reservoir. The removal of wide areas of the upper 8–10 m of the residual cap (Fig. 2c and d) after initial sag and collapse implies that most of those upper layers are also subject to sublimation, especially once the upper surface is fragmented or disturbed. Most of the eight metres that are removed is probably CO_2; the lag might be less volatile water ice. Radar data returns from the south-polar residual cap are very different from the north-polar cap[19], and can be interpreted as indicative of clean ice with large cavities.

The evolution of the topography after collapse (steps 4–7 above) suggests at least two subsequent non-steady-state periods. The wide area of removal of three or more layers (Fig. 2c and d) indicates considerable sublimation. The circular and near-circular forms may develop because at these high latitudes, ablational removal can become nearly azimuthally isotropic owing to the small change in solar elevation during the day. The remnant landforms suggest lag deposits from ablation and further collapse. The small proportion remaining as lag (Fig. 2a and c) suggests a small non-volatile (or less volatile) component in the upper layers of the residual cap area. The moats (Fig. 2c and d) also require some sequence of renewed deposition and subsequent partial removal, or a period of debris apron growth followed by apron retreat or compression of underlying layers. Additionally, some areas show probable burial and partial exhumation of the depressions (Fig. 2e).

The MOC data show that the southern residual cap is not simply a temporary anomaly of residual summer CO_2 frost; it is a geological feature indicative of depositional and ablational events recorded neither in the north nor in the outlying southern polar layered deposits. The geographical restriction to the CO_2 residual cap strongly suggests that CO_2 ice is involved, as does the apparent requirement for a component distinct from water ice. The most obvious environmental distinctions of the southern residual cap area are its elevation, about 6 km above the northern one[11,20], and its presence in the hemisphere that has very low atmospheric water content[2].

Indications of burial and exhumation (Fig. 2e, and steps 6 and 7 above) suggest repetitive, or even periodic formation of the distinctive southern residual cap morphology. Change in hemispheric asymmetry in polar processes has been attributed to periodic variations[21–24] in the orbital eccentricity, obliquity and season of perihelion of Mars. Even the shortest of these cycles, 51,000 years for season of perihelion, could easily allow the build-up (or sublimation) of the whole of the south residual cap unit. Eight metres of solid CO_2 accumulated as net residual in 1,000 years would require only about 2% of the current seasonal total[8] CO_2 to be retained each year. However, elucidation of the time scale represented by the residual deposits rests on detection of the net atmospheric CO_2 budget and on mapping cycles recorded in the layered deposits as a whole. ☐

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Impossibility of deleting an unknown quantum state

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A photon in an arbitrary polarization state cannot be cloned perfectly[1,2]. But suppose that at our disposal we have several copies of a photon in an unknown state. Is it possible to delete the information content of one or more of these photons by a physical process? Specifically, if two photons are in the same initial polarization state, is there a mechanism that produces one photon in the same initial state and the other in some standard polarization state? If this could be done, then one would create a standard blank state onto which one could copy an unknown state approximately, by deterministic cloning[3,4] or exactly, by probabilistic cloning[5,6]. This could in principle be useful in quantum computation, where one could store new information in an already computed state by deleting the old information. Here we show, however, that the linearity of quantum theory does not allow us to delete a copy of an arbitrary quantum state perfectly. Though in a classical computer information can be deleted (reversibly) against a copy[7], the analogous task cannot be accomplished, even irreversibly, with quantum information.

Quantum information has the unique property that it cannot be amplified accurately. If an arbitrary state could be cloned, then by using non-local resources one could send signals faster than light[1,2]. However, orthogonal quantum states can be perfectly copied. Although two non-orthogonal photon-states cannot be copied perfectly by a unitary process[8], they can be copied by a unitary-reduction process[5]. More interestingly, non-orthogonal states from a linearly independent set can evolve into a linear superposition of multiple copy states using a new cloning machine[9]. With the recent advances in quantum information theory, such as quantum cryptography[10], quantum teleportation[11–14] and quantum computing[15], we would like to know what we can do with the vast amount of information contained in an unknown state and what we cannot.

As is now well understood, erasing a single copy of some classical information is an irreversible operation. It may only be done with some energy cost; a result known as the Landauer erasure principle[7]. In quantum theory, erasure of a single unknown state may be thought of as swapping it with some standard state and then dumping it into the environment. The deletion we wish to study is not the same as irreversible erasure; it is more like reversible 'uncopying' of an unknown quantum state. We will show that there is no quantum deleting machine that can delete one unknown state against a copy in either a reversible or an irreversible manner.

Let us suppose that we have several copies of some unknown information. Classically we may delete one copy against the others, uncopying it in a perfectly reversible manner[7]. The situation is very different in quantum theory. Such a quantum deleting machine would involve two initially identical qubits (for example, photons of arbitrary polarization) in some state $|\psi\rangle$ and an ancilla in some initial state $|A\rangle$, which corresponds to the 'ready' state of the deleting apparatus. The aim of this machine is to delete one of two copies of $|\psi\rangle$ and replace it with some standard state of a qubit $|\Sigma\rangle$. The quantum deleting operation is defined for an input $|\psi\rangle|\psi\rangle$ such that the linear operator acts on the combined Hilbert space of input and ancilla. That is, it is defined by:

$$|\psi\rangle|\psi\rangle|A\rangle| \longrightarrow |\psi\rangle|\Sigma\rangle|A_\psi\rangle \tag{1}$$

Here $|A_\psi\rangle$ is the final state of the ancilla, which may in general depend on the polarization of the original photon. (If we knew that this process was unitary, it might work like the time-reverse of cloning.) One obvious solution to this equation is to swap the second and third states. However, this reduces to the standard erasure result where the extra copies have played no role. We will therefore explicitly exclude swapping as describing quantum deleting.

Consider the action of this deleting machine, equation (1), on a pair of horizontally or vertically polarized photons:

$$\begin{aligned} |H\rangle|H\rangle A\rangle &\longrightarrow |H\rangle|\Sigma\rangle|A_H\rangle \\ |V\rangle|V\rangle A\rangle &\longrightarrow |V\rangle|\Sigma\rangle|A_V\rangle \end{aligned} \tag{2}$$

We note that the transformation defining our deleting machine, equation (1), does not completely specify its action when the input states are non-identical. This is in contrast to the Wootters–Zurek cloning transformation[1], whose definition specifies its action for all possible inputs. Because of this the transformation corresponding to our machine is not the time-reverse of cloning. In fact, the transformation (1), defines a whole class of possible deleting machines which could behave differently if the two inputs are unequal or even entangled, for example,

$$\frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle)|A\rangle \longrightarrow |\Phi\rangle \tag{3}$$

where $|\Phi\rangle$ might be any state of the combined input–ancilla system.

Now for an arbitrary input qubit $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$ (where α and β are unknown complex numbers with $|\alpha|^2 + |\beta|^2 = 1$), linearity and the transformations (2) and (3) show that the deleting machine yields

$$\begin{aligned} &|\psi\rangle|\psi\rangle|A\rangle \\ &= [\alpha^2|H\rangle|H\rangle + \beta^2|V\rangle|V\rangle + \alpha\beta(|H\rangle|V\rangle + |V\rangle|H\rangle)]|A\rangle \\ &\longrightarrow \alpha^2|H\rangle|\Sigma\rangle|A_H\rangle + \beta^2|V\rangle|\Sigma\rangle|A_V\rangle + \sqrt{2}\alpha\beta|\Phi\rangle \end{aligned} \tag{4}$$

which is a quadratic polynomial in α and β. However, if transformation (1) is to hold, transformation (4) must reduce to

$$(\alpha|H\rangle + \beta|V\rangle)|\Sigma\rangle|A_\psi\rangle \tag{5}$$

for all α and β. As $|\Phi\rangle$ is independent of α and β then $|A_\psi\rangle$ must be linear in α and β with the only solutions being $|\Phi\rangle = (|H\rangle|\Sigma\rangle|A_V\rangle + |V\rangle|\Sigma\rangle|A_H\rangle)/\sqrt{}2$ and $|A_\psi\rangle = \alpha|A_H\rangle + \beta|A_V\rangle$. Further, since the final state (5) must be normalized for all possible α and β, it follows that the ancilla states $|A_H\rangle$ and $|A_V\rangle$ are orthogonal. However, as discussed above, transformation (1) is therefore not uncopying at all, but merely swapping onto a two-dimensional subspace of the ancilla. It appears that there is no option but to move the information around without deleting it. That is, the linearity of quantum theory forbids deleting one unknown state against a copy. This we call the 'quantum no-deleting' principle. This principle is complementary in spirit to the no-cloning principle, and we expect it to play a fundamental role in future understanding of quantum information theory.

We emphasize that copying and deleting of information in a classical computer are inevitable operations whereas similar operations cannot be realized perfectly in quantum computers. This may have potential applications in information processing because it provides intrinsic security to quantum files in a quantum computer. No one can obliterate a copy of an unknown file from a collection of several copies in a quantum computer. In spite of the quantum no-deleting principle one might try to construct a universal and optimal approximate quantum deleting machine by analogy with optimal quantum cloning machines[16]. When memory in a quantum computer is scarce (at least for a finite number of qubits), approxi mate deleting may play an important role in its own way. Although at first glance quantum deleting may seem the reverse of quantum cloning, it is not so. Despite the distinction between these two operations there may be some link between the optimal fidelities of approximate deleting and cloning. Nevertheless, nature seems to put another limitation on quantum information imposed by the linearity of quantum mechanics. ☐

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