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Quantum coherence: myth or fact?

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Abstract

It has recently been argued that the inability to measure the absolute phase of an electromagnetic field prohibits the representation of a laser's output as a quantum optical coherent state. This argument has generally been considered technically correct but conceptually disturbing. Indeed, it would seem to place in question the very concept of the coherent state. Here we show that this argument fails to take into account a fundamental principle that not only re-admits the coherent state as legitimate, but formalizes a fundamental concept about model building in general, and in quantum mechanics in particular.

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There is sometimes a clash between theorists and experimentalists which is so deep that it seems to be based on some fundamental difference in approach. One such debate revolves around the ability to create coherent states in real devices, e.g., in lasers. While experimentalists have been interpreting their work in terms of coherent states for decades, some theorists now argue that this language is invalid [1]. A resolution to this debate is urgent not only in consideration of the last forty years of experimental achievements but also to provide a precise interpretation of present and future experimental results.

The representation of a state and its associated interpretation are fundamental issues. For example, in quantum theory there is an infinite number of ensembles $\{\hat{P}_j\}$ for decomposing a mixed state $\hat{\rho}$ via[1]

$$\hat{\rho} = \sum_j p_j \hat{P}_j, \quad p_j \geqslant 0, \qquad \hat{P}_j^2 = \hat{P}_j. \tag{1}$$

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[1] For an example of different ensembles, see Eqs. (2.4) and (2.5) in Ref. [3]. Also see Ref. [2].

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Only when the state is pure does this representation become unique. The laws of quantum mechanics say that (in the absence of any additional information other than the state's identity) no physical interpretation can be based on a *preferred* choice of an ensemble for this decomposition [3]. This result has been coined the partition ensemble fallacy (PEF) [3].

For decades quantum mechanical aspects of laser science have been explained in terms of the coherent state formalism [4,5]. Recently, however, PEF has been used to attack the very notion of the coherent state in the context of continuous variable teleportation [1]. In these experiments coherent states were chosen as the 'alphabet' transmitted from sender to receiver [6–8]. However, if Rudolph and Sanders' argument [1] holds then the implications are significantly more far reaching than for just teleportation. In fact, the formalism of coherent states is a basic tool in quantum optics and the use of laser light to produce so-called coherent states is all pervasive.

Let us go through Rudolph and Sanders' argument. It has long been argued, though without rigorous proof, that the absolute phase of an electromagnetic field is not observable [9,10]. This difficulty is typically circumvented by requiring that the nominal description of laser light as a coherent state $\big||\alpha|e^{-i\phi}\big\rangle$ should be averaged over the unknowable quantity ϕ [1]. The resulting description of the laser state then becomes

$$\hat{\rho}_{\mathrm{PEF}} = \int_0^{2\pi} \frac{d\phi}{2\pi}\,\Pr(\phi)\big||\alpha|e^{-i\phi}\big\rangle\big\langle|\alpha|e^{-i\phi}\big| \tag{2}$$

$$= \int_0^{2\pi} \frac{d\phi}{2\pi}\big||\alpha|e^{-i\phi}\big\rangle\big\langle|\alpha|e^{-i\phi}\big| \tag{3}$$

$$= e^{-|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}|n\rangle\langle n|, \tag{4}$$

where following Ref. [1] we have taken the prior probability $\Pr(\phi)$ to be flat for the latter two forms. Thus, the ensemble of states describing a laser's output could as easily be chosen as a collection of number states $|n\rangle$, Eq. (4), instead of a collection of coherent states, Eq. (3). Recalling that the PEF disallows interpretations for states based on a preferred choice of ensemble, *we should apparently infer that experiments using lasers cannot be reliably interpreted as demonstrating features or properties of coherent states*. This is the logic behind the argument of Rudolph and Sanders [1].

It seems in the field of laser science that this logic has generally been accepted to be technically correct, but recognized as conceptually disturbing. Indeed it is hard to see how this difficulty would not infect the coherent state as a general concept. However, theoretical physicists have given different reasons why this logic is conceptually troublesome and hence not physically applicable. One attack is from Wiseman who although agreeing with the argument, claims it is unacceptably pedantic, since it implies that we could never write down a time t or a phase ϕ if its intrinsic resolution were beyond that of direct human experience [11]. A different objection has been directed towards the applicability of the PEF to a real laser. Here, Gea-Banacloche [10] and later Wiseman and Vaccaro [12] have argued that detailed knowledge of laser dynamics *should* give extra information about the identity of the underlying states created by a laser. This suggests that only a preferred ensemble is physically realizable. However, their analyses require the untested assumption of a perfectly Markovian dynamics for a laser. The Markovian assumption seems unlikely to be a fundamental truth. Yet another direction of attack has been made by van Enk and Fuchs [13]. They claim that the actual state of a laser should be represented as a tensor product of repeated identical states. With such a restriction, the laser's state is enforced to be uniquely a coherent state. Unfortunately, this restriction invokes again an untestable assumption.

Given so many attempts to resolve the conflict by introduction of untestable assumptions, let us revisit the argument of Rudolph and Sanders to see whether it itself is free from them. In particular, the automatic assumption that the prior distribution of phases $\Pr(\phi)$ should be taken as flat appears straightforward. Ordinarily, when one has an unknown quantity, one assigns a prior distribution based on whatever prior information is available. If one lacks *any* information then one tries to rely on symmetries in the problem. Thus, since any choice of absolute phase ϕ leads to the same observable results, the flat prior distribution appears to be the canonical choice.

In fact, there is something fishy about this reasoning. That a prior distribution is a meaningful summary of our knowledge (or lack thereof) depends on the full procedure of inference. Here the quantity at hand is not simply unknown, but claimed unknowable. If we believe the claim that a laser's phase is unmeasurable then no inference can ever be made from the prior. In other words, *absolutely any* choice of $\Pr(\phi)$ will give identical predictions. To this extent, one's choice of a prior distribution for an unobservable quantity is a matter of 'religion'. It lies outside the realm of science.

The flip side of this argument can be found if we pick a delta function for the prior. Such a choice reduces the density matrix to a pure (coherent) state, thus rendering the entire application of the PEF inadmissible. Nonetheless, the fact remains that the two choices for the prior (flat or delta function) are not amenable to any physical test that will distinguish between them. Since the application of a principle cannot depend upon an untestable choice, this logic confirms our claim that the PEF cannot be invoked for *any* choice of prior.

A more precise language for the states of the form Eq. (2) is that they are cosets of operators on the Hilbert space. We are already familiar with treating states as cosets of vectors on Hilbert space: since the absolute phase φ of a wavefunction is unobservable, all states $e^{i\varphi}|\psi\rangle$ are equivalent. Mathematically this coset structure corresponds to a projective Hilbert space. Indeed, *the formation of cosets of indistinguishable states is a universal feature of unobservability which induces an equivalence relation among states*. Following tradition, we may then label a coset by any of its members. The realization of the underlying coset structure means that *any* preferred label is equally valid. This is tantamount to freedom of choice of a prior. For experimentalists the natural choice would then be a delta function, reducing to the familiar coherent-state language.

The unobservability of optical phase guarantees experimentalists the freedom to continue talking about a laser's output in terms of coherent states. In fact, with the state represented in the form of (2) there is nothing to prevent experimentalists from using coherent states for their state representation (provided any additional knowledge remains inaccessible). Physically then the usual coherent-state language is unfalsifiable. Mathematically, the freedom to choose the prior due to the unobservability of ϕ induces an equivalence relation among states (2) over all choices of $\Pr(\phi)$.

This equivalence relation is fundamental and so the quantum theory of the laser needs to be formulated on top of it. It is well known in standard laser theory, that due to the random spontaneous emission events, the laser output field shows a random walk on the phase angle [14]. Under the application of the equivalence relation, such a diffusion process cannot be represented on the absolute phase of a laser output field. So how do we reconcile these two claims, that a laser's phase diffuses, yet it is meaningless to attribute it to a diffusion of absolute phase? The resolution comes when we realize that to observe a laser's phase diffusion we must actually measure it relative to some other phase reference—another laser for example. However, in that case, the diffusion observed is not the absolute phase diffusion, but instead the relative phase diffusion between two (or more systems). For that larger, doubled system, the relative phase is observable, but the equivalence relation forbids measurements or inferences about the global absolute phase. The equivalence relation guarantees that the choice of the state representation for the reference does not affect any physical understanding about observable quantities, while standard laser theory explains the phase diffusion on the relative phase of the laser output field.

The principle that unobservability induces equivalence (UIE) is, in fact, more fundamental than quantum mechanics itself. We would claim that any attempt to build models about the world (quantum mechanical or otherwise) must conform to this principle. By comparison, PEF is only meaningful within quantum theory. We demonstrated that the conventional interpretation of PEF as universally applicable is flawed. In particular, whenever PEF invokes inference, UIE must first be applied to ensure that inference is possible. Thus, for the class of inference problems considered here the applicability of PEF is dictated by UIE. It is the hierarchical ordering of principles which allows UIE to trump PEF. This hierarchy then allows us to pin-point the flaw in the argument of Rudolph and Sanders; their invocation of PEF is invalid precisely in the case to which they apply it: namely where a laser's phase would be unobservable.

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