

Equivalent efficiency of a simulated photon-number detector

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(Received 14 November 2001; published 16 September 2002)

Homodyne detection is considered as a way to improve the efficiency of communication near the single-photon level. The current lack of commercially available *infrared* photon-number detectors significantly reduces the mutual information accessible in such a communication channel. We consider simulating direct detection via homodyne detection. We find that our particular simulated direct detection strategy could provide limited improvement in the classical information transfer. However, we argue that homodyne detectors (and a polynomial number of linear optical elements) cannot simulate photocounters arbitrarily well, since otherwise the exponential gap between quantum and classical computers would vanish.

DOI: 10.1103/PhysRevA.66.032306

PACS number(s): 03.67.Lx, 42.50.Dv, 89.70.+c

The fundamental limitations to classical communication in optical channels are due to the quantum nature of the signals being transmitted. These limitations are well understood for *ideal* optical communication channels [1,2]. The capacity of a communication channel is defined to be the maximum mutual information (optimized over the choice of source alphabet used by the sender and the detection strategy used by the receiver) across the communication channel with respect to some physical channel constraint—such as the mean energy throughput. This characterization of a communication channel is important because Shannon's noisy coding theorem proves that any attempt at communication beyond this capacity necessarily fails due to unrecoverable errors [3]. A corollary to this theorem states that, even for a nonoptimal source alphabet or detection strategy, the mutual information of the communication channel is (asymptotically) achievable using error correction [3,4].

For a single-mode optical communication channel the optimal capacity, under a mean energy constraint, is achieved with a source alphabet of photon-number states and ideal photon-number detectors [1,2]. In this ideal case the orthogonality of the signals and hence their perfect distinguishability makes error correction unnecessary. Unfortunately, however, *such ideal operation is currently impractical*, since neither ideal photon-number state preparation nor ideal photon-number detection is achievable.

The detection of weak signals (few photons) is especially difficult at communications wavelengths (1.3–1.55 μm). Ideally, we would wish to achieve this by simply counting the photons. Now the process of photocounting is often synonymous with using an avalanching device with saturated gain, since each photon produces a strong and standard signal at the output. Unfortunately, it is a technological fact that both at infrared and optical frequencies, the best avalanche photodiodes never have as high a quantum efficiency as the best available linear detectors (having linear gain, such as PIN photodiodes). In fact, currently, no *commercial* photocounters are available at communications frequencies. Thus, at these frequencies PIN photodiodes are routinely used despite their high dark count rates. Partly because of this, the traditional solution at communications wavelengths has been to encode signals on intense (many-photon) pulses.

At communication frequencies, photon counting has been achieved with InGaAs or Ge avalanche photodiodes operating in so-called Geiger mode [5,6]. Due to the high dark count rate, the performance of these detectors as photon counter is very low. The best efficiency reported is around 20% at 1.3 μm with the optimal temperature 77 K [5], and is around 10% at 1.5 μm with the optimal temperature 213 K [6].

In this paper we consider an alternate encoding and detection strategy which is suitable for truly weak signals and current technological limitations. The basic idea is to *simulate* direct detection via a dual-homodyne scheme. Because strong local oscillators are continuously producing a strong output photocurrent, even high dark count rate detectors like PIN photodiodes, which have the highest quantum efficiencies, may be used. For example, at communication frequencies such devices can have efficiencies approaching 90% [7] and even 98% at optical frequencies [8]. We determine the mutual information for inefficient direct detectors and compare it to that of efficient homodyne detectors for a source alphabet preferring direct detection strategies. In doing so we are able to compute an equivalent efficiency for our simulated photon-number detectors for communication purposes.

As we shall see, there is a communication penalty for simulating direct detection in this way. Notwithstanding this penalty, the reduced signal-to-noise ratio due to high dark counts suggests that our indirect detection strategy is worth consideration.

Let us first consider two random variables A and B , with individual values labeled a and b , respectively, and a joint probability of $P_{a,b}$. The mutual information between these variables is defined by

$$\mathcal{I}(A:B) = \sum_{a,b} P_{a,b} \ln \left(\frac{P_{a,b}}{P_a P_b} \right), \quad (1)$$

and is, in some sense, a measure of the information content that is common to both variables. This quantity of mutual information is important in communication theory, because it can be used to quantify the information content that a receiver, observing variable B , learns about the sender's message represented by variable A .

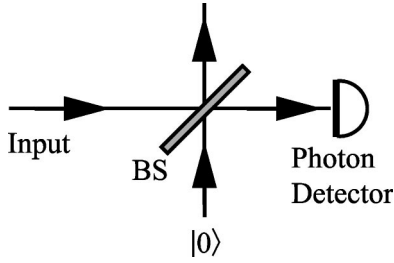


FIG. 1. The nonperfect measurement scheme: This models the nonperfect direct photon-number detection with a perfect photon counter and the beam splitter (BS), which determines the finite efficiency as η^2 .

The optimal capacity of an ideal bosonic communication channel with a mean energy constraint is achievable with a mean-channel state that is thermal and is calculated from Eq. (1), yielding [1]

$$\mathcal{I}^{pd}(A:B) = (1 + \bar{n}) \ln(1 + \bar{n}) - \bar{n} \ln \bar{n}, \quad (2)$$

where \bar{n} is the mean photon number of the thermal state

$$P_n = \frac{1}{1 + \bar{n}} \left(\frac{\bar{n}}{1 + \bar{n}} \right)^n. \quad (3)$$

We model loss by introducing extra beam splitters into the channel or in front of the detector, discarding photons in the unused port. A schematic representation of this is shown in Fig. 1. The parameter η is the *amplitude* efficiency corresponding to the amplitude-reflection coefficient of the beam splitter, so η^2 represents the quantum efficiency of the overall detector.

Assume that the input signal is characterized by a mean-channel state that is thermal (3), with vacuum entering the second input of the beam splitter, then the probability distribution for detecting m photons with our model of an inefficient detector is given by

$$P_m = \frac{1}{1 + \eta^2 \bar{n}} \left(\frac{\eta^2 \bar{n}}{1 + \eta^2 \bar{n}} \right)^m. \quad (4)$$

This corresponds to a Poisson distribution with reduced mean number of detected photons, down to $\eta^2 \bar{n}$. The mutual information for a source alphabet of number states and inefficient detection is then simply

$$\begin{aligned} \mathcal{I}^\eta(A:B) &= \ln[(1 - \eta^2)^{\bar{n}} (1 + \eta^2 \bar{n})] + \eta^2 \bar{n} \ln \left[\frac{(1 + \eta^2 \bar{n})}{(1 - \eta^2)^{\bar{n}}} \right] \\ &+ \frac{1}{1 + \bar{n}} \sum_{n=0}^{\infty} \sum_{m=0}^n \eta^{2m} (1 - \eta^2)^{(n-m)} \\ &\times \left(\frac{\bar{n}}{1 + \bar{n}} \right)^n \binom{n}{m} \ln \binom{n}{m}. \end{aligned} \quad (5)$$

We observe in Fig. 2 that the mutual information decreases for finite loss of photons. In particular, a small amount of

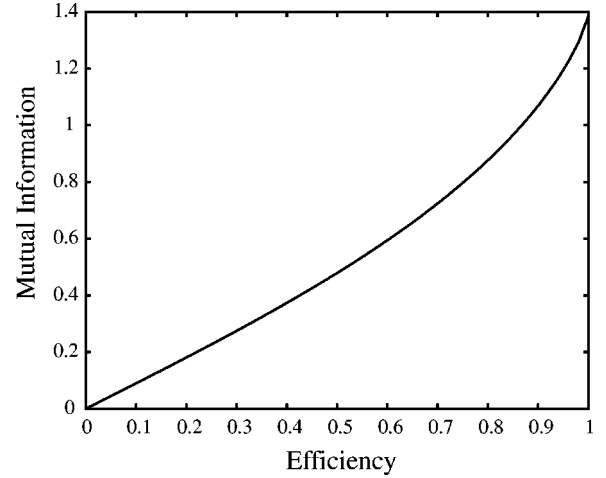


FIG. 2. The mutual information nats versus quantum efficiency η^2 of a nonperfect measurement scheme is plotted for a mean photon number of 1. This curve shows that a slight reduction below perfect efficiency causes a significant reduction in mutual information.

loss away from perfect detection results in a significant decrease in the mutual information, while the loss of mutual information is almost linear in the low efficiency regime ≤ 0.5 . This implies that a small improvement in the photon detection efficiency of current technology cannot be expected to bring a significant increase in the information throughput.

We now consider near-ideal homodyne measurements in comparison to nonideal photon counting. As we have discussed, such measurements can achieve very high efficiencies because homodyne detectors may be operated without regard to dark current. Hence as a first approximation we will treat the homodyne measurements as ideal. Our detection strategy is based on dual homodyne detection, which can simultaneously detect both quadrature-phase amplitudes. A schematic representation of dual homodyne detection is shown in Fig. 3.

Suppose that a number state is sent down the channel as an input signal and vacuum enters the first beam splitter as its second mode (as shown in Fig. 3). This specifies an input A as $|n\rangle \otimes |0\rangle$, or simply denoted $|n0\rangle_A$. Similarly the output B is collapsed into eigenstates of quadrature-phase ampli-

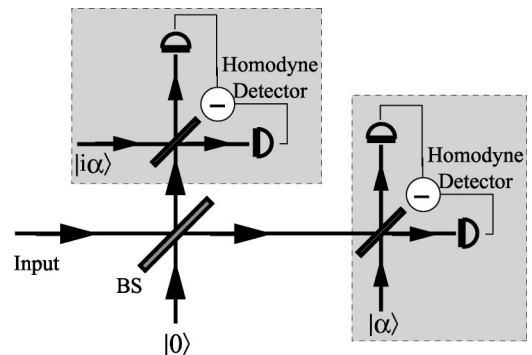


FIG. 3. Dual homodyne measurement strategy: Each gray box shows an individual homodyne detector to measure a different quadrature-phase amplitude of the input signal.

tudes which are analogs of position and momentum and we denote by $|XP\rangle_B$ states. The probability of measuring X and P at the output B , given an input A specified by $|n0\rangle_A$, is given by the square of the inner product ${}_A\langle n0|XP\rangle_B$. Taking $|k\rangle$ as number states, this probability may be written

$$\begin{aligned} P_{X,P|n} &= \left| \sum_{k=0}^n \binom{n}{k}^{1/2} \sqrt{2}^{-n} \langle X|k\rangle \langle P|n-k\rangle \right|^2 \\ &= \frac{1}{2^{2n} \pi n!} e^{X^2+P^2} \left| \left(\frac{\partial}{\partial X} - i \frac{\partial}{\partial P} \right)^n e^{-(X^2+P^2)} \right|^2 \\ &= \frac{1}{\pi n!} e^{uv} \left| \left(2 \frac{\partial}{\partial v} \right)^n e^{-uv} \right|^2_{(u,v)=(X-iP, X+iP)}. \end{aligned} \quad (6)$$

Clearly, this probability $P_{(X,P)|n}$ is dependent only on the product $uv (=X^2+P^2)$; hence we change the variables to the polar coordinates $(I^{1/2}, \theta)$ where the intensity $I \equiv X^2 + P^2$, yielding

$$P_{(X,P)|n} = \frac{1}{\pi n!} I^n e^{-I}. \quad (7)$$

Finally, integrating this over all angles θ gives

$$P_{I|n} = \int_0^{2\pi} d\theta P_{(I,\theta)|n} = \frac{1}{n!} I^n e^{-I}. \quad (8)$$

Now the quantity $P_{I|n}$ is a conditional probability for a given input photon number n . From it we can compute the unconditioned probability averaged over the mean-channel thermal state with a mean photon number \bar{n} . The resulting distribution is

$$P_I = \sum_{n=0}^{\infty} P_{I|n} P_{\bar{n}} = \frac{1}{1+\bar{n}} e^{-I/(1+\bar{n})}. \quad (9)$$

These probabilities from Eqs. (8) and (9) allow us to calculate the mutual information between sender and receiver for this dual-homodyne detection scheme. It is given by

$$\mathcal{I}^{hd}(A:B) = (1+\bar{n}) \ln(1+\bar{n}) - \gamma \bar{n} - \sum_{n=1}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n \ln n, \quad (10)$$

where $\gamma = 0.5772 \dots$ is Euler's constant. We note here that we could have replaced the dual homodyne detection by heterodyne measurement [9,10].

We now introduce a measure of efficiency for the dual-homodyne measurement. There are a number of possible ways to evaluate efficiency of detection schemes. The measure of efficiency that we introduce here is based on a comparison with the direct detection scheme with finite efficiency. In particular, we will equate the mutual information achievable in each of these two schemes with an alphabet chosen to prefer direct detection. The choice of finite efficiency η^{*2} in the direct detection scheme for which this equivalence holds is dubbed by us the *equivalent efficiency*

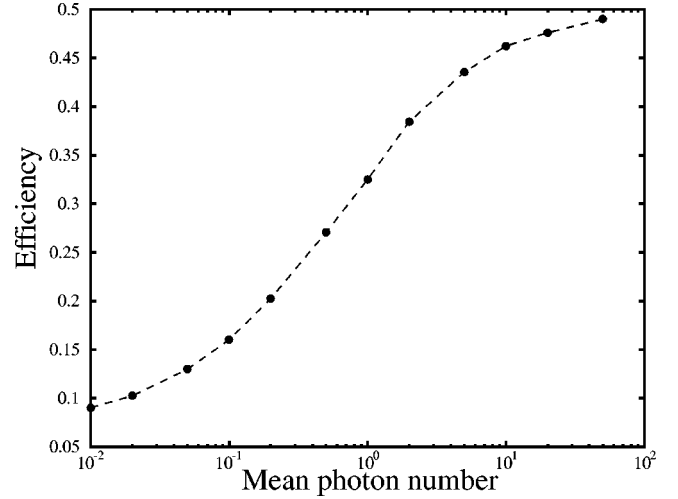


FIG. 4. The equivalent efficiency η^{*2} determined by inverting Eq. (11); shown on a semilogarithmic scale in the mean photon number.

of the dual-homodyne detection scheme (at least for the purposes of classical communication as analyzed here). Thus, the equivalent efficiency may be determined by inverting the relation

$$I \eta^{*2}(\bar{n}) = \mathcal{I}^{hd}(\bar{n}) \quad (11)$$

for the efficiency η^{*2} . (In other words, the photon detection efficiency is chosen to be that efficiency for which the mutual information obtained by direct detection is exactly that given by dual-homodyne detection.) This efficiency is obviously dependent on the mean photon number \bar{n} for the source alphabet. For instance, the equivalent efficiency of the dual-homodyne measurement for a mean photon number of 1 is just $\eta_1^{*2} \approx 0.327$. More general cases are shown on the graph of equivalent efficiency versus mean photon number in Fig. 4. The region $0.1 \leq \bar{n} \leq 1$ is the most sensitive to the mean photon number in the growth of the equivalent efficiency η^{*2} . For smaller mean photon numbers, the mutual information $\mathcal{I}^{hd}(\bar{n})$ of Eq. (10) does not gain much by small increases in the mean photon number. Similarly, for $\bar{n} \geq 1$ no significant gain in equivalent efficiency is obtained for small changes in the mean photon number of the source alphabet.

For an input alphabet of photon-number states, it is clear that schemes based upon homodyne measurement cannot be expected to perform as well as ideal direct photon detection. Nonetheless, such ideal direct photodetectors are not currently technologically realistic, especially at communications wavelengths. By contrast, since homodyne detectors may be operated without regard to dark current a significantly higher quantum efficiency is readily available for them. We have found that replacing inefficient direct detectors with homodyne-based simulated direct detectors can yield reasonable improvements, even near the single-photon level of operation. In this paper, we have shown that this improvement is theoretically possible for the purposes of classical communication through a single-mode bosonic channel. Our analy-

sis used a communication alphabet of photon-number eigenstates, which were thus optimized for (ideal) direct detection schemes. This choice is strongly prejudiced toward direct detection and against our homodyne scheme. Thus, our estimate of equivalent efficiency is likely to be an underestimate of the performance of homodyne-based schemes in general. If, instead, we had optimized the input alphabet for homodyne detection, we would have seen a significant improvement in capacity [2]. However, since this capacity is very likely to be larger than that possible for the imperfect direct detection schemes our strategy for computing a figure of “equivalent efficiency” would not be applicable.

Another aspect that is missing from the analysis given here is a detailed consideration of error correction coding schemes that would be required to achieve the performance promised by Shannon’s measure of communication throughput, namely, the mutual information. In general, more complicated encoding will be required to achieve the information transfer given by this measure.

Finally, it remains to be considered whether the approach studied here really has any applicability to either quantum communication or computation. The maximum 50% equivalent efficiency of the simulated photon detection here might rule out these possibilities for detecting quantum information represented within discrete photonic Hilbert spaces. Thus, if we hope to use these ideas beyond classical communication this low efficiency implies that discrete Hilbert spaces will

need to be abandoned. This suggests using homodyne detectors within some kind of continuous quantum variable scenario. For such variables a generalized Gottesman-Knill theorem has been derived [11]. This theorem states, under relatively mild assumptions, that quantum computational circuits consisting of gate operations made from quadratic Hamiltonians and homodyne-based measurements can be efficiently simulated on a classical computer. Further, including direct photodetection is sufficient to provide these circuits with the capability to perform universal quantum computations [12]. This observation suggests that our particular simulation strategy cannot be improved arbitrarily. For if homodyne-based measurements (and linear optics) could come arbitrarily close to simulating photocounting with only a polynomial number of components then a universal quantum computer could be simulated classically in polynomial time by this above theorem, which would imply $BQP=BPP$. (BPP is the class of problems that can be solved using randomized algorithms in polynomial time, while BQP is the class of all computational problems which can be solved efficiently on a quantum computer.) This outrageously unlikely outcome suggests that simulating direct detection using homodyne detectors must have limited efficiency.

This work is funded by the Research Foundation for Opto-Science and Technology.

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