus should be made of its charge and EDM distributions, in order to obtain a more reliable limit of the proton EDM from the experimental data of Hinds and Sandars (1979).

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